MPFR

The Multiple Precision Floating-Point Reliable Library
Edition 2.3.0
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This manual documents how to install and use the Multiple Precision Floating-Point Reliable Library, version 2.3.0.

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1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to extend the class of floating-point numbers provided by the GNU MP library by a precise semantics. The main differences with the `mpf` class from GNU MP are:

- the `mpfr` code is portable, i.e. the result of any operation does not depend (or should not) on the machine word size `mp_bits_per_limb` (32 or 64 on most machines);
- the precision in bits can be set exactly to any valid value for each variable (including very small precision);
- `mpfr` provides the four rounding modes from the IEEE 754-1985 standard.

In particular, with a precision of 53 bits, `mpfr` should be able to exactly reproduce all computations with double-precision machine floating-point numbers (`double` type in C), except the default exponent range is much wider and subnormal numbers are not implemented but can be emulated.

This version of MPFR is released under the GNU Lesser General Public License. It is permitted to link MPFR to non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

1.1 How to Use This Manual

Everyone should read Chapter 4 [MPFR Basics], page 6. If you need to install the library yourself, you need to read Chapter 2 [Installing MPFR], page 3, too.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.

2 Installing MPFR

2.1 How to Install

Here are the steps needed to install the library on Unix systems (more details are provided in the '`INSTALL`' file):

1. To build MPFR, you first have to install GNU MP (version 4.1 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need a standard Unix '`make`' program, plus some other standard Unix utility programs.
2. In the MPFR build directory, type '`./configure`'

   This will prepare the build and setup the options according to your system. If you get error messages, you might check that you use the same compiler and compile options as for GNU MP (see the '`INSTALL`' file).
3. '`make`'

   This will compile MPFR, and create a library archive file '`libmpfr.a`'. A dynamic library may be produced too (see configure).
4. '`make check`'

   This will make sure MPFR was built correctly. If you get error messages, please report this to '`mpfr@loria.fr`'. (See Chapter 3 [Reporting Bugs], page 5, for information on what to include in useful bug reports.)
5. '`make install`'

   This will copy the files '`mpfr.h`' and '`mpf2mpfr.h`' to the directory '`/usr/local/include`', the file '`libmpfr.a`' to the directory '`/usr/local/lib`', and the file '`mpfr.info`' to the directory '`/usr/local/share/info`' (or if you passed the '`--prefix`' option to '`configure`', using the prefix directory given as argument to '`--prefix`' instead of '`/usr/local`').

2.2 Other 'make' Targets

There are some other useful make targets:

- '`mpfr.info`' or '`info`'

  Create an info version of the manual, in '`mpfr.info`'.
- '`mpfr.dvi`' or '`dvi`'

  Create a DVI version of the manual, in '`mpfr.dvi`'.
- '`mpfr.ps`'

  Create a Postscript version of the manual, in '`mpfr.ps`'.
- '`clean`'

  Delete all object files and archive files, but not the configuration files.
- '`distclean`'

  Delete all files not included in the distribution.
- '`uninstall`'

  Delete all files copied by '`make install`'.

2.3 Known Build Problems

MPFR suffers from all bugs from the GNU MP library, plus many more.

Please report other problems to '`mpfr@loria.fr`'. See Chapter 3 [Reporting Bugs], page 5. Some bug fixes are available on the MPFR web page `http://www.mpfr.org/`.

2.4 Getting the Latest Version of MPFR

The latest version of MPFR is available from `http://www.mpfr.org/`.

3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR web page http://www.mpfr.org/: perhaps this bug is already known, in which case you may find there a workaround for it. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results printed are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using '`cc -V`' on some machines, or, if you're using gcc, '`gcc -v`'. Also, include the output from '`uname -a`' and the MPFR version (the GMP version may be useful too).

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we won't do anything about it (aside of chiding you to send better bug reports).

Send your bug report to: '`mpfr@loria.fr`'.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

4 MPFR Basics

All declarations needed to use MPFR are collected in the include file '`mpfr.h`'. It is designed to work with both C and C++ compilers. You should include that file in any program using the MPFR library:

```
#include <mpfr.h>
```

4.1 Nomenclature and Types

A *floating-point number* or *float* for short, is an arbitrary precision mantissa with a limited precision exponent. The C data type for such objects is `mpfr_t` (internally defined as a one-element array of a structure, and `mpfr_ptr` is the C data type representing a pointer to this structure). A floating-point number can have three special values: Not-a-Number (NaN) or plus or minus Infinity. NaN represents an uninitialized object, the result of an invalid operation (like 0 divided by 0), or a value that cannot be determined (like +Infinity minus +Infinity). Moreover, like in the IEEE 754-1985 standard, zero is signed, i.e. there are both +0 and -0; the behavior is the same as in the IEEE 754-1985 standard and it is generalized to the other functions supported by MPFR.

The *precision* is the number of bits used to represent the mantissa of a floating-point number; the corresponding C data type is `mp_prec_t`. The precision can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. In the current implementation, `MPFR_PREC_MIN` is equal to 2.

The *rounding mode* specifies the way to round the result of a floating-point operation, in case the exact result can not be represented exactly in the destination mantissa; the corresponding C data type is `mp_rnd_t`.

A *limb* means the part of a multi-precision number that fits in a single word. (We chose this word because a limb of the human body is analogous to a digit, only larger, and containing several digits.) Normally a limb contains 32 or 64 bits. The C data type for a limb is `mp_limb_t`.

4.2 Function Classes

There is only one class of functions in the MPFR library:

1. Functions for floating-point arithmetic, with names beginning with `mpfr_`. The associated type is `mpfr_t`.

4.3 MPFR Variable Conventions

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator.

MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication, `mpfr_mul`, can be used like this: `mpfr_mul (x, x, x, rnd_mode)`. This computes the square of x with rounding mode `rnd_mode` and puts the result back in x.

Before you can assign to an MPFR variable, you need to initialize it by calling one of the special initialization functions. When you're done with a variable, you need to clear it out, using one of the functions for that purpose.

A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned to any number of times.

For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited.

You don't need to be concerned about allocating additional space for MPFR variables, since any variable has a mantissa of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

4.4 Rounding Modes

The following four rounding modes are supported:

- `GMP_RNDN`: round to nearest
- `GMP_RNDZ`: round towards zero
- `GMP_RNDU`: round towards plus infinity
- `GMP_RNDD`: round towards minus infinity

The '`round to nearest`' mode works as in the IEEE 754-1985 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 5/2, which is represented by (10.1) in binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the *drift* phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value of type `int`, called the *ternary value*. The value stored in the destination variable is correctly rounded, i.e. MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

As a consequence, in case of a non-zero real rounded result, the error on the result is less or equal to 1/2 ulp (unit in the last place) of the target in the rounding to nearest mode, and less than 1 ulp of the target in the directed rounding modes (a ulp is the weight of the least significant represented bit of the target after rounding).

Unless documented otherwise, functions returning an `int` return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the `GMP_RNDU` rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an `int`.

Unless documented otherwise, functions returning a `1` (or any other value specified in this manual) for special cases (like `acos(0)`) should return an overflow or an underflow if `1` is not representable in the current exponent range.

4.5 Floating-Point Values on Special Numbers

This section specifies the floating-point values (of type `mpfr_t`) returned by MPFR functions. For functions returning several values (like `mpfr_sin_cos`), the rules apply to each result separately.

Functions can have one or several input arguments. An input point is a mapping from these input arguments to the set of the MPFR numbers. When none of its components are NaN, an input point can also be seen as a tuple in the extended real numbers (the set of the real numbers with both infinities).

When the input point is in the domain of the mathematical function, the result is rounded as described in Section "Rounding Modes" (but see below for the specification of the sign of an exact zero). Otherwise the general rules from this section apply unless stated otherwise in the description of the MPFR function (Chapter 5 [MPFR Interface], page 10).

When the input point is not in the domain of the mathematical function but is in its closure in the extended real numbers and the function can be extended by continuity, the result is the obtained limit. Examples: `mpfr_hypot` on (+Inf,0) gives +Inf. But `mpfr_pow` cannot be defined on (1,+Inf) using this rule, as one can find sequences (x_n,y_n) such that x_n goes to 1, y_n goes to +Inf and $(x_n)^{y_n}$ goes to any positive value when n goes to the infinity.

When the input point is in the closure of the domain of the mathematical function and an input argument is +0 (resp. −0), one considers the limit when the corresponding argument approaches 0 from above (resp. below). If the limit is not defined (e.g., `mpfr_log` on −0), the behavior must be specified in the description of the MPFR function.

When the result is equal to 0, its sign is determined by considering the limit as if the input point were not in the domain: If one approaches 0 from above (resp. below), the result is +0 (resp. −0). In the other cases, the sign must be specified in the description of the MPFR function. Example: `mpfr_sin` on +0 gives +0.

When the input point is not in the closure of the domain of the function, the result is NaN. Example: `mpfr_sqrt` on −17 gives NaN.

