

# GNU MPFR

---

The Multiple Precision Floating-Point Reliable Library  
Edition 3.1.3  
June 2015

**The MPFR team**

[mpfr@inria.fr](mailto:mpfr@inria.fr)

---

This manual documents how to install and use the Multiple Precision Floating-Point Reliable Library, version 3.1.3.

Copyright 1991, 1993-2015 Free Software Foundation, Inc.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, with no Front-Cover Texts, and with no Back-Cover Texts. A copy of the license is included in [Appendix A \[GNU Free Documentation License\]](#), page 47.

# Table of Contents

MPFR Copying Conditions.....	<b>1</b>
<b>1 Introduction to MPFR.....</b>	<b>2</b>
1.1 How to Use This Manual.....	2
<b>2 Installing MPFR.....</b>	<b>3</b>
2.1 How to Install.....	3
2.2 Other ‘make’ Targets.....	4
2.3 Build Problems.....	4
2.4 Getting the Latest Version of MPFR.....	4
<b>3 Reporting Bugs.....</b>	<b>5</b>
<b>4 MPFR Basics.....</b>	<b>6</b>
4.1 Headers and Libraries.....	6
4.2 Nomenclature and Types.....	6
4.3 MPFR Variable Conventions.....	7
4.4 Rounding Modes.....	7
4.5 Floating-Point Values on Special Numbers.....	8
4.6 Exceptions.....	9
4.7 Memory Handling.....	10
<b>5 MPFR Interface.....</b>	<b>11</b>
5.1 Initialization Functions.....	11
5.2 Assignment Functions.....	13
5.3 Combined Initialization and Assignment Functions.....	16
5.4 Conversion Functions.....	16
5.5 Basic Arithmetic Functions.....	18
5.6 Comparison Functions.....	21
5.7 Special Functions.....	22
5.8 Input and Output Functions.....	26
5.9 Formatted Output Functions.....	26
5.9.1 Requirements.....	26
5.9.2 Format String.....	27
5.9.3 Functions.....	29
5.10 Integer and Remainder Related Functions.....	29
5.11 Rounding Related Functions.....	31
5.12 Miscellaneous Functions.....	32
5.13 Exception Related Functions.....	35
5.14 Compatibility With MPF.....	37
5.15 Custom Interface.....	37
5.16 Internals.....	39

<b>6</b>	<b>API Compatibility</b> .....	<b>40</b>
6.1	Type and Macro Changes .....	40
6.2	Added Functions .....	41
6.3	Changed Functions .....	42
6.4	Removed Functions .....	43
6.5	Other Changes .....	43
	<b>Contributors</b> .....	<b>45</b>
	<b>References</b> .....	<b>46</b>
	<b>Appendix A GNU Free Documentation License</b> .....	<b>47</b>
A.1	ADDENDUM: How to Use This License For Your Documents .....	52
	<b>Concept Index</b> .....	<b>53</b>
	<b>Function and Type Index</b> .....	<b>54</b>

## MPFR Copying Conditions

The GNU MPFR library (or MPFR for short) is *free*; this means that everyone is free to use it and free to redistribute it on a free basis. The library is not in the public domain; it is copyrighted and there are restrictions on its distribution, but these restrictions are designed to permit everything that a good cooperating citizen would want to do. What is not allowed is to try to prevent others from further sharing any version of this library that they might get from you.

Specifically, we want to make sure that you have the right to give away copies of the library, that you receive source code or else can get it if you want it, that you can change this library or use pieces of it in new free programs, and that you know you can do these things.

To make sure that everyone has such rights, we have to forbid you to deprive anyone else of these rights. For example, if you distribute copies of the GNU MPFR library, you must give the recipients all the rights that you have. You must make sure that they, too, receive or can get the source code. And you must tell them their rights.

Also, for our own protection, we must make certain that everyone finds out that there is no warranty for the GNU MPFR library. If it is modified by someone else and passed on, we want their recipients to know that what they have is not what we distributed, so that any problems introduced by others will not reflect on our reputation.

The precise conditions of the license for the GNU MPFR library are found in the Lesser General Public License that accompanies the source code. See the file `COPYING.LESSER`.

# 1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to provide a class of floating-point numbers with precise semantics. The main characteristics of MPFR, which make it differ from most arbitrary precision floating-point software tools, are:

- the MPFR code is portable, i.e., the result of any operation does not depend on the machine word size `mp_bits_per_limb` (64 on most current processors);
- the precision in bits can be set *exactly* to any valid value for each variable (including very small precision);
- MPFR provides the four rounding modes from the IEEE 754-1985 standard, plus away-from-zero, as well as for basic operations as for other mathematical functions.

In particular, with a precision of 53 bits, MPFR is able to exactly reproduce all computations with double-precision machine floating-point numbers (e.g., `double` type in C, with a C implementation that rigorously follows Annex F of the ISO C99 standard and `FP_CONTRACT` pragma set to `OFF`) on the four arithmetic operations and the square root, except the default exponent range is much wider and subnormal numbers are not implemented (but can be emulated).

This version of MPFR is released under the GNU Lesser General Public License, version 3 or any later version. It is permitted to link MPFR to most non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

## 1.1 How to Use This Manual

Everyone should read [Chapter 4 \[MPFR Basics\]](#), page 6. If you need to install the library yourself, you need to read [Chapter 2 \[Installing MPFR\]](#), page 3, too. To use the library you will need to refer to [Chapter 5 \[MPFR Interface\]](#), page 11.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.

## 2 Installing MPFR

The MPFR library is already installed on some GNU/Linux distributions, but the development files necessary to the compilation such as `mpfr.h` are not always present. To check that MPFR is fully installed on your computer, you can check the presence of the file `mpfr.h` in `/usr/include`, or try to compile a small program having `#include <mpfr.h>` (since `mpfr.h` may be installed somewhere else). For instance, you can try to compile:

```
#include <stdio.h>
#include <mpfr.h>
int main (void)
{
    printf ("MPFR library: %s\nMPFR header:  %s (based on %d.%d.%d)\n",
           mpfr_get_version (), MPFR_VERSION_STRING, MPFR_VERSION_MAJOR,
           MPFR_VERSION_MINOR, MPFR_VERSION_PATCHLEVEL);
    return 0;
}
```

with

```
cc -o version version.c -lmpfr -lgmp
```

and if you get errors whose first line looks like

```
version.c:2:19: error: mpfr.h: No such file or directory
```

then MPFR is probably not installed. Running this program will give you the MPFR version.

If MPFR is not installed on your computer, or if you want to install a different version, please follow the steps below.

### 2.1 How to Install

Here are the steps needed to install the library on Unix systems (more details are provided in the `INSTALL` file):

1. To build MPFR, you first have to install GNU MP (version 4.1 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need the standard Unix ‘make’ command, plus some other standard Unix utility commands. Then, in the MPFR build directory, type the following commands.
2. ‘./configure’  
This will prepare the build and setup the options according to your system. You can give options to specify the install directories (instead of the default `/usr/local`), threading support, and so on. See the `INSTALL` file and/or the output of ‘./configure --help’ for more information, in particular if you get error messages.
3. ‘make’  
This will compile MPFR, and create a library archive file `libmpfr.a`. On most platforms, a dynamic library will be produced too.
4. ‘make check’  
This will make sure that MPFR was built correctly. If any test fails, information about this failure can be found in the `tests/test-suite.log` file. If you want the contents of this file to be automatically output in case of failure, you can set the ‘VERBOSE’ environment variable to 1 before running ‘make check’, for instance by typing:  
‘VERBOSE=1 make check’

In case of failure, you may want to check whether the problem is already known. If not, please report this failure to the MPFR mailing-list ‘[mpfr@inria.fr](mailto:mpfr@inria.fr)’. For details, See [Chapter 3 \[Reporting Bugs\], page 5](#).

#### 5. ‘make install’

This will copy the files `mpfr.h` and `mpf2mpfr.h` to the directory `/usr/local/include`, the library files (`libmpfr.a` and possibly others) to the directory `/usr/local/lib`, the file `mpfr.info` to the directory `/usr/local/share/info`, and some other documentation files to the directory `/usr/local/share/doc/mpfr` (or if you passed the ‘`--prefix`’ option to `configure`, using the prefix directory given as argument to ‘`--prefix`’ instead of `/usr/local`).

## 2.2 Other ‘make’ Targets

There are some other useful make targets:

- ‘`mpfr.info`’ or ‘`info`’  
Create or update an info version of the manual, in `mpfr.info`.  
This file is already provided in the MPFR archives.
- ‘`mpfr.pdf`’ or ‘`pdf`’  
Create a PDF version of the manual, in `mpfr.pdf`.
- ‘`mpfr.dvi`’ or ‘`dvi`’  
Create a DVI version of the manual, in `mpfr.dvi`.
- ‘`mpfr.ps`’ or ‘`ps`’  
Create a Postscript version of the manual, in `mpfr.ps`.
- ‘`mpfr.html`’ or ‘`html`’  
Create a HTML version of the manual, in several pages in the directory `doc/mpfr.html`; if you want only one output HTML file, then type ‘`makeinfo --html --no-split mpfr.texi`’ from the ‘`doc`’ directory instead.
- ‘`clean`’  
Delete all object files and archive files, but not the configuration files.
- ‘`distclean`’  
Delete all generated files not included in the distribution.
- ‘`uninstall`’  
Delete all files copied by ‘`make install`’.

## 2.3 Build Problems

In case of problem, please read the `INSTALL` file carefully before reporting a bug, in particular section “In case of problem”. Some problems are due to bad configuration on the user side (not specific to MPFR). Problems are also mentioned in the FAQ <http://www.mpfr.org/faq.html>.

Please report problems to the MPFR mailing-list ‘[mpfr@inria.fr](mailto:mpfr@inria.fr)’. See [Chapter 3 \[Reporting Bugs\], page 5](#). Some bug fixes are available on the MPFR 3.1.3 web page <http://www.mpfr.org/mpfr-3.1.3/>.

## 2.4 Getting the Latest Version of MPFR

The latest version of MPFR is available from <ftp://ftp.gnu.org/gnu/mpfr/> or <http://www.mpfr.org/>.



## 3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR 3.1.3 web page <http://www.mpfr.org/mpfr-3.1.3/> and the FAQ <http://www.mpfr.org/faq.html>: perhaps this bug is already known, in which case you may find there a workaround for it. You might also look in the archives of the MPFR mailing-list: <https://sympa.inria.fr/sympa/arc/mpfr>. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug, i.e., a small self-content program, using no other library than MPFR. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results you get are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using ‘`cc -V`’ on some machines, or, if you’re using GCC, ‘`gcc -v`’. Also, include the output from ‘`uname -a`’ and the MPFR version (the GMP version may be useful too). If you get a failure while running ‘`make`’ or ‘`make check`’, please include the `config.log` file in your bug report, and in case of test failure, the `tests/test-suite.log` file too.

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we will not do anything about it (aside of chiding you to send better bug reports).

Send your bug report to the MPFR mailing-list ‘`mpfr@inria.fr`’.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

## 4 MPFR Basics

### 4.1 Headers and Libraries

All declarations needed to use MPFR are collected in the include file `mpfr.h`. It is designed to work with both C and C++ compilers. You should include that file in any program using the MPFR library:

```
#include <mpfr.h>
```

Note however that prototypes for MPFR functions with `FILE *` parameters are provided only if `<stdio.h>` is included too (before `mpfr.h`):

```
#include <stdio.h>
#include <mpfr.h>
```

Likewise `<stdarg.h>` (or `<varargs.h>`) is required for prototypes with `va_list` parameters, such as `mpfr_vprintf`.

And for any functions using `intmax_t`, you must include `<stdint.h>` or `<inttypes.h>` before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions. Moreover, users of C++ compilers under some platforms may need to define `MPFR_USE_INTMAX_T` (and should do it for portability) before `mpfr.h` has been included; of course, it is possible to do that on the command line, e.g., with `-DMPFR_USE_INTMAX_T`.

Note: If `mpfr.h` and/or `gmp.h` (used by `mpfr.h`) are included several times (possibly from another header file), `<stdio.h>` and/or `<stdarg.h>` (or `<varargs.h>`) should be included **before the first inclusion** of `mpfr.h` or `gmp.h`. Alternatively, you can define `MPFR_USE_FILE` (for MPFR I/O functions) and/or `MPFR_USE_VA_LIST` (for MPFR functions with `va_list` parameters) anywhere before the last inclusion of `mpfr.h`. As a consequence, if your file is a public header that includes `mpfr.h`, you need to use the latter method.

When calling a MPFR macro, it is not allowed to have previously defined a macro with the same name as some keywords (currently `do`, `while` and `sizeof`).

You can avoid the use of MPFR macros encapsulating functions by defining the `MPFR_USE_NO_MACRO` macro before `mpfr.h` is included. In general this should not be necessary, but this can be useful when debugging user code: with some macros, the compiler may emit spurious warnings with some warning options, and macros can prevent some prototype checking.

All programs using MPFR must link against both `libmpfr` and `libgmp` libraries. On a typical Unix-like system this can be done with `'-lmpfr -lgmp'` (in that order), for example:

```
gcc myprogram.c -lmpfr -lgmp
```

MPFR is built using Libtool and an application can use that to link if desired, see *GNU Libtool*.

If MPFR has been installed to a non-standard location, then it may be necessary to set up environment variables such as `'C_INCLUDE_PATH'` and `'LIBRARY_PATH'`, or use `'-I'` and `'-L'` compiler options, in order to point to the right directories. For a shared library, it may also be necessary to set up some sort of run-time library path (e.g., `'LD_LIBRARY_PATH'`) on some systems. Please read the `INSTALL` file for additional information.

### 4.2 Nomenclature and Types

A *floating-point number*, or *float* for short, is an arbitrary precision significand (also called mantissa) with a limited precision exponent. The C data type for such objects is `mpfr_t`

(internally defined as a one-element array of a structure, and `mpfr_ptr` is the C data type representing a pointer to this structure). A floating-point number can have three special values: Not-a-Number (NaN) or plus or minus Infinity. NaN represents an uninitialized object, the result of an invalid operation (like 0 divided by 0), or a value that cannot be determined (like  $+\text{Infinity} - \text{Infinity}$ ). Moreover, like in the IEEE 754 standard, zero is signed, i.e., there are both  $+0$  and  $-0$ ; the behavior is the same as in the IEEE 754 standard and it is generalized to the other functions supported by MPFR. Unless documented otherwise, the sign bit of a NaN is unspecified.

The *precision* is the number of bits used to represent the significand of a floating-point number; the corresponding C data type is `mpfr_prec_t`. The precision can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. In the current implementation, `MPFR_PREC_MIN` is equal to 2.

Warning! MPFR needs to increase the precision internally, in order to provide accurate results (and in particular, correct rounding). Do not attempt to set the precision to any value near `MPFR_PREC_MAX`, otherwise MPFR will abort due to an assertion failure. Moreover, you may reach some memory limit on your platform, in which case the program may abort, crash or have undefined behavior (depending on your C implementation).

The *rounding mode* specifies the way to round the result of a floating-point operation, in case the exact result can not be represented exactly in the destination significand; the corresponding C data type is `mpfr_rnd_t`.

### 4.3 MPFR Variable Conventions

Before you can assign to an MPFR variable, you need to initialize it by calling one of the special initialization functions. When you're done with a variable, you need to clear it out, using one of the functions for that purpose. A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned to any number of times. For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited. You do not need to be concerned about allocating additional space for MPFR variables, since any variable has a significand of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator. MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication, `mpfr_mul`, can be used like this: `mpfr_mul(x, x, x, rnd)`. This computes the square of  $x$  with rounding mode `rnd` and puts the result back in  $x$ .