When an input argument is NaN, the result is NaN, possibly except when a partial function is constant on the finite floating-point numbers; such a case is always explicitly specified in Chapter 5 [MPFR Interface], page 10. Example: `mpfr_hypot` on (NaN,0) gives NaN, but `mpfr_hypot` on (NaN,+Inf) gives +Inf (as specified in Section 5.7 [Special Functions], page 19), since for any finite input x, `mpfr_hypot` on (x,+Inf) gives +Inf.

4.6 Exceptions

MPFR supports 5 exception types:

- Underflow: An underflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent smaller than the minimum exponent of the current range. In the round-to-nearest mode, the halfway case is rounded toward zero.

  Note: This is not the single definition of the underflow. MPFR chooses to consider the underflow after rounding. The underflow before rounding can also be defined. For instance, consider a function that has the exact result $7 \times 2^{e-4}$, where e is the smallest exponent (for a mantissa between 1/2 and 1) in the current range, with a 2-bit target precision and rounding towards plus infinity. The exact result has the exponent $e-1$. With the underflow before rounding, such a function call would yield an underflow, as $e-1$ is outside the current exponent range. However, MPFR first considers the rounded result assuming an unbounded exponent range. The exact result cannot be represented exactly in precision 2, and here, it is rounded to 0.5×2^e, which is representable in the current exponent range. As a consequence, this will not yield an underflow in MPFR.
- Overflow: An overflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for

the rounding), has an exponent larger than the maximum exponent of the current range. In the round-to-nearest mode, the result is infinite.

- NaN: A NaN exception occurs when the result of a function is a NaN.
- Inexact: An inexact exception occurs when the result of a function cannot be represented exactly and must be rounded.
- Range error: A range exception occurs when a function that does not return a MPFR number (such as comparisons and conversions to an integer) has an invalid result (e.g. an argument is NaN in `mpfr_cmp` or in a conversion to an integer).

MPFR has a global flag for each exception, which can be cleared, set or tested by functions described in Section 5.12 [Exception Related Functions], page 26.

Differences with the ISO C99 standard:

- In C, only quiet NaNs are specified, and a NaN propagation does not raise an invalid exception. Unless explicitly stated otherwise, MPFR sets the NaN flag whenever a NaN is generated, even when a NaN is propagated (e.g. in NaN `+` NaN), as if all NaNs were signaling.
- An invalid exception in C corresponds to either a NaN exception or a range error in MPFR.

5 MPFR Interface

The floating-point functions expect arguments of type `mpfr_t`.

The MPFR floating-point functions have an interface that is similar to the GNU MP integer functions. The function prefix for floating-point operations is `mpfr_`.

There is one significant characteristic of floating-point numbers that has motivated a difference between this function class and other GNU MP function classes: the inherent inexactness of floating-point arithmetic. The user has to specify the precision for each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend from the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with "infinite accuracy"), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754-1985 arithmetic. The results obtained on one computer should not differ from the results obtained on a computer with a different word size.

MPFR does not keep track of the accuracy of a computation. This is left to the user or to a higher layer. As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with large precision, then MPFR will still compute the result with full precision.

The value of the standard C macro `errno` may be set to non-zero by any MPFR function or macro, whether or not there is an error.

5.1 Initialization Functions

An `mpfr_t` object must be initialized before storing the first value in it. The functions `mpfr_init` and `mpfr_init2` are used for that purpose.

`void mpfr_init2` (*mpfr_t* `x`, *mp_prec_t* `prec`) [Function]

Initialize *x*, set its precision to be **exactly** *prec* bits and its value to NaN. (Warning: the corresponding `mpf` functions initialize to zero instead.)

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. To change the precision of a variable which has already been initialized, use `mpfr_set_prec`. The precision *prec* must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined).

`void mpfr_clear` (*mpfr_t* `x`) [Function]

Free the space occupied by *x*. Make sure to call this function for all `mpfr_t` variables when you are done with them.

`void mpfr_init` (*mpfr_t* `x`) [Function]

Initialize *x* and set its value to NaN.

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. The precision of *x* is the default precision, which can be changed by a call to `mpfr_set_default_prec`.

`void mpfr_set_default_prec` (*mp_prec_t* `prec`) [Function]

Set the default precision to be **exactly** *prec* bits. The precision of a variable means the number of bits used to store its mantissa. All subsequent calls to `mpfr_init` will use this precision, but previously initialized variables are unaffected. This default precision is set to 53 bits initially. The precision can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`.

`mp_prec_t mpfr_get_default_prec` (*void*) [Function]
Return the default MPFR precision in bits.

Here is an example on how to initialize floating-point variables:

```
{
  mpfr_t x, y;
  mpfr_init (x);                /* use default precision */
  mpfr_init2 (y, 256);          /* precision exactly 256 bits */
  ...
  /* When the program is about to exit, do ... */
  mpfr_clear (x);
  mpfr_clear (y);
}
```

The following functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

`void mpfr_set_prec` (*mpfr_t* `x`, *mp_prec_t* `prec`) [Function]
Reset the precision of *x* to be **exactly** *prec* bits, and set its value to NaN. The previous value stored in *x* is lost. It is equivalent to a call to `mpfr_clear(x)` followed by a call to `mpfr_init2(x, prec)`, but more efficient as no allocation is done in case the current allocated space for the mantissa of *x* is enough. The precision *prec* can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`.

In case you want to keep the previous value stored in *x*, use `mpfr_prec_round` instead.

`mp_prec_t mpfr_get_prec` (*mpfr_t* `x`) [Function]
Return the precision actually used for assignments of *x*, i.e. the number of bits used to store its mantissa.

5.2 Assignment Functions

These functions assign new values to already initialized floats (see Section 5.1 [Initialization Functions], page 10). When using any functions using `intmax_t`, you must include `<stdint.h>` or `<inttypes.h>` before '`mpfr.h`', to allow '`mpfr.h`' to define prototypes for these functions.

`int mpfr_set` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_ui` (*mpfr_t* `rop`, *unsigned long int* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_si` (*mpfr_t* `rop`, *long int* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_uj` (*mpfr_t* `rop`, *uintmax_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_sj` (*mpfr_t* `rop`, *intmax_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_d` (*mpfr_t* `rop`, *double* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_ld` (*mpfr_t* `rop`, *long double* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_decimal64` (*mpfr_t* `rop`, *_Decimal64* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_z` (*mpfr_t* `rop`, *mpz_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_q` (*mpfr_t* `rop`, *mpq_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_set_f` (*mpfr_t* `rop`, *mpf_t* `op`, *mp_rnd_t* `rnd`) [Function]
Set the value of *rop* from *op*, rounded towards the given direction *rnd*. Note that the input 0 is converted to +0 by `mpfr_set_ui`, `mpfr_set_si`, `mpfr_set_sj`, `mpfr_set_uj`, `mpfr_set_`

`z`, `mpfr_set_q` and `mpfr_set_f`, regardless of the rounding mode. If the system doesn't support the IEEE-754 standard, `mpfr_set_d`, `mpfr_set_ld` and `mpfr_set_decimal64` might not preserve the signed zeros. The `mpfr_set_decimal64` function is built only with the configure option '`--enable-decimal-float`', which also requires '`--with-gmp-build`', and when the compiler or system provides the '`_Decimal64`' data type (GCC version 4.2.0 is known to support this data type, but only when configured with '`--enable-decimal-float`' too). `mpfr_set_q` might not be able to work if the numerator (or the denominator) can not be representable as a `mpfr_t`.

Note: If you want to store a floating-point constant to a `mpfr_t`, you should use `mpfr_set_str` (or one of the MPFR constant functions, such as `mpfr_const_pi` for π) instead of `mpfr_set_d`, `mpfr_set_ld` or `mpfr_set_decimal64`. Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary number before MPFR can work with it.

`int mpfr_set_ui_2exp` (*mpfr_t* **`rop`**, *unsigned long int* **`op`**, *mp_exp_t* **`e`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_set_si_2exp` (*mpfr_t* **`rop`**, *long int* **`op`**, *mp_exp_t* **`e`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_set_uj_2exp` (*mpfr_t* **`rop`**, *uintmax_t* **`op`**, *intmax_t* **`e`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_set_sj_2exp` (*mpfr_t* **`rop`**, *intmax_t* **`op`**, *intmax_t* **`e`**, *mp_rnd_t* **`rnd`**) [Function]

Set the value of *rop* from $op \times 2^e$, rounded towards the given direction *rnd*. Note that the input 0 is converted to `+0`.

`int mpfr_set_str` (*mpfr_t* **`rop`**, *const char* ***`s`**, *int* **`base`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the value of the whole string *s* in base *base*, rounded in the direction *rnd*. See the documentation of `mpfr_strtofr` for a detailed description of the valid string formats. This function returns 0 if the entire string up to the final null character is a valid number in base *base*; otherwise it returns -1, and *rop* may have changed.

`int mpfr_strtofr` (*mpfr_t* **`rop`**, *const char* ***`nptr`**, *char* ****`endptr`**, *int* **`base`**, *mp_rnd_t* **`rnd`**) [Function]

Read a floating-point number from a string *nptr* in base *base*, rounded in the direction *rnd*. If successful, the result is stored in *rop* and `*endptr` points to the character just after those parsed. If *str* doesn't start with a valid number then *rop* is set to zero and the value of *nptr* is stored in the location referenced by *endptr*.