### 4.4 Rounding Modes

The following five rounding modes are supported:

- `MPFR_RNDN`: round to nearest (roundTiesToEven in IEEE 754-2008),
- `MPFR_RNDZ`: round toward zero (roundTowardZero in IEEE 754-2008),
- `MPFR_RNDU`: round toward plus infinity (roundTowardPositive in IEEE 754-2008),
- `MPFR_RNDD`: round toward minus infinity (roundTowardNegative in IEEE 754-2008),
- `MPFR_RNDA`: round away from zero.

The 'round to nearest' mode works as in the IEEE 754 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 2.5, which is represented by (10.1) in

binary, is rounded to  $(10.0)=2$  with a precision of two bits, and not to  $(11.0)=3$ . This rule avoids the *drift* phenomenon mentioned by Knuth in volume 2 of *The Art of Computer Programming* (Section 4.2.2).

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value of type `int`, called the *ternary value*. The value stored in the destination variable is correctly rounded, i.e., MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

As a consequence, in case of a non-zero real rounded result, the error on the result is less or equal to  $1/2$  ulp (unit in the last place) of that result in the rounding to nearest mode, and less than 1 ulp of that result in the directed rounding modes (a ulp is the weight of the least significant represented bit of the result after rounding).

Unless documented otherwise, functions returning an `int` return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the `MPFR_RNDU` rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an `int`.

Unless documented otherwise, functions returning as result the value 1 (or any other value specified in this manual) for special cases (like `acos(0)`) yield an overflow or an underflow if that value is not representable in the current exponent range.

## 4.5 Floating-Point Values on Special Numbers

This section specifies the floating-point values (of type `mpfr_t`) returned by MPFR functions (where by “returned” we mean here the modified value of the destination object, which should not be mixed with the ternary return value of type `int` of those functions). For functions returning several values (like `mpfr_sin_cos`), the rules apply to each result separately.

Functions can have one or several input arguments. An input point is a mapping from these input arguments to the set of the MPFR numbers. When none of its components are NaN, an input point can also be seen as a tuple in the extended real numbers (the set of the real numbers with both infinities).

When the input point is in the domain of the mathematical function, the result is rounded as described in Section “Rounding Modes” (but see below for the specification of the sign of an exact zero). Otherwise the general rules from this section apply unless stated otherwise in the description of the MPFR function ([Chapter 5 \[MPFR Interface\], page 11](#)).

When the input point is not in the domain of the mathematical function but is in its closure in the extended real numbers and the function can be extended by continuity, the result is the obtained limit. Examples: `mpfr_hypot` on  $(+\text{Inf}, 0)$  gives  $+\text{Inf}$ . But `mpfr_pow` cannot be defined on  $(1, +\text{Inf})$  using this rule, as one can find sequences  $(x_n, y_n)$  such that  $x_n$  goes to 1,  $y_n$  goes to  $+\text{Inf}$  and  $(x_n)^{y_n}$  goes to any positive value when  $n$  goes to the infinity.

When the input point is in the closure of the domain of the mathematical function and an input argument is  $+0$  (resp.  $-0$ ), one considers the limit when the corresponding argument approaches

0 from above (resp. below). If the limit is not defined (e.g., `mpfr_log` on  $-0$ ), the behavior is specified in the description of the MPFR function.

When the result is equal to 0, its sign is determined by considering the limit as if the input point were not in the domain: If one approaches 0 from above (resp. below), the result is  $+0$  (resp.  $-0$ ); for example, `mpfr_sin` on  $+0$  gives  $+0$ . In the other cases, the sign is specified in the description of the MPFR function; for example `mpfr_max` on  $-0$  and  $+0$  gives  $+0$ .

When the input point is not in the closure of the domain of the function, the result is NaN. Example: `mpfr_sqrt` on  $-17$  gives NaN.

When an input argument is NaN, the result is NaN, possibly except when a partial function is constant on the finite floating-point numbers; such a case is always explicitly specified in [Chapter 5 \[MPFR Interface\], page 11](#). Example: `mpfr_hypot` on  $(\text{NaN}, 0)$  gives NaN, but `mpfr_hypot` on  $(\text{NaN}, +\text{Inf})$  gives  $+\text{Inf}$  (as specified in [Section 5.7 \[Special Functions\], page 22](#)), since for any finite input  $x$ , `mpfr_hypot` on  $(x, +\text{Inf})$  gives  $+\text{Inf}$ .

## 4.6 Exceptions

MPFR supports 6 exception types:

- Underflow: An underflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent smaller than the minimum value of the current exponent range. (In the round-to-nearest mode, the halfway case is rounded toward zero.)

Note: This is not the single possible definition of the underflow. MPFR chooses to consider the underflow *after* rounding. The underflow before rounding can also be defined. For instance, consider a function that has the exact result  $7 \times 2^{e-4}$ , where  $e$  is the smallest exponent (for a significand between  $1/2$  and  $1$ ), with a 2-bit target precision and rounding toward plus infinity. The exact result has the exponent  $e-1$ . With the underflow before rounding, such a function call would yield an underflow, as  $e-1$  is outside the current exponent range. However, MPFR first considers the rounded result assuming an unbounded exponent range. The exact result cannot be represented exactly in precision 2, and here, it is rounded to  $0.5 \times 2^e$ , which is representable in the current exponent range. As a consequence, this will not yield an underflow in MPFR.

- Overflow: An overflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent larger than the maximum value of the current exponent range. In the round-to-nearest mode, the result is infinite. Note: unlike the underflow case, there is only one possible definition of overflow here.
- Divide-by-zero: An exact infinite result is obtained from finite inputs.
- NaN: A NaN exception occurs when the result of a function is NaN.
- Inexact: An inexact exception occurs when the result of a function cannot be represented exactly and must be rounded.
- Range error: A range exception occurs when a function that does not return a MPFR number (such as comparisons and conversions to an integer) has an invalid result (e.g., an argument is NaN in `mpfr_cmp`, or a conversion to an integer cannot be represented in the target type).

MPFR has a global flag for each exception, which can be cleared, set or tested by functions described in [Section 5.13 \[Exception Related Functions\], page 35](#).

Differences with the ISO C99 standard:

- In C, only quiet NaNs are specified, and a NaN propagation does not raise an invalid exception. Unless explicitly stated otherwise, MPFR sets the NaN flag whenever a NaN is generated, even when a NaN is propagated (e.g., in NaN + NaN), as if all NaNs were signaling.
- An invalid exception in C corresponds to either a NaN exception or a range error in MPFR.

## 4.7 Memory Handling

MPFR functions may create caches, e.g., when computing constants such as  $\pi$ , either because the user has called a function like `mpfr_const_pi` directly or because such a function was called internally by the MPFR library itself to compute some other function.

At any time, the user can free the various caches with `mpfr_free_cache`. It is strongly advised to do that before terminating a thread, or before exiting when using tools like ‘`valgrind`’ (to avoid memory leaks being reported).

MPFR internal data such as flags, the exponent range, the default precision and rounding mode, and caches (i.e., data that are not accessed via parameters) are either global (if MPFR has not been compiled as thread safe) or per-thread (thread local storage, TLS). The initial values of TLS data after a thread is created entirely depend on the compiler and thread implementation (MPFR simply does a conventional variable initialization, the variables being declared with an implementation-defined TLS specifier).

## 5 MPFR Interface

The floating-point functions expect arguments of type `mpfr_t`.

The MPFR floating-point functions have an interface that is similar to the GNU MP functions. The function prefix for floating-point operations is `mpfr_`.

The user has to specify the precision of each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend on the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with “infinite accuracy”), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754 arithmetic. The results obtained on a given computer are identical to those obtained on a computer with a different word size, or with a different compiler or operating system.

MPFR *does not keep track* of the accuracy of a computation. This is left to the user or to a higher layer (for example the MPFI library for interval arithmetic). As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with large precision, then MPFR will still compute the result with full precision.

The value of the standard C macro `errno` may be set to non-zero by any MPFR function or macro, whether or not there is an error.

### 5.1 Initialization Functions

An `mpfr_t` object must be initialized before storing the first value in it. The functions `mpfr_init` and `mpfr_init2` are used for that purpose.

`void mpfr_init2 (mpfr_t x, mpfr_prec_t prec)` [Function]  
 Initialize `x`, set its precision to be **exactly** `prec` bits and its value to NaN. (Warning: the corresponding MPF function initializes to zero instead.)

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. To change the precision of a variable which has already been initialized, use `mpfr_set_prec`. The precision `prec` must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined).

`void mpfr_inits2 (mpfr_prec_t prec, mpfr_t x, ...)` [Function]  
 Initialize all the `mpfr_t` variables of the given variable argument `va_list`, set their precision to be **exactly** `prec` bits and their value to NaN. See `mpfr_init2` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from `x`, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).

`void mpfr_clear (mpfr_t x)` [Function]  
 Free the space occupied by the significand of `x`. Make sure to call this function for all `mpfr_t` variables when you are done with them.

`void mpfr_clears (mpfr_t x, ...)` [Function]  
 Free the space occupied by all the `mpfr_t` variables of the given `va_list`. See `mpfr_clear` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from `x`, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).

Here is an example of how to use multiple initialization functions (since `NULL` is not necessarily defined in this context, we use `(mpfr_ptr) 0` instead, but `(mpfr_ptr) NULL` is also correct).

```
{
  mpfr_t x, y, z, t;
  mpfr_inits2 (256, x, y, z, t, (mpfr_ptr) 0);
  ...
  mpfr_clears (x, y, z, t, (mpfr_ptr) 0);
}
```

`void mpfr_init (mpfr_t x)` [Function]  
 Initialize `x`, set its precision to the default precision, and set its value to NaN. The default precision can be changed by a call to `mpfr_set_default_prec`.

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use `mpfr_init2`.

`void mpfr_inits (mpfr_t x, ...)` [Function]  
 Initialize all the `mpfr_t` variables of the given `va_list`, set their precision to the default precision and their value to NaN. See `mpfr_init` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from `x`, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use `mpfr_inits2`.

`MPFR_DECL_INIT (name, prec)` [Macro]  
 This macro declares `name` as an automatic variable of type `mpfr_t`, initializes it and sets its precision to be **exactly** `prec` bits and its value to NaN. `name` must be a valid identifier. You must use this macro in the declaration section. This macro is much faster than using `mpfr_init2` but has some drawbacks:

- You **must not** call `mpfr_clear` with variables created with this macro (the storage is allocated at the point of declaration and deallocated when the brace-level is exited).
- You **cannot** change their precision.
- You **should not** create variables with huge precision with this macro.
- Your compiler must support ‘Non-Constant Initializers’ (standard in C++ and ISO C99) and ‘Token Pasting’ (standard in ISO C89). If `prec` is not a constant expression, your compiler must support ‘variable-length automatic arrays’ (standard in ISO C99). GCC 2.95.3 and above supports all these features. If you compile your program with GCC in C89 mode and with ‘-pedantic’, you may want to define the `MPFR_USE_EXTENSION` macro to avoid warnings due to the `MPFR_DECL_INIT` implementation.

`void mpfr_set_default_prec (mpfr_prec_t prec)` [Function]  
 Set the default precision to be **exactly** `prec` bits, where `prec` can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. The precision of a variable means the number of bits used to store its significand. All subsequent calls to `mpfr_init` or `mpfr_inits` will use this precision, but previously initialized variables are unaffected. The default precision is set to 53 bits initially.

Note: when MPFR is built with the `--enable-thread-safe` configure option, the default precision is local to each thread. See [Section 4.7 \[Memory Handling\]](#), page 10, for more information.



`mpfr_prec_t mpfr_get_default_prec (void)` [Function]  
 Return the current default MPFR precision in bits. See the documentation of `mpfr_set_default_prec`.

Here is an example on how to initialize floating-point variables:

```
{
    mpfr_t x, y;
    mpfr_init (x);           /* use default precision */
    mpfr_init2 (y, 256);     /* precision exactly 256 bits */
    ...
    /* When the program is about to exit, do ... */
    mpfr_clear (x);
    mpfr_clear (y);
    mpfr_free_cache ();     /* free the cache for constants like pi */
}
```

The following functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

`void mpfr_set_prec (mpfr_t x, mpfr_prec_t prec)` [Function]  
 Reset the precision of `x` to be **exactly** `prec` bits, and set its value to NaN. The previous value stored in `x` is lost. It is equivalent to a call to `mpfr_clear(x)` followed by a call to `mpfr_init2(x, prec)`, but more efficient as no allocation is done in case the current allocated space for the significand of `x` is enough. The precision `prec` can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. In case you want to keep the previous value stored in `x`, use `mpfr_prec_round` instead.

Warning! You must not use this function if `x` was initialized with `MPFR_DECL_INIT` or with `mpfr_custom_init_set` (see [Section 5.15 \[Custom Interface\]](#), page 37).

`mpfr_prec_t mpfr_get_prec (mpfr_t x)` [Function]  
 Return the precision of `x`, i.e., the number of bits used to store its significand.

## 5.2 Assignment Functions

These functions assign new values to already initialized floats (see [Section 5.1 \[Initialization Functions\]](#), page 11).

`int mpfr_set (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_si (mpfr_t rop, long int op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_uj (mpfr_t rop, uintmax_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_sj (mpfr_t rop, intmax_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_setflt (mpfr_t rop, float op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_setd (mpfr_t rop, double op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_setld (mpfr_t rop, long double op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_decimal64 (mpfr_t rop, _Decimal64 op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_z (mpfr_t rop, mpz_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_q (mpfr_t rop, mpq_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_set_f (mpfr_t rop, mpf_t op, mpfr_rnd_t rnd)` [Function]  
 Set the value of `rop` from `op`, rounded toward the given direction `rnd`. Note that the input 0 is converted to +0 by `mpfr_set_ui`, `mpfr_set_si`, `mpfr_set_uj`, `mpfr_set_sj`,

`mpfr_set_z`, `mpfr_set_q` and `mpfr_set_f`, regardless of the rounding mode. If the system does not support the IEEE 754 standard, `mpfr_setflt`, `mpfr_set_d`, `mpfr_set_ld` and `mpfr_set_decimal64` might not preserve the signed zeros. The `mpfr_set_decimal64` function is built only with the configure option ‘`--enable-decimal-float`’, which also requires ‘`--with-gmp-build`’, and when the compiler or system provides the ‘`_Decimal64`’ data type (recent versions of GCC support this data type); to use `mpfr_set_decimal64`, one should define the macro `MPFR_WANT_DECIMAL_FLOATS` before including `mpfr.h`. `mpfr_set_q` might fail if the numerator (or the denominator) can not be represented as a `mpfr_t`.

Note: If you want to store a floating-point constant to a `mpfr_t`, you should use `mpfr_set_str` (or one of the MPFR constant functions, such as `mpfr_const_pi` for  $\pi$ ) instead of `mpfr_setflt`, `mpfr_set_d`, `mpfr_set_ld` or `mpfr_set_decimal64`. Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary (or decimal, for `mpfr_set_decimal64`) number before MPFR can work with it.

`int mpfr_set_ui_2exp (mpfr_t rop, unsigned long int op, mpfr_exp_t e, mpfr_rnd_t rnd)` [Function]

`int mpfr_set_si_2exp (mpfr_t rop, long int op, mpfr_exp_t e, mpfr_rnd_t rnd)` [Function]

`int mpfr_set_uj_2exp (mpfr_t rop, uintmax_t op, intmax_t e, mpfr_rnd_t rnd)` [Function]

`int mpfr_set_sj_2exp (mpfr_t rop, intmax_t op, intmax_t e, mpfr_rnd_t rnd)` [Function]

`int mpfr_set_z_2exp (mpfr_t rop, mpz_t op, mpfr_exp_t e, mpfr_rnd_t rnd)` [Function]

Set the value of `rop` from  $op \times 2^e$ , rounded toward the given direction `rnd`. Note that the input 0 is converted to +0.

`int mpfr_set_str (mpfr_t rop, const char *s, int base, mpfr_rnd_t rnd)` [Function]

Set `rop` to the value of the string `s` in base `base`, rounded in the direction `rnd`. See the documentation of `mpfr_strtofr` for a detailed description of the valid string formats. Contrary to `mpfr_strtofr`, `mpfr_set_str` requires the *whole* string to represent a valid floating-point number.

The meaning of the return value differs from other MPFR functions: it is 0 if the entire string up to the final null character is a valid number in base `base`; otherwise it is `-1`, and `rop` may have changed (users interested in the [\[ternary value\]](#), page 8 should use `mpfr_strtofr` instead).

Note: it is preferable to use `mpfr_strtofr` if one wants to distinguish between an infinite `rop` value coming from an infinite `s` or from an overflow.

`int mpfr_strtofr (mpfr_t rop, const char *nptr, char **endptr, int base, mpfr_rnd_t rnd)` [Function]

Read a floating-point number from a string `nptr` in base `base`, rounded in the direction `rnd`; `base` must be either 0 (to detect the base, as described below) or a number from 2 to 62 (otherwise the behavior is undefined). If `nptr` starts with valid data, the result is stored in `rop` and `*endptr` points to the character just after the valid data (if `endptr` is not a null pointer); otherwise `rop` is set to zero (for consistency with `strtod`) and the value of `nptr` is stored in the location referenced by `endptr` (if `endptr` is not a null pointer). The usual ternary value is returned.

Parsing follows the standard C `strtod` function with some extensions. After optional leading whitespace, one has a subject sequence consisting of an optional sign (+ or -), and either nu-

meric data or special data. The subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-whitespace character, that is of the expected form.

The form of numeric data is a non-empty sequence of significand digits with an optional decimal point, and an optional exponent consisting of an exponent prefix followed by an optional sign and a non-empty sequence of decimal digits. A significand digit is either a decimal digit or a Latin letter (62 possible characters), with `A = 10`, `B = 11`, `...`, `Z = 35`; case is ignored in bases less or equal to 36, in bases larger than 36, `a = 36`, `b = 37`, `...`, `z = 61`. The value of a significand digit must be strictly less than the base. The decimal point can be either the one defined by the current locale or the period (the first one is accepted for consistency with the C standard and the practice, the second one is accepted to allow the programmer to provide MPFR numbers from strings in a way that does not depend on the current locale). The exponent prefix can be `e` or `E` for bases up to 10, or `@` in any base; it indicates a multiplication by a power of the base. In bases 2 and 16, the exponent prefix can also be `p` or `P`, in which case the exponent, called *binary exponent*, indicates a multiplication by a power of 2 instead of the base (there is a difference only for base 16); in base 16 for example `1p2` represents 4 whereas `1@2` represents 256. The value of an exponent is always written in base 10.

If the argument `base` is 0, then the base is automatically detected as follows. If the significand starts with `0b` or `0B`, base 2 is assumed. If the significand starts with `0x` or `0X`, base 16 is assumed. Otherwise base 10 is assumed.

Note: The exponent (if present) must contain at least a digit. Otherwise the possible exponent prefix and sign are not part of the number (which ends with the significand). Similarly, if `0b`, `0B`, `0x` or `0X` is not followed by a binary/hexadecimal digit, then the subject sequence stops at the character 0, thus 0 is read.

Special data (for infinities and NaN) can be `@inf@` or `@nan@(n-char-sequence-opt)`, and if `base ≤ 16`, it can also be `infinity`, `inf`, `nan` or `nan(n-char-sequence-opt)`, all case insensitive. A `n-char-sequence-opt` is a possibly empty string containing only digits, Latin letters and the underscore (0, 1, 2, `...`, 9, a, b, `...`, z, A, B, `...`, Z, `_`). Note: one has an optional sign for all data, even NaN. For example, `-@NaN@(This_Is_Not_17)` is a valid representation for NaN in base 17.

```
void mpfr_set_nan (mpfr_t x) [Function]
void mpfr_set_inf (mpfr_t x, int sign) [Function]
void mpfr_set_zero (mpfr_t x, int sign) [Function]
```

Set the variable `x` to NaN (Not-a-Number), infinity or zero respectively. In `mpfr_set_inf` or `mpfr_set_zero`, `x` is set to plus infinity or plus zero iff `sign` is nonnegative; in `mpfr_set_nan`, the sign bit of the result is unspecified.

```
void mpfr_swap (mpfr_t x, mpfr_t y) [Function]
```

Swap the structures pointed to by `x` and `y`. In particular, the values are exchanged without rounding (this may be different from three `mpfr_set` calls using a third auxiliary variable).

Warning! Since the precisions are exchanged, this will affect future assignments. Moreover, since the significand pointers are also exchanged, you must not use this function if the allocation method used for `x` and/or `y` does not permit it. This is the case when `x` and/or `y` were declared and initialized with `MPFR_DECL_INIT`, and possibly with `mpfr_custom_init_set` (see [Section 5.15 \[Custom Interface\]](#), page 37).

### 5.3 Combined Initialization and Assignment Functions

```
int mpfr_init_set (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_si (mpfr_t rop, long int op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_d (mpfr_t rop, double op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_ld (mpfr_t rop, long double op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_z (mpfr_t rop, mpz_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_q (mpfr_t rop, mpq_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_f (mpfr_t rop, mpf_t op, mpfr_rnd_t rnd) [Macro]
```

Initialize *rop* and set its value from *op*, rounded in the direction *rnd*. The precision of *rop* will be taken from the active default precision, as set by `mpfr_set_default_prec`.

```
int mpfr_init_set_str (mpfr_t x, const char *s, int base, mpfr_rnd_t rnd) [Function]
    Initialize x and set its value from the string s in base base, rounded in the direction rnd. See mpfr_set_str.
```

### 5.4 Conversion Functions

```
float mpfr_get_flt (mpfr_t op, mpfr_rnd_t rnd) [Function]
double mpfr_get_d (mpfr_t op, mpfr_rnd_t rnd) [Function]
long double mpfr_get_ld (mpfr_t op, mpfr_rnd_t rnd) [Function]
_Decimal64 mpfr_get_decimal64 (mpfr_t op, mpfr_rnd_t rnd) [Function]
```

Convert *op* to a float (respectively double, long double or `_Decimal64`), using the rounding mode *rnd*. If *op* is NaN, some fixed NaN (either quiet or signaling) or the result of 0.0/0.0 is returned. If *op* is  $\pm\text{Inf}$ , an infinity of the same sign or the result of  $\pm 1.0/0.0$  is returned. If *op* is zero, these functions return a zero, trying to preserve its sign, if possible. The `mpfr_get_decimal64` function is built only under some conditions: see the documentation of `mpfr_set_decimal64`.

```
long mpfr_get_si (mpfr_t op, mpfr_rnd_t rnd) [Function]
unsigned long mpfr_get_ui (mpfr_t op, mpfr_rnd_t rnd) [Function]
intmax_t mpfr_get_sj (mpfr_t op, mpfr_rnd_t rnd) [Function]
uintmax_t mpfr_get_uj (mpfr_t op, mpfr_rnd_t rnd) [Function]
```

Convert *op* to a long, an unsigned long, an `intmax_t` or an `uintmax_t` (respectively) after rounding it with respect to *rnd*. If *op* is NaN, 0 is returned and the *erange* flag is set. If *op* is too big for the return type, the function returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow; the *erange* flag is set too. See also `mpfr_fits_slong_p`, `mpfr_fits_ulong_p`, `mpfr_fits_intmax_p` and `mpfr_fits_uintmax_p`.

```
double mpfr_get_d_2exp (long *exp, mpfr_t op, mpfr_rnd_t rnd) [Function]
long double mpfr_get_ld_2exp (long *exp, mpfr_t op, mpfr_rnd_t rnd) [Function]
```

Return *d* and set *exp* (formally, the value pointed to by *exp*) such that  $0.5 \leq |d| < 1$  and  $d \times 2^{\text{exp}}$  equals *op* rounded to double (resp. long double) precision, using the given rounding mode. If *op* is zero, then a zero of the same sign (or an unsigned zero, if the implementation does not have signed zeros) is returned, and *exp* is set to 0. If *op* is NaN or an infinity, then the corresponding double precision (resp. long-double precision) value is returned, and *exp* is undefined.

```
int mpfr_frexp (mpfr_exp_t *exp, mpfr_t y, mpfr_t x, mpfr_rnd_t rnd) [Function]
    Set exp (formally, the value pointed to by exp) and y such that  $0.5 \leq |y| < 1$  and  $y \times 2^{\text{exp}}$  equals x rounded to the precision of y, using the given rounding mode. If x is zero, then y
```

is set to a zero of the same sign and *exp* is set to 0. If *x* is NaN or an infinity, then *y* is set to the same value and *exp* is undefined.

`mpfr_exp_t mpfr_get_z_2exp (mpz_t rop, mpfr_t op)` [Function]  
 Put the scaled significand of *op* (regarded as an integer, with the precision of *op*) into *rop*, and return the exponent *exp* (which may be outside the current exponent range) such that *op* exactly equals  $rop \times 2^{\text{exp}}$ . If *op* is zero, the minimal exponent `emin` is returned. If *op* is NaN or an infinity, the *erange* flag is set, *rop* is set to 0, and the the minimal exponent `emin` is returned. The returned exponent may be less than the minimal exponent `emin` of MPFR numbers in the current exponent range; in case the exponent is not representable in the `mpfr_exp_t` type, the *erange* flag is set and the minimal value of the `mpfr_exp_t` type is returned.

`int mpfr_get_z (mpz_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Convert *op* to a `mpz_t`, after rounding it with respect to *rnd*. If *op* is NaN or an infinity, the *erange* flag is set, *rop* is set to 0, and 0 is returned.

`int mpfr_get_f (mpf_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Convert *op* to a `mpf_t`, after rounding it with respect to *rnd*. The *erange* flag is set if *op* is NaN or an infinity, which do not exist in MPF. If *op* is NaN, then *rop* is undefined. If *op* is +Inf (resp. -Inf), then *rop* is set to the maximum (resp. minimum) value in the precision of the MPF number; if a future MPF version supports infinities, this behavior will be considered incorrect and will change (portable programs should assume that *rop* is set either to this finite number or to an infinite number). Note that since MPFR currently has the same exponent type as MPF (but not with the same radix), the range of values is much larger in MPF than in MPFR, so that an overflow or underflow is not possible.

`char * mpfr_get_str (char *str, mpfr_exp_t *exp_ptr, int b, size_t n, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Convert *op* to a string of digits in base *b*, with rounding in the direction *rnd*, where *n* is either zero (see below) or the number of significant digits output in the string; in the latter case, *n* must be greater or equal to 2. The base may vary from 2 to 62; otherwise the function does nothing and immediately returns a null pointer. If the input number is an ordinary number, the exponent is written through the pointer *exp\_ptr* (for input 0, the current minimal exponent is written); the type `mpfr_exp_t` is large enough to hold the exponent in all cases.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number  $-3.1416$  would be returned as `"-31416"` in the string and 1 written at *exp\_ptr*. If *rnd* is to nearest, and *op* is exactly in the middle of two consecutive possible outputs, the one with an even significand is chosen, where both significands are considered with the exponent of *op*. Note that for an odd base, this may not correspond to an even last digit: for example with 2 digits in base 7, (14) and a half is rounded to (15) which is 12 in decimal, (16) and a half is rounded to (20) which is 14 in decimal, and (26) and a half is rounded to (26) which is 20 in decimal.

If *n* is zero, the number of digits of the significand is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding to nearest, will recover the original value of *op*. More precisely, in most cases, the chosen precision of *str* is the minimal precision *m* depending only on  $p = \text{PREC}(op)$  and *b* that satisfies the above property, i.e.,  $m = 1 + \lceil p \frac{\log 2}{\log b} \rceil$ , with *p* replaced by *p*-1 if *b* is a power of 2, but in some very rare cases, it might be *m* + 1 (the smallest case for bases up to 62 is when *p* equals 186564318007 for bases 7 and 49).

If *str* is a null pointer, space for the significand is allocated using the current allocation function and a pointer to the string is returned (unless the base is invalid). To free the returned string, you must use `mpfr_free_str`.

If *str* is not a null pointer, it should point to a block of storage large enough for the significand, i.e., at least  $\max(n + 2, 7)$ . The extra two bytes are for a possible minus sign, and for the terminating null character, and the value 7 accounts for `-@Inf@` plus the terminating null character. The pointer to the string *str* is returned (unless the base is invalid).

Note: The NaN and inexact flags are currently not set when need be; this will be fixed in future versions. Programmers should currently assume that whether the flags are set by this function is unspecified.

```
void mpfr_free_str (char *str) [Function]
    Free a string allocated by mpfr_get_str using the current unallocation function. The block is assumed to be strlen(str)+1 bytes. For more information about how it is done: see Section “Custom Allocation” in GNU MP.
```

```
int mpfr_fits_ulong_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_fits_slong_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_fits_uint_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_fits_sint_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_fits_ushort_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_fits_sshort_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_fits_uintmax_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_fits_intmax_p (mpfr_t op, mpfr_rnd_t rnd) [Function]
    Return non-zero if op would fit in the respective C data type, respectively unsigned long, long, unsigned int, int, unsigned short, short, uintmax_t, intmax_t, when rounded to an integer in the direction rnd.
```

## 5.5 Basic Arithmetic Functions

```
int mpfr_add (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_add_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, [Function]
                mpfr_rnd_t rnd)
int mpfr_add_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_add_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd) [Function]
int mpfr_add_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_add_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd) [Function]
    Set rop to op1 + op2 rounded in the direction rnd. For types having no signed zero, it is considered unsigned (i.e.,  $(+0) + 0 = (+0)$  and  $(-0) + 0 = (-0)$ ). The mpfr_add_d function assumes that the radix of the double type is a power of 2, with a precision at most that declared by the C implementation (macro IEEE_DBL_MANT_DIG, and if not defined 53 bits).
```

```
int mpfr_sub (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_ui_sub (mpfr_t rop, unsigned long int op1, mpfr_t op2, [Function]
                mpfr_rnd_t rnd)
int mpfr_sub_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, [Function]
                mpfr_rnd_t rnd)
int mpfr_si_sub (mpfr_t rop, long int op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_sub_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_d_sub (mpfr_t rop, double op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_sub_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd) [Function]
int mpfr_z_sub (mpfr_t rop, mpz_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
```

`int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to  $op1 - op2$  rounded in the direction *rnd*. For types having no signed zero, it is considered unsigned (i.e.,  $(+0) - 0 = (+0)$ ,  $(-0) - 0 = (-0)$ ,  $0 - (+0) = (-0)$  and  $0 - (-0) = (+0)$ ). The same restrictions than for `mpfr_add_d` apply to `mpfr_d_sub` and `mpfr_sub_d`.

`int mpfr_mul (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_mul_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_mul_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_mul_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_mul_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_mul_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to  $op1 \times op2$  rounded in the direction *rnd*. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zero, it is considered positive). The same restrictions than for `mpfr_add_d` apply to `mpfr_mul_d`.

`int mpfr_sqr (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to  $op^2$  rounded in the direction *rnd*.