Parsing follows the standard C `strtod` function. This means optional leading whitespace, an optional `+` or `-`, mantissa digits with an optional decimal point, and an optional exponent consisting of an `e` or `E` (if $base \leq 10$) or `@`, an optional sign, and digits. The decimal point can be either the one defined by the current locale or the period (the first one is accepted for consistency with the C standard and the practice, the second one is accepted to allow the programmer to provide MPFR numbers from strings in a way that does not depend on the current locale). A hexadecimal mantissa can be given with a leading `0x` or `0X`, in which case `p` or `P` may introduce an optional binary exponent, indicating the power of 2 by which the mantissa is to be scaled. A binary mantissa can be given with a leading `0b` or `0B`, in which case `e`, `E`, `p`, `P` or `@` may introduce the binary exponent. The exponent is always written in base 10.

In addition, `infinity`, `inf` (if $base \leq 10$) or `@inf@` with an optional sign, or `nan`, `nan(n-char-sequence)` (if $base \leq 10$), `@nan@` or `@nan@(n-char-sequence)` all case insensitive (as

Latin letters), can be given. A `n-char-sequence` is a non-empty string containing only digits, Latin letters and the underscore (0, 1, 2, ..., 9, a, b, ..., z, A, B, ..., Z, _).

There must be at least one digit in the mantissa for the number to be valid. If an exponent has no digits it's ignored and parsing stops after the mantissa. If an `0x`, `0X`, `0b` or `0B` is not followed by hexadecimal/binary digits, parsing stops after the first `0`: the subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-white-space character, that is of the expected form. The subject sequence contains no characters if the input string is not of the expected form.

Note that in the hex format the exponent `P` represents a power of 2, whereas `@` represents a power of the base (i.e. 16).

If the argument *base* is different from 0, it must be in the range 2 to 36. Case is ignored; uppercase and lowercase letters have the same value.

If `base` is 0, then it tries to identify the used base: if the mantissa begins with the `0x` prefix, it assumes that *base* is 16. If it begins with `0b`, it assumes that *base* is 2. Otherwise, it assumes it is 10.

It returns a usual ternary value. If *endptr* is not a null pointer, a pointer to the character after the last character used in the conversion is stored in the location referenced by *endptr*.

`void mpfr_set_inf` (*mpfr_t* `x`, *int* `sign`) [Function]
`void mpfr_set_nan` (*mpfr_t* `x`) [Function]

Set the variable *x* to infinity or NaN (Not-a-Number) respectively. In `mpfr_set_inf`, *x* is set to plus infinity iff *sign* is nonnegative.

`void mpfr_swap` (*mpfr_t* `x`, *mpfr_t* `y`) [Function]

Swap the values *x* and *y* efficiently. Warning: the precisions are exchanged too; in case the precisions are different, `mpfr_swap` is thus not equivalent to three `mpfr_set` calls using a third auxiliary variable.

5.3 Combined Initialization and Assignment Functions

`int mpfr_init_set` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Macro]
`int mpfr_init_set_ui` (*mpfr_t* `rop`, *unsigned long int* `op`, *mp_rnd_t* `rnd`) [Macro]
`int mpfr_init_set_si` (*mpfr_t* `rop`, *signed long int* `op`, *mp_rnd_t* `rnd`) [Macro]
`int mpfr_init_set_d` (*mpfr_t* `rop`, *double* `op`, *mp_rnd_t* `rnd`) [Macro]
`int mpfr_init_set_ld` (*mpfr_t* `rop`, *long double* `op`, *mp_rnd_t* `rnd`) [Macro]
`int mpfr_init_set_z` (*mpfr_t* `rop`, *mpz_t* `op`, *mp_rnd_t* `rnd`) [Macro]
`int mpfr_init_set_q` (*mpfr_t* `rop`, *mpq_t* `op`, *mp_rnd_t* `rnd`) [Macro]
`int mpfr_init_set_f` (*mpfr_t* `rop`, *mpf_t* `op`, *mp_rnd_t* `rnd`) [Macro]

Initialize *rop* and set its value from *op*, rounded in the direction *rnd*. The precision of *rop* will be taken from the active default precision, as set by `mpfr_set_default_prec`.

`int mpfr_init_set_str` (*mpfr_t* `x`, *const char* `*s`, *int* `base`, *mp_rnd_t* `rnd`) [Function]

Initialize *x* and set its value from the string *s* in base *base*, rounded in the direction *rnd*. See `mpfr_set_str`.

5.4 Conversion Functions

`double mpfr_get_d` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`long double mpfr_get_ld` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

`_Decimal64 mpfr_get_decimal64` (*mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Convert *op* to a `double` (respectively `_Decimal64` or `long double`), using the rounding mode *rnd*. If *op* is NaN, some fixed NaN (either quiet or signaling) or the result of 0.0/0.0 is returned. If *op* is ±Inf, an infinity of the same sign or the result of ±1.0/0.0 is returned. If *op* is zero, these functions return a zero, trying to preserve its sign, if possible. The `mpfr_get_decimal64` function is built only under some conditions: see the documentation of `mpfr_set_decimal64`.

`double mpfr_get_d_2exp` (*long **`exp`, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`long double mpfr_get_ld_2exp` (*long **`exp`, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Return d and set *exp* such that $0.5 \le |d| < 1$ and $d \times 2^{exp}$ equals *op* rounded to double (resp. long double) precision, using the given rounding mode. If *op* is zero, then a zero of the same sign (or an unsigned zero, if the implementation does not have signed zeros) is returned, and *exp* is set to 0. If *op* is NaN or an infinity, then the corresponding double precision (resp. long-double precision) value is returned, and *exp* is undefined.

`long mpfr_get_si` (*mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`unsigned long mpfr_get_ui` (*mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`intmax_t mpfr_get_sj` (*mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`uintmax_t mpfr_get_uj` (*mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Convert *op* to a `long`, an `unsigned long`, an `intmax_t` or an `uintmax_t` (respectively) after rounding it with respect to *rnd*. If *op* is NaN, the result is undefined. If *op* is too big for the return type, it returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow. The flag erange is set too. See also `mpfr_fits_slong_p`, `mpfr_fits_ulong_p`, `mpfr_fits_intmax_p` and `mpfr_fits_uintmax_p`.

`mp_exp_t mpfr_get_z_exp` (*mpz_t* **`rop`**, *mpfr_t* **`op`**) [Function]

Put the scaled mantissa of *op* (regarded as an integer, with the precision of *op*) into *rop*, and return the exponent *exp* (which may be outside the current exponent range) such that *op* exactly equals $rop \times 2^{\mathrm{exp}}$. If the exponent is not representable in the `mp_exp_t` type, the behavior is undefined.

`void mpfr_get_z` (*mpz_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Convert *op* to a `mpz_t`, after rounding it with respect to *rnd*. If *op* is NaN or Inf, the result is undefined.

`int mpfr_get_f` (*mpf_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Convert *op* to a `mpf_t`, after rounding it with respect to *rnd*. Return zero iff no error occurred, in particular a non-zero value is returned if *op* is NaN or Inf, which do not exist in `mpf`.

`char * mpfr_get_str` (*char **`str`, *mp_exp_t **`expptr`, *int* **`b`**, *size_t* **`n`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Convert *op* to a string of digits in base *b*, with rounding in the direction *rnd*, where *n* is either zero (see below) or the number of significant digits; in the latter case, *n* must be greater or equal to 2. The base may vary from 2 to 36.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number −3.1416 would be returned as "−31416" in the string and 1 written at *expptr*. If *rnd* is to nearest, and *op* is exactly in the middle of two possible outputs, the one with an even last digit is chosen (for an odd base, this may not correspond to an even mantissa).

If *n* is zero, the number of digits of the mantissa is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding to nearest, will recover the original value of *op*. More precisely, in most cases, the chosen precision of *str* is the minimal precision depending on *n* and *b* only that satisfies the above property, i.e., $m = 1 + \lceil n \frac{\log 2}{\log b} \rceil$, but in some very rare cases, it might be $m + 1$.

If *str* is a null pointer, space for the mantissa is allocated using the current allocation function, and a pointer to the string is returned. To free the returned string, you must use `mpfr_free_str`.

If *str* is not a null pointer, it should point to a block of storage large enough for the mantissa, i.e., at least `max(n + 2, 7)`. The extra two bytes are for a possible minus sign, and for the terminating null character.

If the input number is an ordinary number, the exponent is written through the pointer *expptr* (the current minimal exponent for 0).

A pointer to the string is returned, unless there is an error, in which case a null pointer is returned.

`void mpfr_free_str` (*char* *`str`) [Function]

Free a string allocated by `mpfr_get_str` using the current unallocation function (preliminary interface). The block is assumed to be `strlen(str)+1` bytes. For more information about how it is done: see section "Custom Allocation" in *GNU MP*.