`int mpfr_div (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_ui_div (mpfr_t rop, unsigned long int op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_div_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_si_div (mpfr_t rop, long int op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_div_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_d_div (mpfr_t rop, double op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_div_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_div_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_div_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to  $op1/op2$  rounded in the direction *rnd*. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zero, it is considered positive). The same restrictions than for `mpfr_add_d` apply to `mpfr_d_div` and `mpfr_div_d`.

`int mpfr_sqrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_sqrt_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to  $\sqrt{op}$  rounded in the direction *rnd* (set *rop* to  $-0$  if *op* is  $-0$ , to be consistent with the IEEE 754 standard). Set *rop* to NaN if *op* is negative.

`int mpfr_rec_sqrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to  $1/\sqrt{op}$  rounded in the direction *rnd*. Set *rop* to  $+\text{Inf}$  if *op* is  $\pm 0$ ,  $+0$  if *op* is  $+\text{Inf}$ , and NaN if *op* is negative.

`int mpfr_cbrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_root (mpfr_t rop, mpfr_t op, unsigned long int k, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the cubic root (resp. the *k*th root) of *op* rounded in the direction *rnd*. For *k* odd (resp. even) and *op* negative (including  $-\text{Inf}$ ), set *rop* to a negative number (resp. NaN). The *k*th root of  $-0$  is defined to be  $-0$ , whatever the parity of *k*.

`int mpfr_pow (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]  
`int mpfr_pow_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)` [Function]

```

int mpfr_pow_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)      [Function]
int mpfr_pow_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)      [Function]
int mpfr_ui_pow_ui (mpfr_t rop, unsigned long int op1, unsigned long int  [Function]
                    op2, mpfr_rnd_t rnd)
int mpfr_ui_pow (mpfr_t rop, unsigned long int op1, mpfr_t op2,          [Function]
                 mpfr_rnd_t rnd)

```

Set *rop* to  $op1^{op2}$ , rounded in the direction *rnd*. Special values are handled as described in the ISO C99 and IEEE 754-2008 standards for the `pow` function:

- `pow( $\pm 0$ , y)` returns plus or minus infinity for *y* a negative odd integer.
- `pow( $\pm 0$ , y)` returns plus infinity for *y* negative and not an odd integer.
- `pow( $\pm 0$ , y)` returns plus or minus zero for *y* a positive odd integer.
- `pow( $\pm 0$ , y)` returns plus zero for *y* positive and not an odd integer.
- `pow(-1,  $\pm$ Inf)` returns 1.
- `pow(+1, y)` returns 1 for any *y*, even a NaN.
- `pow(x,  $\pm 0$ )` returns 1 for any *x*, even a NaN.
- `pow(x, y)` returns NaN for finite negative *x* and finite non-integer *y*.
- `pow(x, -Inf)` returns plus infinity for  $0 < |x| < 1$ , and plus zero for  $|x| > 1$ .
- `pow(x, +Inf)` returns plus zero for  $0 < |x| < 1$ , and plus infinity for  $|x| > 1$ .
- `pow(-Inf, y)` returns minus zero for *y* a negative odd integer.
- `pow(-Inf, y)` returns plus zero for *y* negative and not an odd integer.
- `pow(-Inf, y)` returns minus infinity for *y* a positive odd integer.
- `pow(-Inf, y)` returns plus infinity for *y* positive and not an odd integer.
- `pow(+Inf, y)` returns plus zero for *y* negative, and plus infinity for *y* positive.

```

int mpfr_neg (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)                      [Function]
int mpfr_abs (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)                      [Function]

```

Set *rop* to  $-op$  and the absolute value of *op* respectively, rounded in the direction *rnd*. Just changes or adjusts the sign if *rop* and *op* are the same variable, otherwise a rounding might occur if the precision of *rop* is less than that of *op*.

```

int mpfr_dim (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)        [Function]

```

Set *rop* to the positive difference of *op1* and *op2*, i.e.,  $op1 - op2$  rounded in the direction *rnd* if  $op1 > op2$ , +0 if  $op1 \leq op2$ , and NaN if *op1* or *op2* is NaN.

```

int mpfr_mul_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2,         [Function]
                 mpfr_rnd_t rnd)

```

```

int mpfr_mul_2si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)  [Function]

```

Set *rop* to  $op1 \times 2^{op2}$  rounded in the direction *rnd*. Just increases the exponent by *op2* when *rop* and *op1* are identical.

```

int mpfr_div_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2,         [Function]
                 mpfr_rnd_t rnd)

```

```

int mpfr_div_2si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)  [Function]

```

Set *rop* to  $op1/2^{op2}$  rounded in the direction *rnd*. Just decreases the exponent by *op2* when *rop* and *op1* are identical.



## 5.6 Comparison Functions

```

int mpfr_cmp (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_cmp_ui (mpfr_t op1, unsigned long int op2) [Function]
int mpfr_cmp_si (mpfr_t op1, long int op2) [Function]
int mpfr_cmp_d (mpfr_t op1, double op2) [Function]
int mpfr_cmp_ld (mpfr_t op1, long double op2) [Function]
int mpfr_cmp_z (mpfr_t op1, mpz_t op2) [Function]
int mpfr_cmp_q (mpfr_t op1, mpq_t op2) [Function]
int mpfr_cmp_f (mpfr_t op1, mpf_t op2) [Function]

```

Compare  $op1$  and  $op2$ . Return a positive value if  $op1 > op2$ , zero if  $op1 = op2$ , and a negative value if  $op1 < op2$ . Both  $op1$  and  $op2$  are considered to their full own precision, which may differ. If one of the operands is NaN, set the *erange* flag and return zero.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., `mpfr_equal_p` for the equality) described below; they behave like the IEEE 754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).

```

int mpfr_cmp_ui_2exp (mpfr_t op1, unsigned long int op2, mpfr_exp_t e) [Function]
int mpfr_cmp_si_2exp (mpfr_t op1, long int op2, mpfr_exp_t e) [Function]

```

Compare  $op1$  and  $op2 \times 2^e$ . Similar as above.

```

int mpfr_cmpabs (mpfr_t op1, mpfr_t op2) [Function]

```

Compare  $|op1|$  and  $|op2|$ . Return a positive value if  $|op1| > |op2|$ , zero if  $|op1| = |op2|$ , and a negative value if  $|op1| < |op2|$ . If one of the operands is NaN, set the *erange* flag and return zero.

```

int mpfr_nan_p (mpfr_t op) [Function]
int mpfr_inf_p (mpfr_t op) [Function]
int mpfr_number_p (mpfr_t op) [Function]
int mpfr_zero_p (mpfr_t op) [Function]
int mpfr_regular_p (mpfr_t op) [Function]

```

Return non-zero if  $op$  is respectively NaN, an infinity, an ordinary number (i.e., neither NaN nor an infinity), zero, or a regular number (i.e., neither NaN, nor an infinity nor zero). Return zero otherwise.

```

int mpfr_sgn (mpfr_t op) [Macro]

```

Return a positive value if  $op > 0$ , zero if  $op = 0$ , and a negative value if  $op < 0$ . If the operand is NaN, set the *erange* flag and return zero. This is equivalent to `mpfr_cmp_ui (op, 0)`, but more efficient.

```

int mpfr_greater_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_greaterequal_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_less_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_lessequal_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_equal_p (mpfr_t op1, mpfr_t op2) [Function]

```

Return non-zero if  $op1 > op2$ ,  $op1 \geq op2$ ,  $op1 < op2$ ,  $op1 \leq op2$ ,  $op1 = op2$  respectively, and zero otherwise. Those functions return zero whenever  $op1$  and/or  $op2$  is NaN.

`int mpfr_lessgreater_p (mpfr_t op1, mpfr_t op2)` [Function]  
 Return non-zero if  $op1 < op2$  or  $op1 > op2$  (i.e., neither  $op1$ , nor  $op2$  is NaN, and  $op1 \neq op2$ ), zero otherwise (i.e.,  $op1$  and/or  $op2$  is NaN, or  $op1 = op2$ ).

`int mpfr_unordered_p (mpfr_t op1, mpfr_t op2)` [Function]  
 Return non-zero if  $op1$  or  $op2$  is a NaN (i.e., they cannot be compared), zero otherwise.

## 5.7 Special Functions

All those functions, except explicitly stated (for example `mpfr_sin_cos`), return a [ternary value], page 8, i.e., zero for an exact return value, a positive value for a return value larger than the exact result, and a negative value otherwise.

Important note: in some domains, computing special functions (either with correct or incorrect rounding) is expensive, even for small precision, for example the trigonometric and Bessel functions for large argument.

`int mpfr_log (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_log2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_log10 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set  $rop$  to the natural logarithm of  $op$ ,  $\log_2 op$  or  $\log_{10} op$ , respectively, rounded in the direction  $rnd$ . Set  $rop$  to  $-\text{Inf}$  if  $op$  is  $-0$  (i.e., the sign of the zero has no influence on the result).

`int mpfr_exp (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_exp2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_exp10 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set  $rop$  to the exponential of  $op$ , to  $2^{op}$  or to  $10^{op}$ , respectively, rounded in the direction  $rnd$ .

`int mpfr_cos (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_sin (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_tan (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set  $rop$  to the cosine of  $op$ , sine of  $op$ , tangent of  $op$ , rounded in the direction  $rnd$ .

`int mpfr_sin_cos (mpfr_t sop, mpfr_t cop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set simultaneously  $sop$  to the sine of  $op$  and  $cop$  to the cosine of  $op$ , rounded in the direction  $rnd$  with the corresponding precisions of  $sop$  and  $cop$ , which must be different variables. Return 0 iff both results are exact, more precisely it returns  $s + 4c$  where  $s = 0$  if  $sop$  is exact,  $s = 1$  if  $sop$  is larger than the sine of  $op$ ,  $s = 2$  if  $sop$  is smaller than the sine of  $op$ , and similarly for  $c$  and the cosine of  $op$ .

`int mpfr_sec (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_csc (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_cot (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set  $rop$  to the secant of  $op$ , cosecant of  $op$ , cotangent of  $op$ , rounded in the direction  $rnd$ .

`int mpfr_acos (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_asin (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_atan (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

Set  $rop$  to the arc-cosine, arc-sine or arc-tangent of  $op$ , rounded in the direction  $rnd$ . Note that since `acos(-1)` returns the floating-point number closest to  $\pi$  according to the given rounding mode, this number might not be in the output range  $0 \leq rop < \pi$  of the arc-cosine function; still, the result lies in the image of the output range by the rounding function. The

same holds for `asin(-1)`, `asin(1)`, `atan(-Inf)`, `atan(+Inf)` or for `atan(op)` with large `op` and small precision of `rop`.

`int mpfr_atan2 (mpfr_t rop, mpfr_t y, mpfr_t x, mpfr_rnd_t rnd)` [Function]

Set `rop` to the arc-tangent2 of `y` and `x`, rounded in the direction `rnd`: if `x > 0`, `atan2(y, x) = atan (y/x)`; if `x < 0`, `atan2(y, x) = sign(y)*(Pi - atan (|y/x|))`, thus a number from  $-\pi$  to  $\pi$ . As for `atan`, in case the exact mathematical result is  $+\pi$  or  $-\pi$ , its rounded result might be outside the function output range.

`atan2(y, 0)` does not raise any floating-point exception. Special values are handled as described in the ISO C99 and IEEE 754-2008 standards for the `atan2` function:

- `atan2(+0, -0)` returns  $+\pi$ .
- `atan2(-0, -0)` returns  $-\pi$ .
- `atan2(+0, +0)` returns  $+0$ .
- `atan2(-0, +0)` returns  $-0$ .
- `atan2(+0, x)` returns  $+\pi$  for  $x < 0$ .
- `atan2(-0, x)` returns  $-\pi$  for  $x < 0$ .
- `atan2(+0, x)` returns  $+0$  for  $x > 0$ .
- `atan2(-0, x)` returns  $-0$  for  $x > 0$ .
- `atan2(y, 0)` returns  $-\pi/2$  for  $y < 0$ .
- `atan2(y, 0)` returns  $+\pi/2$  for  $y > 0$ .
- `atan2(+Inf, -Inf)` returns  $+3\pi/4$ .
- `atan2(-Inf, -Inf)` returns  $-3\pi/4$ .
- `atan2(+Inf, +Inf)` returns  $+\pi/4$ .
- `atan2(-Inf, +Inf)` returns  $-\pi/4$ .
- `atan2(+Inf, x)` returns  $+\pi/2$  for finite  $x$ .
- `atan2(-Inf, x)` returns  $-\pi/2$  for finite  $x$ .
- `atan2(y, -Inf)` returns  $+\pi$  for finite  $y > 0$ .
- `atan2(y, -Inf)` returns  $-\pi$  for finite  $y < 0$ .
- `atan2(y, +Inf)` returns  $+0$  for finite  $y > 0$ .
- `atan2(y, +Inf)` returns  $-0$  for finite  $y < 0$ .

`int mpfr_cosh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_sinh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_tanh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

Set `rop` to the hyperbolic cosine, sine or tangent of `op`, rounded in the direction `rnd`.

`int mpfr_sinh_cosh (mpfr_t sop, mpfr_t cop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

Set simultaneously `sop` to the hyperbolic sine of `op` and `cop` to the hyperbolic cosine of `op`, rounded in the direction `rnd` with the corresponding precision of `sop` and `cop`, which must be different variables. Return 0 iff both results are exact (see `mpfr_sin_cos` for a more detailed description of the return value).

`int mpfr_sech (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_csch (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_coth (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

Set `rop` to the hyperbolic secant of `op`, cosecant of `op`, cotangent of `op`, rounded in the direction `rnd`.

`int mpfr_acosh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_asinh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_atanh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the inverse hyperbolic cosine, sine or tangent of *op*, rounded in the direction *rnd*.

`int mpfr_fac_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the factorial of *op*, rounded in the direction *rnd*.

`int mpfr_log1p (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the logarithm of one plus *op*, rounded in the direction *rnd*.

`int mpfr_expml (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to  $e^{op} - 1$ , rounded in the direction *rnd*.

`int mpfr_eint (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the exponential integral of *op*, rounded in the direction *rnd*. For positive *op*, the exponential integral is the sum of Euler's constant, of the logarithm of *op*, and of the sum for *k* from 1 to infinity of  $op^k / (k \cdot k!)$ . For negative *op*, *rop* is set to NaN (this definition for negative argument follows formula 5.1.2 from the Handbook of Mathematical Functions from Abramowitz and Stegun, a future version might use another definition).

`int mpfr_li2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to real part of the dilogarithm of *op*, rounded in the direction *rnd*. MPFR defines the dilogarithm function as  $-\int_{t=0}^{op} \log(1-t)/t dt$ .

`int mpfr_gamma (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the value of the Gamma function on *op*, rounded in the direction *rnd*. When *op* is a negative integer, *rop* is set to NaN.

`int mpfr_lngamma (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the value of the logarithm of the Gamma function on *op*, rounded in the direction *rnd*. When  $-2k - 1 \leq op \leq -2k$ , *k* being a non-negative integer, *rop* is set to NaN. See also `mpfr_lgamma`.

`int mpfr_lgamma (mpfr_t rop, int *signp, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the value of the logarithm of the absolute value of the Gamma function on *op*, rounded in the direction *rnd*. The sign (1 or -1) of  $\Gamma(op)$  is returned in the object pointed to by *signp*. When *op* is an infinity or a non-positive integer, set *rop* to +Inf. When *op* is NaN, -Inf or a negative integer, *\*signp* is undefined, and when *op* is  $\pm 0$ , *\*signp* is the sign of the zero.

`int mpfr_digamma (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the value of the Digamma (sometimes also called Psi) function on *op*, rounded in the direction *rnd*. When *op* is a negative integer, set *rop* to NaN.

`int mpfr_zeta (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_zeta_ui (mpfr_t rop, unsigned long op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the value of the Riemann Zeta function on *op*, rounded in the direction *rnd*.

`int mpfr_erf (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
`int mpfr_erfc (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]  
 Set *rop* to the value of the error function on *op* (resp. the complementary error function on *op*) rounded in the direction *rnd*.

`int mpfr_j0 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_j1 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_jn (mpfr_t rop, long n, mpfr_t op, mpfr_rnd_t rnd)` [Function]

Set *rop* to the value of the first kind Bessel function of order 0, (resp. 1 and *n*) on *op*, rounded in the direction *rnd*. When *op* is NaN, *rop* is always set to NaN. When *op* is plus or minus Infinity, *rop* is set to +0. When *op* is zero, and *n* is not zero, *rop* is set to +0 or -0 depending on the parity and sign of *n*, and the sign of *op*.

`int mpfr_y0 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_y1 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_yn (mpfr_t rop, long n, mpfr_t op, mpfr_rnd_t rnd)` [Function]

Set *rop* to the value of the second kind Bessel function of order 0 (resp. 1 and *n*) on *op*, rounded in the direction *rnd*. When *op* is NaN or negative, *rop* is always set to NaN. When *op* is +Inf, *rop* is set to +0. When *op* is zero, *rop* is set to +Inf or -Inf depending on the parity and sign of *n*.

`int mpfr_fma (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_rnd_t rnd)` [Function]

`int mpfr_fms (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_rnd_t rnd)` [Function]

Set *rop* to  $(op1 \times op2) + op3$  (resp.  $(op1 \times op2) - op3$ ) rounded in the direction *rnd*.

`int mpfr_agm (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]

Set *rop* to the arithmetic-geometric mean of *op1* and *op2*, rounded in the direction *rnd*. The arithmetic-geometric mean is the common limit of the sequences  $u_n$  and  $v_n$ , where  $u_0=op1$ ,  $v_0=op2$ ,  $u_{n+1}$  is the arithmetic mean of  $u_n$  and  $v_n$ , and  $v_{n+1}$  is the geometric mean of  $u_n$  and  $v_n$ . If any operand is negative, set *rop* to NaN.

`int mpfr_hypot (mpfr_t rop, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)` [Function]

Set *rop* to the Euclidean norm of *x* and *y*, i.e.,  $\sqrt{x^2 + y^2}$ , rounded in the direction *rnd*. Special values are handled as described in Section F.9.4.3 of the ISO C99 and IEEE 754-2008 standards: If *x* or *y* is an infinity, then +Inf is returned in *rop*, even if the other number is NaN.

`int mpfr_ai (mpfr_t rop, mpfr_t x, mpfr_rnd_t rnd)` [Function]

Set *rop* to the value of the Airy function Ai on *x*, rounded in the direction *rnd*. When *x* is NaN, *rop* is always set to NaN. When *x* is +Inf or -Inf, *rop* is +0. The current implementation is not intended to be used with large arguments. It works with  $-x$  typically smaller than 500. For larger arguments, other methods should be used and will be implemented in a future version.

`int mpfr_const_log2 (mpfr_t rop, mpfr_rnd_t rnd)` [Function]

`int mpfr_const_pi (mpfr_t rop, mpfr_rnd_t rnd)` [Function]

`int mpfr_const_euler (mpfr_t rop, mpfr_rnd_t rnd)` [Function]

`int mpfr_const_catalan (mpfr_t rop, mpfr_rnd_t rnd)` [Function]

Set *rop* to the logarithm of 2, the value of  $\pi$ , of Euler's constant 0.577..., of Catalan's constant 0.915..., respectively, rounded in the direction *rnd*. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use `mpfr_free_cache`.

`void mpfr_free_cache (void)` [Function]

Free various caches used by MPFR internally, in particular the caches used by the functions computing constants (`mpfr_const_log2`, `mpfr_const_pi`, `mpfr_const_euler` and `mpfr_`

`const_catalan`). You should call this function before terminating a thread, even if you did not call these functions directly (they could have been called internally).

```
int mpfr_sum (mpfr_t rop, mpfr_ptr const tab[], unsigned long int n,          [Function]
              mpfr_rnd_t rnd)
```

Set `rop` to the sum of all elements of `tab`, whose size is `n`, rounded in the direction `rnd`. Warning: for efficiency reasons, `tab` is an array of pointers to `mpfr_t`, not an array of `mpfr_t`. If the returned `int` value is zero, `rop` is guaranteed to be the exact sum; otherwise `rop` might be smaller than, equal to, or larger than the exact sum (in accordance to the rounding mode). However, `mpfr_sum` does guarantee the result is correctly rounded.

## 5.8 Input and Output Functions

This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a `stream` to any of these functions will make them read from `stdin` and write to `stdout`, respectively.

When using any of these functions, you must include the `<stdio.h>` standard header before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions.

```
size_t mpfr_out_str (FILE *stream, int base, size_t n, mpfr_t op,          [Function]
                    mpfr_rnd_t rnd)
```

Output `op` on stream `stream`, as a string of digits in base `base`, rounded in the direction `rnd`. The base may vary from 2 to 62. Print `n` significant digits exactly, or if `n` is 0, enough digits so that `op` can be read back exactly (see `mpfr_get_str`).

In addition to the significant digits, a decimal point (defined by the current locale) at the right of the first digit and a trailing exponent in base 10, in the form ‘eNNN’, are printed. If `base` is greater than 10, ‘@’ will be used instead of ‘e’ as exponent delimiter.

Return the number of characters written, or if an error occurred, return 0.

```
size_t mpfr_inp_str (mpfr_t rop, FILE *stream, int base, mpfr_rnd_t      [Function]
                    rnd)
```

Input a string in base `base` from stream `stream`, rounded in the direction `rnd`, and put the read float in `rop`.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using `mpfr_set_str`. See the documentation of `mpfr_strtofr` for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

## 5.9 Formatted Output Functions

### 5.9.1 Requirements

The class of `mpfr_printf` functions provides formatted output in a similar manner as the standard C `printf`. These functions are defined only if your system supports ISO C variadic functions and the corresponding argument access macros.

When using any of these functions, you must include the `<stdio.h>` standard header before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions.

## 5.9.2 Format String

The format specification accepted by `mpfr_printf` is an extension of the `printf` one. The conversion specification is of the form:

```
% [flags] [width] [.[precision]] [type] [rounding] conv
```

‘flags’, ‘width’, and ‘precision’ have the same meaning as for the standard `printf` (in particular, notice that the ‘precision’ is related to the number of digits displayed in the base chosen by ‘conv’ and not related to the internal precision of the `mpfr_t` variable). `mpfr_printf` accepts the same ‘type’ specifiers as GMP (except the non-standard and deprecated ‘q’, use ‘ll’ instead), namely the length modifiers defined in the C standard:

```
‘h’      short
‘hh’     char
‘j’      intmax_t or uintmax_t
‘l’      long or wchar_t
‘ll’     long long
‘L’      long double
‘t’      ptrdiff_t
‘z’      size_t
```

and the ‘type’ specifiers defined in GMP plus ‘R’ and ‘P’ specific to MPFR (the second column in the table below shows the type of the argument read in the argument list and the kind of ‘conv’ specifier to use after the ‘type’ specifier):

```
‘F’      mpf_t, float conversions
‘Q’      mpq_t, integer conversions
‘M’      mp_limb_t, integer conversions
‘N’      mp_limb_t array, integer conversions
‘Z’      mpz_t, integer conversions

‘P’      mpfr_prec_t, integer conversions
‘R’      mpfr_t, float conversions
```

The ‘type’ specifiers have the same restrictions as those mentioned in the GMP documentation: see Section “Formatted Output Strings” in *GNU MP*. In particular, the ‘type’ specifiers (except ‘R’ and ‘P’) are supported only if they are supported by `gmp_printf` in your GMP build; this implies that the standard specifiers, such as ‘t’, must *also* be supported by your C library if you want to use them.

The ‘rounding’ field is specific to `mpfr_t` arguments and should not be used with other types.

With conversion specification not involving ‘P’ and ‘R’ types, `mpfr_printf` behaves exactly as `gmp_printf`.

The ‘P’ type specifies that a following ‘d’, ‘i’, ‘o’, ‘u’, ‘x’, or ‘X’ conversion specifier applies to a `mpfr_prec_t` argument. It is needed because the `mpfr_prec_t` type does not necessarily correspond to an `int` or any fixed standard type. The ‘precision’ field specifies the minimum number of digits to appear. The default ‘precision’ is 1. For example:

```
mpfr_t x;
mpfr_prec_t p;
mpfr_init (x);
...
p = mpfr_get_prec (x);
```

```
mpfr_printf ("variable x with %Pu bits", p);
```

The ‘R’ type specifies that a following ‘a’, ‘A’, ‘b’, ‘e’, ‘E’, ‘f’, ‘F’, ‘g’, ‘G’, or ‘n’ conversion specifier applies to a `mpfr_t` argument. The ‘R’ type can be followed by a ‘rounding’ specifier denoted by one of the following characters:

‘U’	round toward plus infinity
‘D’	round toward minus infinity
‘Y’	round away from zero
‘Z’	round toward zero
‘N’	round to nearest (with ties to even)
‘*’	rounding mode indicated by the <code>mpfr_rnd_t</code> argument just before the corresponding <code>mpfr_t</code> variable.

The default rounding mode is rounding to nearest. The following three examples are equivalent:

```
mpfr_t x;
mpfr_init (x);
...
mpfr_printf ("% .128Rf", x);
mpfr_printf ("% .128RNf", x);
mpfr_printf ("% .128R*f", MPFR_RNDN, x);
```

Note that the rounding away from zero mode is specified with ‘Y’ because ISO C reserves the ‘A’ specifier for hexadecimal output (see below).

The output ‘conv’ specifiers allowed with `mpfr_t` parameter are:

‘a’ ‘A’	hex float, C99 style
‘b’	binary output
‘e’ ‘E’	scientific format float
‘f’ ‘F’	fixed point float
‘g’ ‘G’	fixed or scientific float

The conversion specifier ‘b’ which displays the argument in binary is specific to `mpfr_t` arguments and should not be used with other types. Other conversion specifiers have the same meaning as for a `double` argument.

In case of non-decimal output, only the significand is written in the specified base, the exponent is always displayed in decimal. Special values are always displayed as `nan`, `-inf`, and `inf` for ‘a’, ‘b’, ‘e’, ‘f’, and ‘g’ specifiers and `NAN`, `-INF`, and `INF` for ‘A’, ‘E’, ‘F’, and ‘G’ specifiers.

If the ‘precision’ field is not empty, the `mpfr_t` number is rounded to the given precision in the direction specified by the rounding mode. If the precision is zero with rounding to nearest mode and one of the following ‘conv’ specifiers: ‘a’, ‘A’, ‘b’, ‘e’, ‘E’, tie case is rounded to even when it lies between two consecutive values at the wanted precision which have the same exponent, otherwise, it is rounded away from zero. For instance, 85 is displayed as "8e+1" and 95 is displayed as "1e+2" with the format specification "% .0RNe". This also applies when the ‘g’ (resp. ‘G’) conversion specifier uses the ‘e’ (resp. ‘E’) style. If the precision is set to a value greater than the maximum value for an `int`, it will be silently reduced down to `INT_MAX`.

If the ‘precision’ field is empty (as in %Re or %.RE) with ‘conv’ specifier ‘e’ and ‘E’, the number is displayed with enough digits so that it can be read back exactly, assuming that the input and output variables have the same precision and that the input and output rounding modes are both rounding to nearest (as for `mpfr_get_str`). The default precision for an empty ‘precision’ field with ‘conv’ specifiers ‘f’, ‘F’, ‘g’, and ‘G’ is 6.



### 5.9.3 Functions

For all the following functions, if the number of characters which ought to be written appears to exceed the maximum limit for an `int`, nothing is written in the stream (resp. to `stdout`, to `buf`, to `str`), the function returns `-1`, sets the `erange` flag, and (in POSIX system only) `errno` is set to `E_OVERFLOW`.

`int mpfr_fprintf (FILE *stream, const char *template, ...)` [Function]

`int mpfr_vfprintf (FILE *stream, const char *template, va_list ap)` [Function]

Print to the stream `stream` the optional arguments under the control of the template string `template`. Return the number of characters written or a negative value if an error occurred.

`int mpfr_printf (const char *template, ...)` [Function]

`int mpfr_vprintf (const char *template, va_list ap)` [Function]

Print to `stdout` the optional arguments under the control of the template string `template`. Return the number of characters written or a negative value if an error occurred.

`int mpfr_sprintf (char *buf, const char *template, ...)` [Function]

`int mpfr_vsprintf (char *buf, const char *template, va_list ap)` [Function]

Form a null-terminated string corresponding to the optional arguments under the control of the template string `template`, and print it in `buf`. No overlap is permitted between `buf` and the other arguments. Return the number of characters written in the array `buf` *not counting* the terminating null character or a negative value if an error occurred.

`int mpfr_snprintf (char *buf, size_t n, const char *template, ...)` [Function]

`int mpfr_vsnprintf (char *buf, size_t n, const char *template, va_list ap)` [Function]

Form a null-terminated string corresponding to the optional arguments under the control of the template string `template`, and print it in `buf`. If `n` is zero, nothing is written and `buf` may be a null pointer, otherwise, the `n-1` first characters are written in `buf` and the `n`-th is a null character. Return the number of characters that would have been written had `n` be sufficiently large, *not counting* the terminating null character, or a negative value if an error occurred.

`int mpfr_asprintf (char **str, const char *template, ...)` [Function]

`int mpfr_vasprintf (char **str, const char *template, va_list ap)` [Function]

Write their output as a null terminated string in a block of memory allocated using the current allocation function. A pointer to the block is stored in `str`. The block of memory must be freed using `mpfr_free_str`. The return value is the number of characters written in the string, excluding the null-terminator, or a negative value if an error occurred.

## 5.10 Integer and Remainder Related Functions

`int mpfr_rint (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)` [Function]

`int mpfr_ceil (mpfr_t rop, mpfr_t op)` [Function]

`int mpfr_floor (mpfr_t rop, mpfr_t op)` [Function]

`int mpfr_round (mpfr_t rop, mpfr_t op)` [Function]

`int mpfr_trunc (mpfr_t rop, mpfr_t op)` [Function]

Set `rop` to `op` rounded to an integer. `mpfr_rint` rounds to the nearest representable integer in the given direction `rnd`, `mpfr_ceil` rounds to the next higher or equal representable integer, `mpfr_floor` to the next lower or equal representable integer, `mpfr_round` to the nearest representable integer, rounding halfway cases away from zero (as in the `roundTiesToAway` mode of IEEE 754-2008), and `mpfr_trunc` to the next representable integer toward zero.

The returned value is zero when the result is exact, positive when it is greater than the original value of *op*, and negative when it is smaller. More precisely, the returned value is 0 when *op* is an integer representable in *rop*, 1 or  $-1$  when *op* is an integer that is not representable in *rop*, 2 or  $-2$  when *op* is not an integer.

When *op* is NaN, the NaN flag is set as usual. In the other cases, the inexact flag is set when *rop* differs from *op*, following the ISO C99 rule for the `rint` function. If you want the behavior to be more like IEEE 754 / ISO TS 18661-1, i.e., the usual behavior where the round-to-integer function is regarded as any other mathematical function, you should use one the `mpfr_rint_*` functions instead (however it is not possible to round to nearest with the even rounding rule yet).