`int mpfr_fits_ulong_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_fits_slong_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_fits_uint_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_fits_sint_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_fits_ushort_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_fits_sshort_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_fits_intmax_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_fits_uintmax_p` (*mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Return non-zero if *op* would fit in the respective C data type, when rounded to an integer in the direction *rnd*.

5.5 Basic Arithmetic Functions

`int mpfr_add` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_add_ui` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_add_si` (*mpfr_t* `rop`, *mpfr_t* `op1`, *long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_add_z` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpz_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_add_q` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpq_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to *op1* + *op2* rounded in the direction *rnd*. For types having no signed zero, it is considered unsigned (i.e. $(+0) + 0 = (+0)$ and $(-0) + 0 = (-0)$).

`int mpfr_sub` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_ui_sub` (*mpfr_t* `rop`, *unsigned long int* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_sub_ui` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_si_sub` (*mpfr_t* `rop`, *long int* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_sub_si` (*mpfr_t* `rop`, *mpfr_t* `op1`, *long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_sub_z` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpz_t* `op2`, *mp_rnd_t* `rnd`) [Function]

`int mpfr_sub_q` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpq_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $\mathit{op1} - \mathit{op2}$ rounded in the direction *rnd*. For types having no signed zero, it is considered unsigned (i.e. $(+0) - 0 = (+0)$, $(-0) - 0 = (-0)$, $0 - (+0) = (-0)$ and $0 - (-0) = (+0)$).

`int mpfr_mul` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_mul_ui` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_mul_si` (*mpfr_t* `rop`, *mpfr_t* `op1`, *long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_mul_z` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpz_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_mul_q` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpq_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $\mathit{op1} \times \mathit{op2}$ rounded in the direction *rnd*. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zero, it is considered positive).

`int mpfr_sqr` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to op^2 rounded in the direction *rnd*.

`int mpfr_div` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_ui_div` (*mpfr_t* `rop`, *unsigned long int* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_div_ui` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_si_div` (*mpfr_t* `rop`, *long int* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_div_si` (*mpfr_t* `rop`, *mpfr_t* `op1`, *long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_div_z` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpz_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_div_q` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpq_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $\mathit{op1}/\mathit{op2}$ rounded in the direction *rnd*. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zero, it is considered positive).

`int mpfr_sqrt` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_sqrt_ui` (*mpfr_t* `rop`, *unsigned long int* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $\sqrt{\mathit{op}}$ rounded in the direction *rnd*. Return -0 if *op* is -0 (to be consistent with the IEEE 754-1985 standard). Set *rop* to NaN if *op* is negative.

`int mpfr_cbrt` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_root` (*mpfr_t* `rop`, *mpfr_t* `op`, *unsigned long int* `k`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the cubic root (resp. the kth root) of *op* rounded in the direction *rnd*. An odd (resp. even) root of a negative number (including $-\mathrm{Inf}$) returns a negative number (resp. NaN). The kth root of -0 is defined to be -0, whatever the parity of k.

`int mpfr_pow` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_pow_ui` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_pow_si` (*mpfr_t* `rop`, *mpfr_t* `op1`, *long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_pow_z` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpz_t* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_ui_pow_ui` (*mpfr_t* `rop`, *unsigned long int* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_ui_pow` (*mpfr_t* `rop`, *unsigned long int* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $\mathit{op1}^{\mathit{op2}}$, rounded in the direction *rnd*. Special values are currently handled as described in the ISO C99 standard for the `pow` function (note this may change in future versions):

- `pow(±0, y)` returns plus or minus infinity for y a negative odd integer.
- `pow(±0, y)` returns plus infinity for y negative and not an odd integer.
- `pow(±0, y)` returns plus or minus zero for y a positive odd integer.
- `pow(±0, y)` returns plus zero for y positive and not an odd integer.
- `pow(-1, ±Inf)` returns 1.
- `pow(+1, y)` returns 1 for any y, even a NaN.
- `pow(x, y)` returns NaN for finite negative x and finite non-integer y.
- `pow(x, -Inf)` returns plus infinity for $0 < |x| < 1$, and plus zero for $|x| > 1$.
- `pow(x, +Inf)` returns plus zero for $0 < |x| < 1$, and plus infinity for $|x| > 1$.
- `pow(-Inf, y)` returns minus zero for y a negative odd integer.
- `pow(-Inf, y)` returns plus zero for y negative and not an odd integer.
- `pow(-Inf, y)` returns minus infinity for y a positive odd integer.
- `pow(-Inf, y)` returns plus infinity for y positive and not an odd integer.
- `pow(+Inf, y)` returns plus zero for y negative, and plus infinity for y positive.

`int mpfr_neg` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $-op$ rounded in the direction *rnd*. Just changes the sign if *rop* and *op* are the same variable.

`int mpfr_abs` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the absolute value of *op*, rounded in the direction *rnd*. Just changes the sign if *rop* and *op* are the same variable.

`int mpfr_dim` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the positive difference of *op1* and *op2*, i.e., $op1 - op2$ rounded in the direction *rnd* if $op1 > op2$, and +0 otherwise. Returns NaN when *op1* or *op2* is NaN.

`int mpfr_mul_2ui` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_mul_2si` (*mpfr_t* `rop`, *mpfr_t* `op1`, *long int* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $op1 \times 2^{op2}$ rounded in the direction *rnd*. Just increases the exponent by *op2* when *rop* and *op1* are identical.

`int mpfr_div_2ui` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_div_2si` (*mpfr_t* `rop`, *mpfr_t* `op1`, *long int* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to $op1/2^{op2}$ rounded in the direction *rnd*. Just decreases the exponent by *op2* when *rop* and *op1* are identical.

5.6 Comparison Functions

`int mpfr_cmp` (*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]
`int mpfr_cmp_ui` (*mpfr_t* `op1`, *unsigned long int* `op2`) [Function]
`int mpfr_cmp_si` (*mpfr_t* `op1`, *signed long int* `op2`) [Function]
`int mpfr_cmp_d` (*mpfr_t* `op1`, *double* `op2`) [Function]
`int mpfr_cmp_ld` (*mpfr_t* `op1`, *long double* `op2`) [Function]
`int mpfr_cmp_z` (*mpfr_t* `op1`, *mpz_t* `op2`) [Function]
`int mpfr_cmp_q` (*mpfr_t* `op1`, *mpq_t* `op2`) [Function]

`int mpfr_cmp_f (`*mpfr_t* `op1`, *mpf_t* `op2`) [Function]

Compare *op1* and *op2*. Return a positive value if $op1 > op2$, zero if $op1 = op2$, and a negative value if $op1 < op2$. Both *op1* and *op2* are considered to their full own precision, which may differ. If one of the operands is NaN, set the erange flag and return zero.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., `mpfr_equal_p` for the equality) described below; they behave like the IEEE-754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).

`int mpfr_cmp_ui_2exp (`*mpfr_t* `op1`, *unsigned long int* `op2`, *mp_exp_t* `e`) [Function]
`int mpfr_cmp_si_2exp (`*mpfr_t* `op1`, *long int* `op2`, *mp_exp_t* `e`) [Function]

Compare *op1* and $op2 \times 2^e$. Similar as above.

`int mpfr_cmpabs (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Compare $|op1|$ and $|op2|$. Return a positive value if $|op1| > |op2|$, zero if $|op1| = |op2|$, and a negative value if $|op1| < |op2|$. If one of the operands is NaN, set the erange flag and return zero.

`int mpfr_nan_p (`*mpfr_t* `op`) [Function]
`int mpfr_inf_p (`*mpfr_t* `op`) [Function]
`int mpfr_number_p (`*mpfr_t* `op`) [Function]
`int mpfr_zero_p (`*mpfr_t* `op`) [Function]

Return non-zero if *op* is respectively NaN, an infinity, an ordinary number (i.e. neither NaN nor an infinity) or zero. Return zero otherwise.

`int mpfr_sgn (`*mpfr_t* `op`) [Macro]

Return a positive value if $op > 0$, zero if $op = 0$, and a negative value if $op < 0$. If the operand is NaN, set the erange flag and return zero.

`int mpfr_greater_p (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Return non-zero if $op1 > op2$, zero otherwise.

`int mpfr_greaterequal_p (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Return non-zero if $op1 \geq op2$, zero otherwise.

`int mpfr_less_p (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Return non-zero if $op1 < op2$, zero otherwise.

`int mpfr_lessequal_p (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Return non-zero if $op1 \leq op2$, zero otherwise.

`int mpfr_lessgreater_p (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Return non-zero if $op1 < op2$ or $op1 > op2$ (i.e. neither *op1*, nor *op2* is NaN, and $op1 \neq op2$), zero otherwise (i.e. *op1* and/or *op2* are NaN, or $op1 = op2$).

`int mpfr_equal_p (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Return non-zero if $op1 = op2$, zero otherwise (i.e. *op1* and/or *op2* are NaN, or $op1 \neq op2$).

`int mpfr_unordered_p (`*mpfr_t* `op1`, *mpfr_t* `op2`) [Function]

Return non-zero if *op1* or *op2* is a NaN (i.e. they cannot be compared), zero otherwise.