Note that `mpfr_round` is different from `mpfr_rint` called with the rounding to nearest mode (where halfway cases are rounded to an even integer or significand). Note also that no double rounding is performed; for instance, 10.5 (1010.1 in binary) is rounded by `mpfr_rint` with rounding to nearest to 12 (1100 in binary) in 2-bit precision, because the two enclosing numbers representable on two bits are 8 and 12, and the closest is 12. (If one first rounded to an integer, one would round 10.5 to 10 with even rounding, and then 10 would be rounded to 8 again with even rounding.)

```
int mpfr_rint_ceil (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)      [Function]
int mpfr_rint_floor (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)   [Function]
int mpfr_rint_round (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)   [Function]
int mpfr_rint_trunc (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)   [Function]
```

Set *rop* to *op* rounded to an integer. `mpfr_rint_ceil` rounds to the next higher or equal integer, `mpfr_rint_floor` to the next lower or equal integer, `mpfr_rint_round` to the nearest integer, rounding halfway cases away from zero, and `mpfr_rint_trunc` to the next integer toward zero. If the result is not representable, it is rounded in the direction *rnd*. The returned value is the ternary value associated with the considered round-to-integer function (regarded in the same way as any other mathematical function).

Contrary to `mpfr_rint`, those functions do perform a double rounding: first *op* is rounded to the nearest integer in the direction given by the function name, then this nearest integer (if not representable) is rounded in the given direction *rnd*. Thus these round-to-integer functions behave more like the other mathematical functions, i.e., the returned result is the correct rounding of the exact result of the function in the real numbers.

For example, `mpfr_rint_round` with rounding to nearest and a precision of two bits rounds 6.5 to 7 (halfway cases away from zero), then 7 is rounded to 8 by the round-even rule, despite the fact that 6 is also representable on two bits, and is closer to 6.5 than 8.

```
int mpfr_frac (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)          [Function]
```

Set *rop* to the fractional part of *op*, having the same sign as *op*, rounded in the direction *rnd* (unlike in `mpfr_rint`, *rnd* affects only how the exact fractional part is rounded, not how the fractional part is generated).

```
int mpfr_modf (mpfr_t iop, mpfr_t fop, mpfr_t op, mpfr_rnd_t rnd) [Function]
```

Set simultaneously *iop* to the integral part of *op* and *fop* to the fractional part of *op*, rounded in the direction *rnd* with the corresponding precision of *iop* and *fop* (equivalent to `mpfr_trunc(iop, op, rnd)` and `mpfr_frac(fop, op, rnd)`). The variables *iop* and *fop* must be different. Return 0 iff both results are exact (see `mpfr_sin_cos` for a more detailed description of the return value).

```
int mpfr_fmod (mpfr_t r, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)   [Function]
int mpfr_remainder (mpfr_t r, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd) [Function]
```

`int mpfr_remquo (mpfr_t r, long* q, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)` [Function]  
 Set  $r$  to the value of  $x - ny$ , rounded according to the direction  $rnd$ , where  $n$  is the integer quotient of  $x$  divided by  $y$ , defined as follows:  $n$  is rounded toward zero for `mpfr_fmod`, and to the nearest integer (ties rounded to even) for `mpfr_remainder` and `mpfr_remquo`.

Special values are handled as described in Section F.9.7.1 of the ISO C99 standard: If  $x$  is infinite or  $y$  is zero,  $r$  is NaN. If  $y$  is infinite and  $x$  is finite,  $r$  is  $x$  rounded to the precision of  $r$ . If  $r$  is zero, it has the sign of  $x$ . The return value is the ternary value corresponding to  $r$ .

Additionally, `mpfr_remquo` stores the low significant bits from the quotient  $n$  in  $*q$  (more precisely the number of bits in a `long` minus one), with the sign of  $x$  divided by  $y$  (except if those low bits are all zero, in which case zero is returned). Note that  $x$  may be so large in magnitude relative to  $y$  that an exact representation of the quotient is not practical. The `mpfr_remainder` and `mpfr_remquo` functions are useful for additive argument reduction.

`int mpfr_integer_p (mpfr_t op)` [Function]  
 Return non-zero iff  $op$  is an integer.

## 5.11 Rounding Related Functions

`void mpfr_set_default_rounding_mode (mpfr_rnd_t rnd)` [Function]  
 Set the default rounding mode to  $rnd$ . The default rounding mode is to nearest initially.

`mpfr_rnd_t mpfr_get_default_rounding_mode (void)` [Function]  
 Get the default rounding mode.

`int mpfr_prec_round (mpfr_t x, mpfr_prec_t prec, mpfr_rnd_t rnd)` [Function]  
 Round  $x$  according to  $rnd$  with precision  $prec$ , which must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined). If  $prec$  is greater or equal to the precision of  $x$ , then new space is allocated for the significand, and it is filled with zeros. Otherwise, the significand is rounded to precision  $prec$  with the given direction. In both cases, the precision of  $x$  is changed to  $prec$ .

Here is an example of how to use `mpfr_prec_round` to implement Newton's algorithm to compute the inverse of  $a$ , assuming  $x$  is already an approximation to  $n$  bits:

```
mpfr_set_prec (t, 2 * n);
mpfr_set (t, a, MPFR_RNDN);          /* round a to 2n bits */
mpfr_mul (t, t, x, MPFR_RNDN);      /* t is correct to 2n bits */
mpfr_ui_sub (t, 1, t, MPFR_RNDN);    /* high n bits cancel with 1 */
mpfr_prec_round (t, n, MPFR_RNDN);  /* t is correct to n bits */
mpfr_mul (t, t, x, MPFR_RNDN);      /* t is correct to n bits */
mpfr_prec_round (x, 2 * n, MPFR_RNDN); /* exact */
mpfr_add (x, x, t, MPFR_RNDN);      /* x is correct to 2n bits */
```

Warning! You must not use this function if  $x$  was initialized with `MPFR_DECL_INIT` or with `mpfr_custom_init_set` (see [Section 5.15 \[Custom Interface\]](#), page 37).

`int mpfr_can_round (mpfr_t b, mpfr_exp_t err, mpfr_rnd_t rnd1, mpfr_rnd_t rnd2, mpfr_prec_t prec)` [Function]

Assuming  $b$  is an approximation of an unknown number  $x$  in the direction  $rnd1$  with error at most two to the power  $E(b)-err$  where  $E(b)$  is the exponent of  $b$ , return a non-zero value if one is able to round correctly  $x$  to precision  $prec$  with the direction  $rnd2$ , and 0 otherwise (including for NaN and Inf). This function **does not modify** its arguments.

If *rnd1* is `MPFR_RNDN`, then the sign of the error is unknown, but its absolute value is the same, so that the possible range is twice as large as with a directed rounding for *rnd1*.

Note: if one wants to also determine the correct [ternary value], page 8 when rounding *b* to precision *prec* with rounding mode *rnd*, a useful trick is the following:

```
if (mpfr_can_round (b, err, MPFR_RNDN, MPFR_RNDZ,
    prec + (rnd == MPFR_RNDN)))
    ...
```

Indeed, if *rnd* is `MPFR_RNDN`, this will check if one can round to *prec*+1 bits with a directed rounding: if so, one can surely round to nearest to *prec* bits, and in addition one can determine the correct ternary value, which would not be the case when *b* is near from a value exactly representable on *prec* bits.

`mpfr_prec_t mpfr_min_prec (mpfr_t x)` [Function]

Return the minimal number of bits required to store the significand of *x*, and 0 for special values, including 0. (Warning: the returned value can be less than `MPFR_PREC_MIN`.)

The function name is subject to change.

`const char * mpfr_print_rnd_mode (mpfr_rnd_t rnd)` [Function]

Return a string ("`MPFR_RNDD`", "`MPFR_RNDU`", "`MPFR_RNDN`", "`MPFR_RNDZ`", "`MPFR_RNDA`") corresponding to the rounding mode *rnd*, or a null pointer if *rnd* is an invalid rounding mode.

## 5.12 Miscellaneous Functions

`void mpfr_nexttoward (mpfr_t x, mpfr_t y)` [Function]

If *x* or *y* is NaN, set *x* to NaN. If *x* and *y* are equal, *x* is unchanged. Otherwise, if *x* is different from *y*, replace *x* by the next floating-point number (with the precision of *x* and the current exponent range) in the direction of *y* (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow or overflow is generated.

`void mpfr_nextabove (mpfr_t x)` [Function]

`void mpfr_nextbelow (mpfr_t x)` [Function]

Equivalent to `mpfr_nexttoward` where *y* is plus infinity (resp. minus infinity).

`int mpfr_min (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]

`int mpfr_max (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]

Set *rop* to the minimum (resp. maximum) of *op1* and *op2*. If *op1* and *op2* are both NaN, then *rop* is set to NaN. If *op1* or *op2* is NaN, then *rop* is set to the numeric value. If *op1* and *op2* are zeros of different signs, then *rop* is set to  $-0$  (resp.  $+0$ ).

`int mpfr_urandomb (mpfr_t rop, gmp_randstate_t state)` [Function]

Generate a uniformly distributed random float in the interval  $0 \leq rop < 1$ . More precisely, the number can be seen as a float with a random non-normalized significand and exponent 0, which is then normalized (thus if *e* denotes the exponent after normalization, then the least  $-e$  significant bits of the significand are always 0).

Return 0, unless the exponent is not in the current exponent range, in which case *rop* is set to NaN and a non-zero value is returned (this should never happen in practice, except in very specific cases). The second argument is a `gmp_randstate_t` structure which should be created using the GMP `gmp_randinit` function (see the GMP manual).

Note: for a given version of MPFR, the returned value of *rop* and the new value of *state* (which controls further random values) do not depend on the machine word size.

**int mpfr\_urandom** (*mpfr\_t rop*, *gmp\_randstate\_t state*, *mpfr\_rnd\_t rnd*) [Function]

Generate a uniformly distributed random float. The floating-point number *rop* can be seen as if a random real number is generated according to the continuous uniform distribution on the interval  $[0, 1]$  and then rounded in the direction *rnd*.

The second argument is a `gmp_randstate_t` structure which should be created using the GMP `gmp_randinit` function (see the GMP manual).

Note: the note for `mpfr_urandomb` holds too. In addition, the exponent range and the rounding mode might have a side effect on the next random state.

**int mpfr\_grandom** (*mpfr\_t rop1*, *mpfr\_t rop2*, *gmp\_randstate\_t state*, *mpfr\_rnd\_t rnd*) [Function]

Generate two random floats according to a standard normal gaussian distribution. If *rop2* is a null pointer, then only one value is generated and stored in *rop1*.

The floating-point number *rop1* (and *rop2*) can be seen as if a random real number were generated according to the standard normal gaussian distribution and then rounded in the direction *rnd*.

The third argument is a `gmp_randstate_t` structure, which should be created using the GMP `gmp_randinit` function (see the GMP manual).

The combination of the ternary values is returned like with `mpfr_sin_cos`. If *rop2* is a null pointer, the second ternary value is assumed to be 0 (note that the encoding of the only ternary value is not the same as the usual encoding for functions that return only one result). Otherwise the ternary value of a random number is always non-zero.

Note: the note for `mpfr_urandomb` holds too. In addition, the exponent range and the rounding mode might have a side effect on the next random state.

**mpfr\_exp\_t mpfr\_get\_exp** (*mpfr\_t x*) [Function]

Return the exponent of *x*, assuming that *x* is a non-zero ordinary number and the significand is considered in  $[1/2, 1)$ . The behavior for NaN, infinity or zero is undefined.

**int mpfr\_set\_exp** (*mpfr\_t x*, *mpfr\_exp\_t e*) [Function]

Set the exponent of *x* if *e* is in the current exponent range, and return 0 (even if *x* is not a non-zero ordinary number); otherwise, return a non-zero value. The significand is assumed to be in  $[1/2, 1)$ .

**int mpfr\_signbit** (*mpfr\_t op*) [Function]

Return a non-zero value iff *op* has its sign bit set (i.e., if it is negative,  $-0$ , or a NaN whose representation has its sign bit set).

**int mpfr\_setsign** (*mpfr\_t rop*, *mpfr\_t op*, *int s*, *mpfr\_rnd\_t rnd*) [Function]

Set the value of *rop* from *op*, rounded toward the given direction *rnd*, then set (resp. clear) its sign bit if *s* is non-zero (resp. zero), even when *op* is a NaN.

**int mpfr\_copysign** (*mpfr\_t rop*, *mpfr\_t op1*, *mpfr\_t op2*, *mpfr\_rnd\_t rnd*) [Function]

Set the value of *rop* from *op1*, rounded toward the given direction *rnd*, then set its sign bit to that of *op2* (even when *op1* or *op2* is a NaN). This function is equivalent to `mpfr_setsign(rop, op1, mpfr_signbit(op2), rnd)`.

`const char * mpfr_get_version (void)` [Function]  
 Return the MPFR version, as a null-terminated string.

`MPFR_VERSION` [Macro]  
`MPFR_VERSION_MAJOR` [Macro]  
`MPFR_VERSION_MINOR` [Macro]  
`MPFR_VERSION_PATCHLEVEL` [Macro]  
`MPFR_VERSION_STRING` [Macro]

`MPFR_VERSION` is the version of MPFR as a preprocessing constant. `MPFR_VERSION_MAJOR`, `MPFR_VERSION_MINOR` and `MPFR_VERSION_PATCHLEVEL` are respectively the major, minor and patch level of MPFR version, as preprocessing constants. `MPFR_VERSION_STRING` is the version (with an optional suffix, used in development and pre-release versions) as a string constant, which can be compared to the result of `mpfr_get_version` to check at run time the header file and library used match:

```
if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))
    fprintf (stderr, "Warning: header and library do not match\n");
```

Note: Obtaining different strings is not necessarily an error, as in general, a program compiled with some old MPFR version can be dynamically linked with a newer MPFR library version (if allowed by the library versioning system).

`long MPFR_VERSION_NUM (major, minor, patchlevel)` [Macro]  
 Create an integer in the same format as used by `MPFR_VERSION` from the given *major*, *minor* and *patchlevel*. Here is an example of how to check the MPFR version at compile time:

```
#if (!defined(MPFR_VERSION) || (MPFR_VERSION < MPFR_VERSION_NUM(3,0,0)))
# error "Wrong MPFR version."
#endif
```

`const char * mpfr_get_patches (void)` [Function]  
 Return a null-terminated string containing the ids of the patches applied to the MPFR library (contents of the `PATCHES` file), separated by spaces. Note: If the program has been compiled with an older MPFR version and is dynamically linked with a new MPFR library version, the identifiers of the patches applied to the old (compile-time) MPFR version are not available (however this information should not have much interest in general).

`int mpfr_buildopt_tls_p (void)` [Function]  
 Return a non-zero value if MPFR was compiled as thread safe using compiler-level Thread Local Storage (that is, MPFR was built with the `--enable-thread-safe` configure option, see `INSTALL` file), return zero otherwise.

`int mpfr_buildopt_decimal_p (void)` [Function]  
 Return a non-zero value if MPFR was compiled with decimal float support (that is, MPFR was built with the `--enable-decimal-float` configure option), return zero otherwise.

`int mpfr_buildopt_gmpinternals_p (void)` [Function]  
 Return a non-zero value if MPFR was compiled with GMP internals (that is, MPFR was built with either `--with-gmp-build` or `--enable-gmp-internals` configure option), return zero otherwise.

`const char * mpfr_buildopt_tune_case (void)` [Function]  
 Return a string saying which thresholds file has been used at compile time. This file is normally selected from the processor type.

### 5.13 Exception Related Functions

`mpfr_exp_t mpfr_get_emin (void)` [Function]

`mpfr_exp_t mpfr_get_emax (void)` [Function]

Return the (current) smallest and largest exponents allowed for a floating-point variable. The smallest positive value of a floating-point variable is  $1/2 \times 2^{\text{emin}}$  and the largest value has the form  $(1 - \varepsilon) \times 2^{\text{emax}}$ , where  $\varepsilon$  depends on the precision of the considered variable.