5.7 Special Functions

All those functions, except explicitly stated, return zero for an exact return value, a positive value for a return value larger than the exact result, and a negative value otherwise.

`int mpfr_log` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_log2` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_log10` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the natural logarithm of *op*, $\log_2 op$ or $\log_{10} op$, respectively, rounded in the direction *rnd*. Return −Inf if *op* is −0 (i.e. the sign of the zero has no influence on the result).

`int mpfr_exp` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_exp2` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_exp10` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the exponential of *op*, to 2^{op} or to 10^{op}, respectively, rounded in the direction *rnd*.

`int mpfr_cos` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_sin` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_tan` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the cosine of *op*, sine of *op*, tangent of *op*, rounded in the direction *rnd*.

`int mpfr_sec` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_csc` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_cot` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the secant of *op*, cosecant of *op*, cotangent of *op*, rounded in the direction *rnd*.

`int mpfr_sin_cos` (*mpfr_t* **`sop`**, *mpfr_t* **`cop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set simultaneously *sop* to the sine of *op* and *cop* to the cosine of *op*, rounded in the direction *rnd* with the corresponding precisions of *sop* and *cop*. Return 0 iff both results are exact.

`int mpfr_acos` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_asin` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_atan` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the arc-cosine, arc-sine or arc-tangent of *op*, rounded in the direction *rnd*.

`int mpfr_atan2` (*mpfr_t* **`rop`**, *mpfr_t* **`y`**, *mpfr_t* **`x`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the arc-tangent2 of y and x, rounded in the direction *rnd*: if `x > 0`, `atan2(y, x) = atan (y/x)`; if `x < 0`, `atan2(y, x) = sign(y)*(Pi - atan (|y/x|))`.

`atan2(y, 0)` does not raise any floating-point exception. Special values are currently handled as described in the ISO C99 standard for the `atan2` function (note this may change in future versions):

- `atan2(+0, -0)` returns $+\pi$.
- `atan2(-0, -0)` returns $-\pi$.
- `atan2(+0, +0)` returns +0.
- `atan2(-0, +0)` returns −0.
- `atan2(+0, x)` returns $+\pi$ for $x < 0$.
- `atan2(-0, x)` returns $-\pi$ for $x < 0$.
- `atan2(+0, x)` returns +0 for $x > 0$.
- `atan2(-0, x)` returns −0 for $x > 0$.

- `atan2(y, 0)` returns $-\pi/2$ for $y < 0$.
- `atan2(y, 0)` returns $+\pi/2$ for $y > 0$.
- `atan2(+Inf, -Inf)` returns $+3 * \pi/4$.
- `atan2(-Inf, -Inf)` returns $-3 * \pi/4$.
- `atan2(+Inf, +Inf)` returns $+\pi/4$.
- `atan2(-Inf, +Inf)` returns $-\pi/4$.
- `atan2(+Inf, x)` returns $+\pi/2$ for finite x.
- `atan2(-Inf, x)` returns $-\pi/2$ for finite x.
- `atan2(y, -Inf)` returns $+\pi$ for finite $y > 0$.
- `atan2(y, -Inf)` returns $-\pi$ for finite $y < 0$.
- `atan2(y, +Inf)` returns +0 for finite $y > 0$.
- `atan2(y, +Inf)` returns -0 for finite $y < 0$.

`int mpfr_cosh` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_sinh` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_tanh` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the hyperbolic cosine, sine or tangent of *op*, rounded in the direction *rnd*.

`int mpfr_sech` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_csch` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_coth` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the hyperbolic secant of *op*, cosecant of *op*, cotangent of *op*, rounded in the direction *rnd*.

`int mpfr_acosh` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_asinh` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]
`int mpfr_atanh` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the inverse hyperbolic cosine, sine or tangent of *op*, rounded in the direction *rnd*.

`int mpfr_fac_ui` (*mpfr_t* `rop`, *unsigned long int* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the factorial of the `unsigned long int` *op*, rounded in the direction *rnd*.

`int mpfr_log1p` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the logarithm of one plus *op*, rounded in the direction *rnd*.

`int mpfr_expm1` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the exponential of *op* minus one, rounded in the direction *rnd*.

`int mpfr_eint` (*mpfr_t* `y`, *mpfr_t* `x`, *mp_rnd_t* `rnd`) [Function]

Set *y* to the exponential integral of *x*, rounded in the direction *rnd*. For positive *x*, the exponential integral is the sum of Euler's constant, of the logarithm of *x*, and of the sum for k from 1 to infinity of $x^k/k/k!$. For negative *x*, the returned value is NaN.

`int mpfr_gamma` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the value of the Gamma function on *op*, rounded in the direction *rnd*. When *op* is a negative integer, NaN is returned.

`int mpfr_lngamma` (*mpfr_t* `rop`, *mpfr_t* `op`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the value of the logarithm of the Gamma function on *op*, rounded in the direction *rnd*. When $-2k-1 \leq x \leq -2k$, k being a non-negative integer, NaN is returned. See also `mpfr_lgamma`.

`int mpfr_lgamma` (*mpfr_t* **`rop`**, *int* **`*signp`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the value of the logarithm of the absolute value of the Gamma function on *op*, rounded in the direction *rnd*. The sign (1 or −1) of Gamma(*op*) is returned in the object pointed to by *signp*. When *op* is an infinity or a non-positive integer, +Inf is returned. When *op* is NaN, −Inf or a negative integer, **signp* is undefined, and when *op* is ±0, **signp* is the sign of the zero.

`int mpfr_zeta` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_zeta_ui` (*mpfr_t* **`rop`**, *unsigned long* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the value of the Riemann Zeta function on *op*, rounded in the direction *rnd*.

`int mpfr_erf` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the value of the error function on *op*, rounded in the direction *rnd*.

`int mpfr_erfc` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the value of the complementary error function on *op*, rounded in the direction *rnd*.

`int mpfr_j0` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_j1` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_jn` (*mpfr_t* **`rop`**, *long* **`n`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the value of the first order Bessel function of order 0, 1 and *n* on *op*, rounded in the direction *rnd*. When *op* is NaN, *rop* is always set to NaN. When *op* is plus or minus Infinity, *rop* is set to +0. When *op* is zero, and *n* is not zero, *rop* is +0 or −0 depending on the parity and sign of *n*, and the sign of *op*.

`int mpfr_y0` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_y1` (*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]
`int mpfr_yn` (*mpfr_t* **`rop`**, *long* **`n`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the value of the second order Bessel function of order 0, 1 and *n* on *op*, rounded in the direction *rnd*. When *op* is NaN or negative, *rop* is always set to NaN. When *op* is +Inf, *rop* is +0. When *op* is zero, *rop* is +Inf or −Inf depending on the parity and sign of *n*.

`int mpfr_fma` (*mpfr_t* **`rop`**, *mpfr_t* **`op1`**, *mpfr_t* **`op2`**, *mpfr_t* **`op3`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to $op1 \times op2 + op3$, rounded in the direction *rnd*.

`int mpfr_fms` (*mpfr_t* **`rop`**, *mpfr_t* **`op1`**, *mpfr_t* **`op2`**, *mpfr_t* **`op3`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to $op1 \times op2 - op3$, rounded in the direction *rnd*.

`int mpfr_agm` (*mpfr_t* **`rop`**, *mpfr_t* **`op1`**, *mpfr_t* **`op2`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the arithmetic-geometric mean of *op1* and *op2*, rounded in the direction *rnd*. The arithmetic-geometric mean is the common limit of the sequences u[n] and v[n], where u[0]=*op1*, v[0]=*op2*, u[n+1] is the arithmetic mean of u[n] and v[n], and v[n+1] is the geometric mean of u[n] and v[n]. If any operand is negative, the return value is NaN.

`int mpfr_hypot` (*mpfr_t* **`rop`**, *mpfr_t* **`x`**, *mpfr_t* **`y`**, *mp_rnd_t* **`rnd`**) [Function]

Set *rop* to the Euclidean norm of *x* and *y*, i.e. $\sqrt{x^2 + y^2}$, rounded in the direction *rnd*. Special values are currently handled as described in Section F.9.4.3 of the ISO C99 standard, for the `hypot` function (note this may change in future versions): If *x* or *y* is an infinity, then plus infinity is returned in *rop*, even if the other number is NaN.

int mpfr_const_log2 (*mpfr_t* **rop**, *mp_rnd_t* **rnd**) [Function]
int mpfr_const_pi (*mpfr_t* **rop**, *mp_rnd_t* **rnd**) [Function]
int mpfr_const_euler (*mpfr_t* **rop**, *mp_rnd_t* **rnd**) [Function]
int mpfr_const_catalan (*mpfr_t* **rop**, *mp_rnd_t* **rnd**) [Function]

Set *rop* to the logarithm of 2, the value of π, of Euler's constant 0.577..., of Catalan's constant 0.915..., respectively, rounded in the direction *rnd*. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use `mpfr_free_cache`.

void mpfr_free_cache (*void*) [Function]

Free various caches used by MPFR internally, in particular the caches used by the functions computing constants (currently `mpfr_const_log2`, `mpfr_const_pi`, `mpfr_const_euler` and `mpfr_const_catalan`). You should call this function when terminating a thread.

int mpfr_sum (*mpfr_t* **rop**, *mpfr_ptr const* **tab**[], *unsigned long* **n**, *mp_rnd_t* **rnd**) [Function]

Set *ret* to the sum of all elements of *tab* whose size is *n*, rounded in the direction *rnd*. Warning, *tab* is a table of pointers to mpfr_t, not a table of mpfr_t (preliminary interface). The returned `int` value is zero when the computed value is the exact value, and non-zero when this cannot be guaranteed, without giving the direction of the error as the other functions do.