`int mpfr_set_emin (mpfr_exp_t exp)` [Function]

`int mpfr_set_emax (mpfr_exp_t exp)` [Function]

Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero value when *exp* is not in the range accepted by the implementation (in that case the smallest or largest exponent is not changed), and zero otherwise. If the user changes the exponent range, it is her/his responsibility to check that all current floating-point variables are in the new allowed range (for example using `mpfr_check_range`), otherwise the subsequent behavior will be undefined, in the sense of the ISO C standard.

`mpfr_exp_t mpfr_get_emin_min (void)` [Function]

`mpfr_exp_t mpfr_get_emin_max (void)` [Function]

`mpfr_exp_t mpfr_get_emax_min (void)` [Function]

`mpfr_exp_t mpfr_get_emax_max (void)` [Function]

Return the minimum and maximum of the exponents allowed for `mpfr_set_emin` and `mpfr_set_emax` respectively. These values are implementation dependent, thus a program using `mpfr_set_emax(mpfr_get_emax_max())` or `mpfr_set_emin(mpfr_get_emin_min())` may not be portable.

`int mpfr_check_range (mpfr_t x, int t, mpfr_rnd_t rnd)` [Function]

This function assumes that *x* is the correctly-rounded value of some real value *y* in the direction *rnd* and some extended exponent range, and that *t* is the corresponding [ternary value], page 8. For example, one performed `t = mpfr_log(x, u, rnd)`, and *y* is the exact logarithm of *u*. Thus *t* is negative if *x* is smaller than *y*, positive if *x* is larger than *y*, and zero if *x* equals *y*. This function modifies *x* if needed to be in the current range of acceptable values: It generates an underflow or an overflow if the exponent of *x* is outside the current allowed range; the value of *t* may be used to avoid a double rounding. This function returns zero if the new value of *x* equals the exact one *y*, a positive value if that new value is larger than *y*, and a negative value if it is smaller than *y*. Note that unlike most functions, the new result *x* is compared to the (unknown) exact one *y*, not the input value *x*, i.e., the ternary value is propagated.

Note: If *x* is an infinity and *t* is different from zero (i.e., if the rounded result is an inexact infinity), then the overflow flag is set. This is useful because `mpfr_check_range` is typically called (at least in MPFR functions) after restoring the flags that could have been set due to internal computations.

`int mpfr_subnormalize (mpfr_t x, int t, mpfr_rnd_t rnd)` [Function]

This function rounds *x* emulating subnormal number arithmetic: if *x* is outside the subnormal exponent range, it just propagates the [ternary value], page 8 *t*; otherwise, it rounds *x* to precision `EXP(x)-emin+1` according to rounding mode *rnd* and previous ternary value *t*, avoiding double rounding problems. More precisely in the subnormal domain, denoting by *e* the value of `emin`, *x* is rounded in fixed-point arithmetic to an integer multiple of  $2^{e-1}$ ; as a consequence,  $1.5 \times 2^{e-1}$  when *t* is zero is rounded to  $2^e$  with rounding to nearest.

`PREC(x)` is not modified by this function. *rnd* and *t* must be the rounding mode and the returned ternary value used when computing *x* (as in `mpfr_check_range`). The subnormal

exponent range is from `emin` to `emin+PREC(x)-1`. If the result cannot be represented in the current exponent range (due to a too small `emax`), the behavior is undefined. Note that unlike most functions, the result is compared to the exact one, not the input value `x`, i.e., the ternary value is propagated.

As usual, if the returned ternary value is non zero, the inexact flag is set. Moreover, if a second rounding occurred (because the input `x` was in the subnormal range), the underflow flag is set.

This is an example of how to emulate binary double IEEE 754 arithmetic (binary64 in IEEE 754-2008) using MPFR:

```
{
  mpfr_t xa, xb; int i; volatile double a, b;

  mpfr_set_default_prec (53);
  mpfr_set_emin (-1073); mpfr_set_emax (1024);

  mpfr_init (xa); mpfr_init (xb);

  b = 34.3; mpfr_set_d (xb, b, MPFR_RNDN);
  a = 0x1.1235P-1021; mpfr_set_d (xa, a, MPFR_RNDN);

  a /= b;
  i = mpfr_div (xa, xa, xb, MPFR_RNDN);
  i = mpfr_subnormalize (xa, i, MPFR_RNDN); /* new ternary value */

  mpfr_clear (xa); mpfr_clear (xb);
}
```

Warning: this emulates a double IEEE 754 arithmetic with correct rounding in the subnormal range, which may not be the case for your hardware.

```
void mpfr_clear_underflow (void) [Function]
void mpfr_clear_overflow (void) [Function]
void mpfr_clear_divby0 (void) [Function]
void mpfr_clear_nanflag (void) [Function]
void mpfr_clear_inexflag (void) [Function]
void mpfr_clear_erangeflag (void) [Function]
```

Clear the underflow, overflow, divide-by-zero, invalid, inexact and *erange* flags.

```
void mpfr_set_underflow (void) [Function]
void mpfr_set_overflow (void) [Function]
void mpfr_set_divby0 (void) [Function]
void mpfr_set_nanflag (void) [Function]
void mpfr_set_inexflag (void) [Function]
void mpfr_set_erangeflag (void) [Function]
```

Set the underflow, overflow, divide-by-zero, invalid, inexact and *erange* flags.

```
void mpfr_clear_flags (void) [Function]
  Clear all global flags (underflow, overflow, divide-by-zero, invalid, inexact, erange).
```

```
int mpfr_underflow_p (void) [Function]
int mpfr_overflow_p (void) [Function]
```



```
int mpfr_divby0_p (void) [Function]
int mpfr_nanflag_p (void) [Function]
int mpfr_inexflag_p (void) [Function]
int mpfr_erangeflag_p (void) [Function]
    Return the corresponding (underflow, overflow, divide-by-zero, invalid, inexact, erange) flag,
    which is non-zero iff the flag is set.
```

## 5.14 Compatibility With MPF

A header file `mpf2mpfr.h` is included in the distribution of MPFR for compatibility with the GNU MP class MPF. By inserting the following two lines after the `#include <gmp.h>` line,

```
#include <mpfr.h>
#include <mpf2mpfr.h>
```

any program written for MPF can be compiled directly with MPFR without any changes (except the `gmp_printf` functions will not work for arguments of type `mpfr_t`). All operations are then performed with the default MPFR rounding mode, which can be reset with `mpfr_set_default_rounding_mode`.

Warning: the `mpf_init` and `mpf_init2` functions initialize to zero, whereas the corresponding MPFR functions initialize to NaN: this is useful to detect uninitialized values, but is slightly incompatible with MPF.

```
void mpfr_set_prec_raw (mpfr_t x, mpfr_prec_t prec) [Function]
    Reset the precision of x to be exactly prec bits. The only difference with mpfr_set_prec is
    that prec is assumed to be small enough so that the significand fits into the current allocated
    memory space for x. Otherwise the behavior is undefined.
```

```
int mpfr_eq (mpfr_t op1, mpfr_t op2, unsigned long int op3) [Function]
    Return non-zero if op1 and op2 are both non-zero ordinary numbers with the same exponent
    and the same first op3 bits, both zero, or both infinities of the same sign. Return zero
    otherwise. This function is defined for compatibility with MPF, we do not recommend to
    use it otherwise. Do not use it either if you want to know whether two numbers are close to
    each other; for instance, 1.011111 and 1.100000 are regarded as different for any value of op3
    larger than 1.
```

```
void mpfr_reldiff (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
    Compute the relative difference between op1 and op2 and store the result in rop. This
    function does not guarantee the correct rounding on the relative difference; it just computes
     $|op1 - op2|/op1$ , using the precision of rop and the rounding mode rnd for all operations.
```

```
int mpfr_mul_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2, [Function]
                  mpfr_rnd_t rnd)
```

```
int mpfr_div_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2, [Function]
                  mpfr_rnd_t rnd)
```

These functions are identical to `mpfr_mul_2ui` and `mpfr_div_2ui` respectively. These functions are only kept for compatibility with MPF, one should prefer `mpfr_mul_2ui` and `mpfr_div_2ui` otherwise.

## 5.15 Custom Interface

Some applications use a stack to handle the memory and their objects. However, the MPFR memory design is not well suited for such a thing. So that such applications are able to use MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows one to use MPFR in two ways:

- Either directly store a floating-point number as a `mpfr_t` on the stack.
- Either store its own representation on the stack and construct a new temporary `mpfr_t` each time it is needed.

Nothing has to be done to destroy the floating-point numbers except garbaging the used memory: all the memory management (allocating, destroying, garbaging) is left to the application.

Each function in this interface is also implemented as a macro for efficiency reasons: for example `mpfr_custom_init (s, p)` uses the macro, while `(mpfr_custom_init) (s, p)` uses the function.

Note 1: MPFR functions may still initialize temporary floating-point numbers using `mpfr_init` and similar functions. See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (`mpfr_const_pi` for example), even if they are not explicitly called. You have to call `mpfr_free_cache` each time you garbage the memory iff `mpfr_init`, through GMP Custom Allocation, allocates its memory on the application stack.

`size_t mpfr_custom_get_size (mpfr_prec_t prec)` [Function]  
Return the needed size in bytes to store the significand of a floating-point number of precision `prec`.

`void mpfr_custom_init (void *significand, mpfr_prec_t prec)` [Function]  
Initialize a significand of precision `prec`, where `significand` must be an area of `mpfr_custom_get_size (prec)` bytes at least and be suitably aligned for an array of `mp_limb_t` (GMP type, see [Section 5.16 \[Internals\], page 39](#)).

`void mpfr_custom_init_set (mpfr_t x, int kind, mpfr_exp_t exp, mpfr_prec_t prec, void *significand)` [Function]  
Perform a dummy initialization of a `mpfr_t` and set it to:

- if `ABS(kind) == MPFR_NAN_KIND`, `x` is set to NaN;
- if `ABS(kind) == MPFR_INF_KIND`, `x` is set to the infinity of sign `sign(kind)`;
- if `ABS(kind) == MPFR_ZERO_KIND`, `x` is set to the zero of sign `sign(kind)`;
- if `ABS(kind) == MPFR_REGULAR_KIND`, `x` is set to a regular number: `x = sign(kind)*significand*2exp`.

In all cases, it uses `significand` directly for further computing involving `x`. It will not allocate anything. A floating-point number initialized with this function cannot be resized using `mpfr_set_prec` or `mpfr_prec_round`, or cleared using `mpfr_clear`! The `significand` must have been initialized with `mpfr_custom_init` using the same precision `prec`.

`int mpfr_custom_get_kind (mpfr_t x)` [Function]  
Return the current kind of a `mpfr_t` as created by `mpfr_custom_init_set`. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`void * mpfr_custom_get_significand (mpfr_t x)` [Function]  
Return a pointer to the significand used by a `mpfr_t` initialized with `mpfr_custom_init_set`. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`mpfr_exp_t mpfr_custom_get_exp (mpfr_t x)` [Function]

Return the exponent of  $x$ , assuming that  $x$  is a non-zero ordinary number. The return value for NaN, Infinity or zero is unspecified but does not produce any trap. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`void mpfr_custom_move (mpfr_t x, void *new_position)` [Function]

Inform MPFR that the significand of  $x$  has moved due to a garbage collect and update its new position to `new_position`. However the application has to move the significand and the `mpfr_t` itself. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

## 5.16 Internals

A *limb* means the part of a multi-precision number that fits in a single word. Usually a limb contains 32 or 64 bits. The C data type for a limb is `mp_limb_t`.

The `mpfr_t` type is internally defined as a one-element array of a structure, and `mpfr_ptr` is the C data type representing a pointer to this structure. The `mpfr_t` type consists of four fields:

- The `_mpfr_prec` field is used to store the precision of the variable (in bits); this is not less than `MPFR_PREC_MIN`.
- The `_mpfr_sign` field is used to store the sign of the variable.
- The `_mpfr_exp` field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values  $n$  are a multiplier  $2^n$  relative to that point. A NaN, an infinity and a zero are indicated by special values of the exponent field.
- Finally, the `_mpfr_d` field is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by `_mpfr_prec`, namely `ceil(_mpfr_prec/mp_bits_per_limb)`. Non-singular (i.e., different from NaN, Infinity or zero) values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zeros.

## 6 API Compatibility

The goal of this section is to describe some API changes that occurred from one version of MPFR to another, and how to write code that can be compiled and run with older MPFR versions. The minimum MPFR version that is considered here is 2.2.0 (released on 20 September 2005).

API changes can only occur between major or minor versions. Thus the patchlevel (the third number in the MPFR version) will be ignored in the following. If a program does not use MPFR internals, changes in the behavior between two versions differing only by the patchlevel should only result from what was regarded as a bug or unspecified behavior.

As a general rule, a program written for some MPFR version should work with later versions, possibly except at a new major version, where some features (described as obsolete for some time) can be removed. In such a case, a failure should occur during compilation or linking. If a result becomes incorrect because of such a change, please look at the various changes below (they are minimal, and most software should be unaffected), at the FAQ and at the MPFR web page for your version (a bug could have been introduced and be already fixed); and if the problem is not mentioned, please send us a bug report (see [Chapter 3 \[Reporting Bugs\]](#), page 5).

However, a program written for the current MPFR version (as documented by this manual) may not necessarily work with previous versions of MPFR. This section should help developers to write portable code.

Note: Information given here may be incomplete. API changes are also described in the NEWS file (for each version, instead of being classified like here), together with other changes.

### 6.1 Type and Macro Changes

The official type for exponent values changed from `mp_exp_t` to `mpfr_exp_t` in MPFR 3.0. The type `mp_exp_t` will remain available as it comes from GMP (with a different meaning). These types are currently the same (`mpfr_exp_t` is defined as `mp_exp_t` with `typedef`), so that programs can still use `mp_exp_t`; but this may change in the future. Alternatively, using the following code after including `mpfr.h` will work with official MPFR versions, as `mpfr_exp_t` was never defined in MPFR 2.x:

```
#if MPFR_VERSION_MAJOR < 3
typedef mp_exp_t mpfr_exp_t;
#endif
```

The official types for precision values and for rounding modes respectively changed from `mp_prec_t` and `mp_rnd_t` to `mpfr_prec_t` and `mpfr_rnd_t` in MPFR 3.0. This change was actually done a long time ago in MPFR, at least since MPFR 2.2.0, with the following code in `mpfr.h`:

```
#ifndef mp_rnd_t
# define mp_rnd_t mpfr_rnd_t
#endif
#ifndef mp_prec_t
# define mp_prec_t mpfr_prec_t
#endif
```

This means that it is safe to use the new official types `mpfr_prec_t` and `mpfr_rnd_t` in your programs. The types `mp_prec_t` and `mp_rnd_t` (defined in MPFR only) may be removed in the future, as the prefix `mp_` is reserved by GMP.

The precision type `mpfr_prec_t` (`mp_prec_t`) was unsigned before MPFR 3.0; it is now signed. `MPFR_PREC_MAX` has not changed, though. Indeed the MPFR code requires that `MPFR_PREC_MAX`

be representable in the exponent type, which may have the same size as `mpfr_prec_t` but has always been signed. The consequence is that valid code that does not assume anything about the signedness of `mpfr_prec_t` should work with past and new MPFR versions. This change was useful as the use of unsigned types tends to convert signed values to unsigned ones in expressions due to the usual arithmetic conversions, which can yield incorrect results if a negative value is converted in such a way. Warning! A program assuming (intentionally or not) that `mpfr_prec_t` is signed may be affected by this problem when it is built and run against MPFR 2.x.