5.8 Input and Output Functions

This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a *stream* argument to any of these functions will make them read from `stdin` and write to `stdout`, respectively.

When using any of these functions, you must include the `<stdio.h>` standard header before '`mpfr.h`', to allow '`mpfr.h`' to define prototypes for these functions.

size_t mpfr_out_str (*FILE* ***stream**, *int* **base**, *size_t* **n**, *mpfr_t* **op**, *mp_rnd_t* **rnd**) [Function]

Output *op* on stream *stream*, as a string of digits in base *base*, rounded in the direction *rnd*. The base may vary from 2 to 36. Print *n* significant digits exactly, or if *n* is 0, enough digits so that *op* can be read back exactly (see `mpfr_get_str`).

In addition to the significant digits, a decimal point (defined by the current locale) at the right of the first digit and a trailing exponent in base 10, in the form '`eNNN`', are printed. If *base* is greater than 10, '`@`' will be used instead of '`e`' as exponent delimiter.

Return the number of bytes written, or if an error occurred, return 0.

size_t mpfr_inp_str (*mpfr_t* **rop**, *FILE* ***stream**, *int* **base**, *mp_rnd_t* **rnd**) [Function]

Input a string in base *base* from stream *stream*, rounded in the direction *rnd*, and put the read float in *rop*.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using `mpfr_set_str` (it may change). See the documentation of `mpfr_strtofr` for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

5.9 Integer Related Functions

`int mpfr_rint (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**`)` [Function]
`int mpfr_ceil (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**`)` [Function]
`int mpfr_floor (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**`)` [Function]
`int mpfr_round (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**`)` [Function]
`int mpfr_trunc (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**`)` [Function]

Set *rop* to *op* rounded to an integer. `mpfr_rint` rounds to the nearest representable integer in the given rounding mode, `mpfr_ceil` rounds to the next higher or equal representable integer, `mpfr_floor` to the next lower or equal representable integer, `mpfr_round` to the nearest representable integer, rounding halfway cases away from zero, and `mpfr_trunc` to the next representable integer towards zero.

The returned value is zero when the result is exact, positive when it is greater than the original value of *op*, and negative when it is smaller. More precisely, the returned value is 0 when *op* is an integer representable in *rop*, 1 or -1 when *op* is an integer that is not representable in *rop*, 2 or -2 when *op* is not an integer.

Note that `mpfr_round` is different from `mpfr_rint` called with the rounding to the nearest mode (where halfway cases are rounded to an even integer or mantissa). Note also that no double rounding is performed; for instance, 4.5 (100.1 in binary) is rounded by `mpfr_round` to 4 (100 in binary) in 2-bit precision, though `round(4.5)` is equal to 5 and 5 (101 in binary) is rounded to 6 (110 in binary) in 2-bit precision.

`int mpfr_rint_ceil (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**`)` [Function]
`int mpfr_rint_floor (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**`)` [Function]
`int mpfr_rint_round (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**`)` [Function]
`int mpfr_rint_trunc (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**`)` [Function]

Set *rop* to *op* rounded to an integer. `mpfr_rint_ceil` rounds to the next higher or equal integer, `mpfr_rint_floor` to the next lower or equal integer, `mpfr_rint_round` to the nearest integer, rounding halfway cases away from zero, and `mpfr_rint_trunc` to the next integer towards zero. If the result is not representable, it is rounded in the direction *rnd*. The returned value is the ternary value associated with the considered round-to-integer function (regarded in the same way as any other mathematical function).

`int mpfr_frac (`*mpfr_t* **`rop`**, *mpfr_t* **`op`**, *mp_rnd_t* **`rnd`**`)` [Function]

Set *rop* to the fractional part of *op*, having the same sign as *op*, rounded in the direction *rnd* (unlike in `mpfr_rint`, *rnd* affects only how the exact fractional part is rounded, not how the fractional part is generated).

`int mpfr_remainder (`*mpfr_t* **`r`**, *mpfr_t* **`x`**, *mpfr_t* **`y`**, *mp_rnd_t* **`rnd`**`)` [Function]
`int mpfr_remquo (`*mpfr_t* **`r`**, *long** **`q`**, *mpfr_t* **`x`**, *mpfr_t* **`y`**, *mp_rnd_t* **`rnd`**`)` [Function]

Set *r* to the remainder of the division of *x* by *y*, with quotient rounded to the nearest integer (ties rounded to even), and *r* rounded according to the direction *rnd*. If *r* is zero, it has the sign of *x*. The return value is the inexact flag corresponding to *r*. Additionally, `mpfr_remquo` stores the low significant bits from the quotient in **q* (more precisely the number of bits in a `long` minus one), with the sign of *x* divided by *y* (except if those low bits are all zero, in which case zero is returned). Note that *x* may be so large in magnitude relative to *y* that an exact representation of the quotient is not practical. These functions are useful for additive argument reduction.

`int mpfr_integer_p (`*mpfr_t* **`op`**`)` [Function]

Return non-zero iff *op* is an integer.

5.10 Miscellaneous Functions

`void mpfr_nexttoward` (*mpfr_t* `x`, *mpfr_t* `y`) [Function]

If x or y is NaN, set x to NaN. Otherwise, if x is different from y, replace x by the next floating-point number (with the precision of x and the current exponent range) in the direction of y, if there is one (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow or overflow is generated.

`void mpfr_nextabove` (*mpfr_t* `x`) [Function]

Equivalent to `mpfr_nexttoward` where y is plus infinity.

`void mpfr_nextbelow` (*mpfr_t* `x`) [Function]

Equivalent to `mpfr_nexttoward` where y is minus infinity.

`int mpfr_min` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the minimum of *op1* and *op2*. If *op1* and *op2* are both NaN, then *rop* is set to NaN. If *op1* or *op2* is NaN, then *rop* is set to the numeric value. If *op1* and *op2* are zeros of different signs, then *rop* is set to -0.

`int mpfr_max` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Set *rop* to the maximum of *op1* and *op2*. If *op1* and *op2* are both NaN, then *rop* is set to NaN. If *op1* or *op2* is NaN, then *rop* is set to the numeric value. If *op1* and *op2* are zeros of different signs, then *rop* is set to +0.

`int mpfr_urandomb` (*mpfr_t* `rop`, *gmp_randstate_t* `state`) [Function]

Generate a uniformly distributed random float in the interval $0 \leq rop < 1$. Return 0, unless the exponent is not in the current exponent range, in which case *rop* is set to NaN and a non-zero value is returned. The second argument is a `gmp_randstate_t` structure which should be created using the GMP `gmp_randinit` function, see the GMP manual.

`void mpfr_random` (*mpfr_t* `rop`) [Function]

Generate a uniformly distributed random float in the interval $0 \leq rop < 1$. This function is deprecated; `mpfr_urandomb` should be used instead.

`void mpfr_random2` (*mpfr_t* `rop`, *mp_size_t* `size`, *mp_exp_t* `exp`) [Function]

Generate a random float of at most *size* limbs, with long strings of zeros and ones in the binary representation. The exponent of the number is in the interval $-exp$ to *exp*. This function is useful for testing functions and algorithms, since this kind of random numbers have proven to be more likely to trigger corner-case bugs. Negative random numbers are generated when *size* is negative. Put +0 in *rop* when size if zero. The internal state of the default pseudorandom number generator is modified by a call to this function (the same one as GMP if MPFR was built using '`--with-gmp-build`').

`mp_exp_t mpfr_get_exp` (*mpfr_t* `x`) [Function]

Get the exponent of x, assuming that x is a non-zero ordinary number. The behavior for NaN, Infinity or Zero is undefined.

`int mpfr_set_exp` (*mpfr_t* `x`, *mp_exp_t* `e`) [Function]

Set the exponent of x if e is in the current exponent range, and return 0 (even if x is not a non-zero ordinary number); otherwise, return a non-zero value.

`int mpfr_signbit (`*mpfr_t* `op)` [Function]

Return a non-zero value iff *op* has its sign bit set (i.e. if it is negative, −0, or a NaN whose representation has its sign bit set).

`int mpfr_setsign (`*mpfr_t* `rop`, *mpfr_t* `op`, *int* `s`, *mp_rnd_t* `rnd)` [Function]

Set the value of *rop* from *op*, rounded towards the given direction *rnd*, then set (resp. clear) its sign bit if *s* is non-zero (resp. zero), even when *op* is a NaN.

`int mpfr_copysign (`*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd)` [Function]

Set the value of *rop* from *op1*, rounded towards the given direction *rnd*, then set its sign bit to that of *op2* (even when *op1* or *op2* is a NaN). This function is equivalent to `mpfr_setsign (rop, op1, mpfr_signbit (op2), rnd)`.

`const char * mpfr_get_version (`*void*`)` [Function]

Return the MPFR version, as a null-terminated string.

`MPFR_VERSION` [Macro]
`MPFR_VERSION_MAJOR` [Macro]
`MPFR_VERSION_MINOR` [Macro]
`MPFR_VERSION_PATCHLEVEL` [Macro]
`MPFR_VERSION_STRING` [Macro]

`MPFR_VERSION` is the version of MPFR as a preprocessing constant. `MPFR_VERSION_MAJOR`, `MPFR_VERSION_MINOR` and `MPFR_VERSION_PATCHLEVEL` are respectively the major, minor and patch level of MPFR version, as preprocessing constants. `MPFR_VERSION_STRING` is the version (with an optional suffix, used in development and pre-release versions) as a string constant, which can be compared to the result of `mpfr_get_version` to check at run time the header file and library used match:

```
if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))
  fprintf (stderr, "Warning: header and library do not match\n");
```

Note: Obtaining different strings is not necessarily an error, as in general, a program compiled with some old MPFR version can be dynamically linked with a newer MPFR library version (if allowed by the library versioning system).

`long MPFR_VERSION_NUM (major, minor, patchlevel)` [Macro]

Create an integer in the same format as used by `MPFR_VERSION` from the given *major*, *minor* and *patchlevel*. Here is an example of how to check the MPFR version at compile time:

```
#if (!defined(MPFR_VERSION) || (MPFR_VERSION<MPFR_VERSION_NUM(2,1,0)))
# error "Wrong MPFR version."
#endif
```

`const char * mpfr_get_patches (`*void*`)` [Function]

Return a null-terminated string containing the ids of the patches applied to the MPFR library (contents of the ‘`PATCHES`’ file), separated by spaces. Note: If the program has been compiled with an older MPFR version and is dynamically linked with a new MPFR library version, the ids of the patches applied to the old (compile-time) MPFR version are not available (however this information should not have much interest in general).

5.11 Rounding Mode Related Functions

`void mpfr_set_default_rounding_mode (`*mp_rnd_t* `rnd)` [Function]

Set the default rounding mode to *rnd*. The default rounding mode is to nearest initially.

`mp_rnd_t mpfr_get_default_rounding_mode` (*void*) [Function]

Get the default rounding mode.

`int mpfr_prec_round` (*mpfr_t* `x`, *mp_prec_t* `prec`, *mp_rnd_t* `rnd`) [Function]

Round *x* according to *rnd* with precision *prec*, which must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined). If *prec* is greater or equal to the precision of *x*, then new space is allocated for the mantissa, and it is filled with zeros. Otherwise, the mantissa is rounded to precision *prec* with the given direction. In both cases, the precision of *x* is changed to *prec*.

`int mpfr_round_prec` (*mpfr_t* `x`, *mp_rnd_t* `rnd`, *mp_prec_t* `prec`) [Function]

[This function is obsolete. Please use `mpfr_prec_round` instead.]

`const char * mpfr_print_rnd_mode` (*mp_rnd_t* `rnd`) [Function]

Return the input string (GMP_RNDD, GMP_RNDU, GMP_RNDN, GMP_RNDZ) corresponding to the rounding mode *rnd* or a null pointer if *rnd* is an invalid rounding mode.

5.12 Exception Related Functions

`mp_exp_t mpfr_get_emin` (*void*) [Function]
`mp_exp_t mpfr_get_emax` (*void*) [Function]

Return the (current) smallest and largest exponents allowed for a floating-point variable. The smallest positive value of a floating-point variable is $1/2 \times 2^{\mathrm{emin}}$ and the largest value has the form $(1 - \varepsilon) \times 2^{\mathrm{emax}}$.

`int mpfr_set_emin` (*mp_exp_t* `exp`) [Function]
`int mpfr_set_emax` (*mp_exp_t* `exp`) [Function]

Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero value when *exp* is not in the range accepted by the implementation (in that case the smallest or largest exponent is not changed), and zero otherwise. If the user changes the exponent range, it is her/his responsibility to check that all current floating-point variables are in the new allowed range (for example using `mpfr_check_range`), otherwise the subsequent behavior will be undefined, in the sense of the ISO C standard.

`mp_exp_t mpfr_get_emin_min` (*void*) [Function]
`mp_exp_t mpfr_get_emin_max` (*void*) [Function]
`mp_exp_t mpfr_get_emax_min` (*void*) [Function]
`mp_exp_t mpfr_get_emax_max` (*void*) [Function]

Return the minimum and maximum of the smallest and largest exponents allowed for `mpfr_set_emin` and `mpfr_set_emax`. These values are implementation dependent; it is possible to create a non portable program by writing `mpfr_set_emax(mpfr_get_emax_max())` and `mpfr_set_emin(mpfr_get_emin_min())` since the values of the smallest and largest exponents become implementation dependent.

`int mpfr_check_range` (*mpfr_t* `x`, *int* `t`, *mp_rnd_t* `rnd`) [Function]

This function forces *x* to be in the current range of acceptable values, *t* being the current ternary value: negative if *x* is smaller than the exact value, positive if *x* is larger than the exact value and zero if *x* is exact (before the call). It generates an underflow or an overflow if the exponent of *x* is outside the current allowed range; the value of *t* may be used to avoid a double rounding. This function returns zero if the rounded result is equal to the exact one, a positive value if the rounded result is larger than the exact one, a negative value if the rounded result is smaller than the exact one. Note that unlike most functions, the result is compared to the exact one, not the input value *x*, i.e. the ternary value is propagated.

`int` **`mpfr_subnormalize`** (*mpfr_t* `x`, *int* `t`, *mp_rnd_t* `rnd`) [Function]

This function rounds *x* emulating subnormal number arithmetic: if *x* is outside the subnormal exponent range, it just propagates the ternary value *t*; otherwise, it rounds *x* to precision `EXP(x)-emin+1` according to rounding mode *rnd* and previous ternary value *t*, avoiding double rounding problems. `PREC(x)` is not modified by this function. *rnd* and *t* must be the used rounding mode for computing *x* and the returned ternary value when computing *x*. The subnormal exponent range is from `emin` to `emin+PREC(x)-1`. This functions assumes that `emax-emin >= PREC(x)`. Note that unlike most functions, the result is compared to the exact one, not the input value *x*, i.e. the ternary value is propagated. This is a preliminary interface.

This is an example of how to emulate double IEEE-754 arithmetic using MPFR:

```
{
  mpfr_t xa, xb;
  int i;
  volatile double a, b;

  mpfr_set_default_prec (53);
  mpfr_set_emin (-1073);
  mpfr_set_emax (1024);

  mpfr_init (xa); mpfr_init (xb);

  b = 34.3; mpfr_set_d (xb, b, GMP_RNDN);
  a = 0x1.1235P-1021; mpfr_set_d (xa, a, GMP_RNDN);

  a /= b;
  i = mpfr_div (xa, xa, xb, GMP_RNDN);
  i = mpfr_subnormalize (xa, i, GMP_RNDN);

  mpfr_clear (xa); mpfr_clear (xb);
}
```

Warning: this emulates a double IEEE-754 arithmetic with correct rounding in the subnormal range, which may not be the case for your hardware.

`void` **`mpfr_clear_underflow`** (*void*) [Function]
`void` **`mpfr_clear_overflow`** (*void*) [Function]
`void` **`mpfr_clear_nanflag`** (*void*) [Function]
`void` **`mpfr_clear_inexflag`** (*void*) [Function]
`void` **`mpfr_clear_erangeflag`** (*void*) [Function]

Clear the underflow, overflow, invalid, inexact and erange flags.

`void` **`mpfr_set_underflow`** (*void*) [Function]
`void` **`mpfr_set_overflow`** (*void*) [Function]
`void` **`mpfr_set_nanflag`** (*void*) [Function]
`void` **`mpfr_set_inexflag`** (*void*) [Function]
`void` **`mpfr_set_erangeflag`** (*void*) [Function]

Set the underflow, overflow, invalid, inexact and erange flags.

`void` **`mpfr_clear_flags`** (*void*) [Function]

Clear all global flags (underflow, overflow, inexact, invalid, erange).

`int` **`mpfr_underflow_p`** (*void*) [Function]

`int mpfr_overflow_p` (*void*) [Function]
`int mpfr_nanflag_p` (*void*) [Function]
`int mpfr_inexflag_p` (*void*) [Function]
`int mpfr_erangeflag_p` (*void*) [Function]

Return the corresponding (underflow, overflow, invalid, inexact, erange) flag, which is non-zero iff the flag is set.

5.13 Advanced Functions

All the given interfaces are preliminary. They might change incompatibly in future revisions.

`MPFR_DECL_INIT` (*name*, *prec*) [Macro]

This macro declares *name* as an automatic variable of type `mpfr_t`, initializes it and sets its precision to be **exactly** *prec* bits and its value to NaN. *name* must be a valid identifier. You must use this macro in the declaration section. This macro is much faster than using `mpfr_init2` but has some drawbacks:

- You **must not** call `mpfr_clear` with variables created with this macro (The storage is allocated at the point of declaration and deallocated when the brace-level is exited.).
- You **can not** change their precision.
- You **should not** create variables with huge precision with this macro.
- Your compiler must support '`Non-Constant Initializers`' (standard in C++ and ISO C99) and '`Token Pasting`' (standard in ISO C89). If *prec* is not a compiler constant, your compiler must support '`Variable-length automatic arrays`' (standard in ISO C99). '`GCC 2.95.3`' supports all these features. If you compile your program with gcc in c89 mode and with '`-pedantic`', you may want to define the `MPFR_USE_EXTENSION` macro to avoid warnings due to the `MPFR_DECL_INIT` implementation.

`void mpfr_inits` (*mpfr_t* `x`, *...*) [Function]

Initialize all the `mpfr_t` variables of the given `va_list`, set their precision to be the default precision and their value to NaN. See `mpfr_init` for more details. The `va_list` is assumed to be composed only of type `mpfr_t`. It begins from *x*. It ends when it encounters a null pointer.

`void mpfr_inits2` (*mp_prec_t* `prec`, *mpfr_t* `x`, *...*) [Function]

Initialize all the `mpfr_t` variables of the given `va_list`, set their precision to be **exactly** *prec* bits and their value to NaN. See `mpfr_init2` for more details. The `va_list` is assumed to be composed only of type `mpfr_t`. It begins from *x*. It ends when it encounters a null pointer.

`void mpfr_clears` (*mpfr_t* `x`, *...*) [Function]

Free the space occupied by all the `mpfr_t` variables of the given `va_list`. See `mpfr_clear` for more details. The `va_list` is assumed to be composed only of type `mpfr_t`. It begins from *x*. It ends when it encounters a null pointer.

Here is an example of how to use multiple initialization functions:

```
{
  mpfr_t x, y, z, t;
  mpfr_inits2 (256, x, y, z, t, (void *) 0);
  ...
  mpfr_clears (x, y, z, t, (void *) 0);
}
```

5.14 Compatibility With MPF

A header file '`mpf2mpfr.h`' is included in the distribution of MPFR for compatibility with the GNU MP class MPF. After inserting the following two lines after the `#include <gmp.h>` line,

```
#include <mpfr.h>
#include <mpf2mpfr.h>
```

any program written for MPF can be compiled directly with MPFR without any changes. All operations are then performed with the default MPFR rounding mode, which can be reset with `mpfr_set_default_rounding_mode`.

Warning: the `mpf_init` and `mpf_init2` functions initialize to zero, whereas the corresponding `mpfr` functions initialize to NaN: this is useful to detect uninitialized values, but is slightly incompatible with `mpf`.

`void mpfr_set_prec_raw` (*mpfr_t* `x`, *mp_prec_t* `prec`) [Function]

Reset the precision of *x* to be **exactly** *prec* bits. The only difference with `mpfr_set_prec` is that *prec* is assumed to be small enough so that the mantissa fits into the current allocated memory space for *x*. Otherwise the behavior is undefined.

`int mpfr_eq` (*mpfr_t* `op1`, *mpfr_t* `op2`, *unsigned long int* `op3`) [Function]

Return non-zero if *op1* and *op2* are both non-zero ordinary numbers with the same exponent and the same first *op3* bits, both zero, or both infinities of the same sign. Return zero otherwise. This function is defined for compatibility with `mpf`. Do not use it if you want to know whether two numbers are close to each other; for instance, 1.011111 and 1.100000 are regarded as different for any value of *op3* larger than 1.

`void mpfr_reldiff` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mp_rnd_t* `rnd`) [Function]

Compute the relative difference between *op1* and *op2* and store the result in *rop*. This function does not guarantee the correct rounding on the relative difference; it just computes $|op1 - op2|/op1$, using the rounding mode *rnd* for all operations and the precision of *rop*.

`int mpfr_mul_2exp` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]

`int mpfr_div_2exp` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mp_rnd_t* `rnd`) [Function]

See `mpfr_mul_2ui` and `mpfr_div_2ui`. These functions are only kept for compatibility with MPF.

5.15 Custom Interface

Some applications use a stack to handle the memory and their objects. However, the MPFR memory design is not well suited for such a thing. So that such applications are able to use MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows them to use MPFR in two ways:

- Either they directly store the MPFR FP number as a `mpfr_t` on the stack.
- Either they store their own representation of a FP number on the stack and construct a new temporary `mpfr_t` each time it is needed.

Nothing has to be done to destroy the FP numbers except garbaging the used memory: all the memory stuff (allocating, destroying, garbaging) is kept to the application.

Each function is this interface is also implemented as a macro for efficiency reasons.

Note 1: MPFR functions may still initialize temporary FP numbers using standard mpfr_init. See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (mpfr_const_pi for example), even if they are not explicitly called. You have to call `mpfr_free_cache` each time you garbage the memory iff mpfr_init, through GMP Custom Allocation, allocates its memory on the application stack.

Note 3: This interface is preliminary.

`size_t` **`mpfr_custom_get_size`** (*mp_prec_t* `prec`) [Function]

Return the needed size in bytes to store the mantissa of a FP number of precision *prec*.

`void` **`mpfr_custom_init`** (*void* `*mantissa`, *mp_prec_t* `prec`) [Function]

Initialize a mantissa of precision *prec*. *mantissa* must be an area of `mpfr_custom_get_size (prec)` bytes at least and be suitably aligned for an array of `mp_limb_t`.

`void` **`mpfr_custom_init_set`** (*mpfr_t* `x`, *int* `kind`, *mp_exp_t* `exp`, *mp_prec_t* `prec`, *void* `*mantissa`) [Function]

Perform a dummy initialization of a `mpfr_t` and set it to:

- if `ABS(kind) == MPFR_NAN_KIND`, *x* is set to NaN;
- if `ABS(kind) == MPFR_INF_KIND`, *x* is set to the infinity of sign `sign(kind)`;
- if `ABS(kind) == MPFR_ZERO_KIND`, *x* is set to the zero of sign `sign(kind)`;
- if `ABS(kind) == MPFR_REGULAR_KIND`, *x* is set to a regular number: `x = sign(kind)*mantissa*2^exp`

In all cases, it uses *mantissa* directly for further computing involving *x*. It will not allocate anything. A FP number initialized with this function cannot be resized using `mpfr_set_prec`, or cleared using `mpfr_clear`! *mantissa* must have been initialized with `mpfr_custom_init` using the same precision *prec*.

`int` **`mpfr_custom_get_kind`** (*mpfr_t* `x`) [Function]

Return the current kind of a `mpfr_t` as used by `mpfr_custom_init_set`. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`void *` **`mpfr_custom_get_mantissa`** (*mpfr_t* `x`) [Function]

Return a pointer to the mantissa used by a `mpfr_t` initialized with `mpfr_custom_init_set`. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`mp_exp_t` **`mpfr_custom_get_exp`** (*mpfr_t* `x`) [Function]

Return the exponent of *x*, assuming that *x* is a non-zero ordinary number. The return value for NaN, Infinity or Zero is unspecified but doesn't produce any trap. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`void` **`mpfr_custom_move`** (*mpfr_t* `x`, *void* `*new_position`) [Function]

Inform MPFR that the mantissa has moved due to a garbage collect and update its new position to `new_position`. However the application has to move the mantissa and the `mpfr_t` itself. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

See the test suite for examples.

5.16 Internals

The following types and functions were mainly designed for the implementation of `mpfr`, but may be useful for users too. However no upward compatibility is guaranteed. You may need to include '`mpfr-impl.h`' to use them.

The `mpfr_t` type consists of four fields.

- The `_mpfr_prec` field is used to store the precision of the variable (in bits); this is not less than `MPFR_PREC_MIN`.
- The `_mpfr_sign` field is used to store the sign of the variable.
- The `_mpfr_exp` field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values n are a multiplier 2^n relative to that point. A NaN, an infinity and a zero are indicated by a special value of the exponent.
- Finally, the `_mpfr_d` is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by `_mpfr_prec`, namely ceil(`_mpfr_prec`/`mp_bits_per_limb`). Non-singular values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zero.

`int mpfr_can_round` (*mpfr_t* **`b`**, *mp_exp_t* **`err`**, *mp_rnd_t* **`rnd1`**, *mp_rnd_t* **`rnd2`**, *mp_prec_t* **`prec`**) [Function]

Assuming *b* is an approximation of an unknown number *x* in the direction *rnd1* with error at most two to the power E(b)-*err* where E(b) is the exponent of *b*, return a non-zero value if one is able to round correctly *x* to precision *prec* with the direction *rnd2*, and 0 otherwise (including for NaN and Inf). This function **does not modify** its arguments.

`double mpfr_get_d1` (*mpfr_t* **`op`**) [Function]

Convert *op* to a `double`, using the default MPFR rounding mode (see function `mpfr_set_default_rounding_mode`). This function is obsolete.

Contributors

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