The rounding modes `GMP_RNDx` were renamed to `MPFR_RNDx` in MPFR 3.0. However the old names `GMP_RNDx` have been kept for compatibility (this might change in future versions), using:

```
#define GMP_RNDN MPFR_RNDN
#define GMP_RNDZ MPFR_RNDZ
#define GMP_RNDU MPFR_RNDU
#define GMP_RNDD MPFR_RNDD
```

The rounding mode “round away from zero” (`MPFR_RNDA`) was added in MPFR 3.0 (however no rounding mode `GMP_RNDA` exists).

## 6.2 Added Functions

We give here in alphabetical order the functions that were added after MPFR 2.2, and in which MPFR version.

- `mpfr_add_d` in MPFR 2.4.
- `mpfr_ai` in MPFR 3.0 (incomplete, experimental).
- `mpfr_asprintf` in MPFR 2.4.
- `mpfr_buildopt_decimal_p` and `mpfr_buildopt_tls_p` in MPFR 3.0.
- `mpfr_buildopt_gmpinternals_p` and `mpfr_buildopt_tune_case` in MPFR 3.1.
- `mpfr_clear_divby0` in MPFR 3.1 (new divide-by-zero exception).
- `mpfr_copysign` in MPFR 2.3. Note: MPFR 2.2 had a `mpfr_copysign` function that was available, but not documented, and with a slight difference in the semantics (when the second input operand is a NaN).
- `mpfr_custom_get_significand` in MPFR 3.0. This function was named `mpfr_custom_get_mantissa` in previous versions; `mpfr_custom_get_mantissa` is still available via a macro in `mpfr.h`:

```
#define mpfr_custom_get_mantissa mpfr_custom_get_significand
```

Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use `mpfr_custom_get_mantissa`.

- `mpfr_d_div` and `mpfr_d_sub` in MPFR 2.4.
- `mpfr_digamma` in MPFR 3.0.
- `mpfr_divby0_p` in MPFR 3.1 (new divide-by-zero exception).
- `mpfr_div_d` in MPFR 2.4.
- `mpfr_fmod` in MPFR 2.4.
- `mpfr_fms` in MPFR 2.3.
- `mpfr_fprintf` in MPFR 2.4.
- `mpfr_frexp` in MPFR 3.1.
- `mpfr_getflt` in MPFR 3.0.
- `mpfr_get_patches` in MPFR 2.3.
- `mpfr_get_z_2exp` in MPFR 3.0. This function was named `mpfr_get_z_exp` in previous versions; `mpfr_get_z_exp` is still available via a macro in `mpfr.h`:

```
#define mpfr_get_z_exp mpfr_get_z_2exp
```

Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use `mpfr_get_z_exp`.

- `mpfr_grandom` in MPFR 3.1.
- `mpfr_j0`, `mpfr_j1` and `mpfr_jn` in MPFR 2.3.
- `mpfr_lgamma` in MPFR 2.3.
- `mpfr_li2` in MPFR 2.4.
- `mpfr_min_prec` in MPFR 3.0.
- `mpfr_modf` in MPFR 2.4.
- `mpfr_mul_d` in MPFR 2.4.
- `mpfr_printf` in MPFR 2.4.
- `mpfr_rec_sqrt` in MPFR 2.4.
- `mpfr_regular_p` in MPFR 3.0.
- `mpfr_remainder` and `mpfr_remquo` in MPFR 2.3.
- `mpfr_set_divby0` in MPFR 3.1 (new divide-by-zero exception).
- `mpfr_set_flt` in MPFR 3.0.
- `mpfr_set_z_2exp` in MPFR 3.0.
- `mpfr_set_zero` in MPFR 3.0.
- `mpfr_setsign` in MPFR 2.3.
- `mpfr_signbit` in MPFR 2.3.
- `mpfr_sinh_cosh` in MPFR 2.4.
- `mpfr_snprintf` and `mpfr_sprintf` in MPFR 2.4.
- `mpfr_sub_d` in MPFR 2.4.
- `mpfr_urandom` in MPFR 3.0.
- `mpfr_vasprintf`, `mpfr_vfprintf`, `mpfr_vprintf`, `mpfr_vsprintf` and `mpfr_vsnprintf` in MPFR 2.4.
- `mpfr_y0`, `mpfr_y1` and `mpfr_yn` in MPFR 2.3.
- `mpfr_z_sub` in MPFR 3.1.

## 6.3 Changed Functions

The following functions have changed after MPFR 2.2. Changes can affect the behavior of code written for some MPFR version when built and run against another MPFR version (older or newer), as described below.

- `mpfr_check_range` changed in MPFR 2.3.2 and MPFR 2.4. If the value is an inexact infinity, the overflow flag is now set (in case it was lost), while it was previously left unchanged. This is really what is expected in practice (and what the MPFR code was expecting), so that the previous behavior was regarded as a bug. Hence the change in MPFR 2.3.2.
- `mpfr_get_f` changed in MPFR 3.0. This function was returning zero, except for NaN and Inf, which do not exist in MPF. The *erange* flag is now set in these cases, and `mpfr_get_f` now returns the usual ternary value.
- `mpfr_get_si`, `mpfr_get_sj`, `mpfr_get_ui` and `mpfr_get_uj` changed in MPFR 3.0. In previous MPFR versions, the cases where the *erange* flag is set were unspecified.
- `mpfr_get_z` changed in MPFR 3.0. The return type was `void`; it is now `int`, and the usual ternary value is returned. Thus programs that need to work with both MPFR 2.x and 3.x must not use the return value. Even in this case, C code using `mpfr_get_z` as the second

or third term of a conditional operator may also be affected. For instance, the following is correct with MPFR 3.0, but not with MPFR 2.x:

```
bool ? mpfr_get_z(...) : mpfr_add(...);
```

On the other hand, the following is correct with MPFR 2.x, but not with MPFR 3.0:

```
bool ? mpfr_get_z(...) : (void) mpfr_add(...);
```

Portable code should cast `mpfr_get_z(...)` to `void` to use the type `void` for both terms of the conditional operator, as in:

```
bool ? (void) mpfr_get_z(...) : (void) mpfr_add(...);
```

Alternatively, `if ... else` can be used instead of the conditional operator.

Moreover the cases where the *erange* flag is set were unspecified in MPFR 2.x.

- `mpfr_get_z_exp` changed in MPFR 3.0. In previous MPFR versions, the cases where the *erange* flag is set were unspecified. Note: this function has been renamed to `mpfr_get_z_2exp` in MPFR 3.0, but `mpfr_get_z_exp` is still available for compatibility reasons.
- `mpfr_strtofr` changed in MPFR 2.3.1 and MPFR 2.4. This was actually a bug fix since the code and the documentation did not match. But both were changed in order to have a more consistent and useful behavior. The main changes in the code are as follows. The binary exponent is now accepted even without the `0b` or `0x` prefix. Data corresponding to NaN can now have an optional sign (such data were previously invalid).
- `mpfr_strtofr` changed in MPFR 3.0. This function now accepts bases from 37 to 62 (no changes for the other bases). Note: if an unsupported base is provided to this function, the behavior is undefined; more precisely, in MPFR 2.3.1 and later, providing an unsupported base yields an assertion failure (this behavior may change in the future).
- `mpfr_subnormalize` changed in MPFR 3.1. This was actually regarded as a bug fix. The `mpfr_subnormalize` implementation up to MPFR 3.0.0 did not change the flags. In particular, it did not follow the generic rule concerning the *inexact* flag (and no special behavior was specified). The case of the *underflow* flag was more a lack of specification.
- `mpfr_urandom` and `mpfr_urandomb` changed in MPFR 3.1. Their behavior no longer depends on the platform (assuming this is also true for GMP's random generator, which is not the case between GMP 4.1 and 4.2 if `gmp_randinit_default` is used). As a consequence, the returned values can be different between MPFR 3.1 and previous MPFR versions. Note: as the reproducibility of these functions was not specified before MPFR 3.1, the MPFR 3.1 behavior is *not* regarded as backward incompatible with previous versions.

## 6.4 Removed Functions

Functions `mpfr_random` and `mpfr_random2` have been removed in MPFR 3.0 (this only affects old code built against MPFR 3.0 or later). (The function `mpfr_random` had been deprecated since at least MPFR 2.2.0, and `mpfr_random2` since MPFR 2.4.0.)

## 6.5 Other Changes

For users of a C++ compiler, the way how the availability of `intmax_t` is detected has changed in MPFR 3.0. In MPFR 2.x, if a macro `INTMAX_C` or `UINTMAX_C` was defined (e.g. when the `__STDC_CONSTANT_MACROS` macro had been defined before `<stdint.h>` or `<inttypes.h>` has been included), `intmax_t` was assumed to be defined. However this was not always the case (more precisely, `intmax_t` can be defined only in the namespace `std`, as with Boost), so that compilations could fail. Thus the check for `INTMAX_C` or `UINTMAX_C` is now disabled for C++ compilers, with the following consequences:

- Programs written for MPFR 2.x that need `intmax_t` may no longer be compiled against MPFR 3.0: a `#define MPFR_USE_INTMAX_T` may be necessary before `mpfr.h` is included.

- The compilation of programs that work with MPFR 3.0 may fail with MPFR 2.x due to the problem described above. Workarounds are possible, such as defining `intmax_t` and `uintmax_t` in the global namespace, though this is not clean.

The divide-by-zero exception is new in MPFR 3.1. However it should not introduce incompatible changes for programs that strictly follow the MPFR API since the exception can only be seen via new functions.

As of MPFR 3.1, the `mpfr.h` header can be included several times, while still supporting optional functions (see [Section 4.1 \[Headers and Libraries\]](#), page 6).



## Contributors

The main developers of MPFR are Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier, Philippe Théveny and Paul Zimmermann.

Sylvie Boldo from ENS-Lyon, France, contributed the functions `mpfr_agm` and `mpfr_log`. Sylvain Chevillard contributed the `mpfr_ai` function. David Daney contributed the hyperbolic and inverse hyperbolic functions, the base-2 exponential, and the factorial function. Alain Delplanque contributed the new version of the `mpfr_get_str` function. Mathieu Dutour contributed the functions `mpfr_acos`, `mpfr_asin` and `mpfr_atan`, and a previous version of `mpfr_gamma`. Laurent Fousse contributed the `mpfr_sum` function. Emmanuel Jeandel, from ENS-Lyon too, contributed the generic hypergeometric code, as well as the internal function `mpfr_exp3`, a first implementation of the sine and cosine, and improved versions of `mpfr_const_log2` and `mpfr_const_pi`. Ludovic Meunier helped in the design of the `mpfr_erf` code. Jean-Luc Rémy contributed the `mpfr_zeta` code. Fabrice Rouillier contributed the `mpfr_xxx_z` and `mpfr_xxx_q` functions, and helped to the Microsoft Windows porting. Damien Stehlé contributed the `mpfr_get_ld_2exp` function.

We would like to thank Jean-Michel Muller and Joris van der Hoeven for very fruitful discussions at the beginning of that project, Torbjörn Granlund and Kevin Ryde for their help about design issues, and Nathalie Revol for her careful reading of a previous version of this documentation. In particular Kevin Ryde did a tremendous job for the portability of MPFR in 2002-2004.

The development of the MPFR library would not have been possible without the continuous support of INRIA, and of the LORIA (Nancy, France) and LIP (Lyon, France) laboratories. In particular the main authors were or are members of the PolKA, Spaces, Cacao and Caramel project-teams at LORIA and of the Arénaire and AriC project-teams at LIP. This project was started during the Fiable (reliable in French) action supported by INRIA, and continued during the AOC action. The development of MPFR was also supported by a grant (202F0659 00 MPN 121) from the Conseil Régional de Lorraine in 2002, from INRIA by an "associate engineer" grant (2003-2005), an "opération de développement logiciel" grant (2007-2009), and the post-doctoral grant of Sylvain Chevillard in 2009-2010. The MPFR-MPC workshop in June 2012 was partly supported by the ERC grant ANTICS of Andreas Enge.

## References

- Richard Brent and Paul Zimmermann, "Modern Computer Arithmetic", Cambridge University Press (to appear), also available from the authors' web pages.
- Laurent Fousse, Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier and Paul Zimmermann, "MPFR: A Multiple-Precision Binary Floating-Point Library With Correct Rounding", ACM Transactions on Mathematical Software, volume 33, issue 2, article 13, 15 pages, 2007, <http://doi.acm.org/10.1145/1236463.1236468>.
- Torbjörn Granlund, "GNU MP: The GNU Multiple Precision Arithmetic Library", version 5.0.1, 2010, <http://gmplib.org>.
- IEEE standard for binary floating-point arithmetic, Technical Report ANSI-IEEE Standard 754-1985, New York, 1985. Approved March 21, 1985: IEEE Standards Board; approved July 26, 1985: American National Standards Institute, 18 pages.
- IEEE Standard for Floating-Point Arithmetic, ANSI-IEEE Standard 754-2008, 2008. Revision of ANSI-IEEE Standard 754-1985, approved June 12, 2008: IEEE Standards Board, 70 pages.
- Donald E. Knuth, "The Art of Computer Programming", vol 2, "Seminumerical Algorithms", 2nd edition, Addison-Wesley, 1981.
- Jean-Michel Muller, "Elementary Functions, Algorithms and Implementation", Birkhäuser, Boston, 2nd edition, 2006.
- Jean-Michel Muller, Nicolas Brisebarre, Florent de Dinechin, Claude-Pierre Jeannerod, Vincent Lefèvre, Guillaume Melquiond, Nathalie Revol, Damien Stehlé and Serge Torrès, "Handbook of Floating-Point Arithmetic", Birkhäuser, Boston, 2009.

# Appendix A GNU Free Documentation License

Version 1.2, November 2002

Copyright © 2000,2001,2002 Free Software Foundation, Inc.  
51 Franklin St, Fifth Floor, Boston, MA 02110-1301, USA

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed.

## 0. PREAMBLE

The purpose of this License is to make a manual, textbook, or other functional and useful document *free* in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or noncommercially. Secondly, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of “copyleft”, which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

## 1. APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The “Document”, below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as “you”. You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A “Modified Version” of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A “Secondary Section” is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document’s overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The “Invariant Sections” are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.

The “Cover Texts” are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A “Transparent” copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not “Transparent” is called “Opaque”.

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD, and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The “Title Page” means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, “Title Page” means the text near the most prominent appearance of the work’s title, preceding the beginning of the body of the text.

A section “Entitled XYZ” means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as “Acknowledgements”, “Dedications”, “Endorsements”, or “History”.) To “Preserve the Title” of such a section when you modify the Document means that it remains a section “Entitled XYZ” according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

## 2. VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or noncommercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section 3.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

## 3. COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document’s license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to

the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.

If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

#### 4. MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of sections 2 and 3 above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

- A. Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.
- B. List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.
- C. State on the Title page the name of the publisher of the Modified Version, as the publisher.
- D. Preserve all the copyright notices of the Document.
- E. Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.
- F. Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.
- G. Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.
- H. Include an unaltered copy of this License.
- I. Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.
- J. Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the

Document for previous versions it was based on. These may be placed in the “History” section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.

- K. For any section Entitled “Acknowledgements” or “Dedications”, Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.
- L. Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.
- M. Delete any section Entitled “Endorsements”. Such a section may not be included in the Modified Version.
- N. Do not retitle any existing section to be Entitled “Endorsements” or to conflict in title with any Invariant Section.
- O. Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version’s license notice. These titles must be distinct from any other section titles.

You may add a section Entitled “Endorsements”, provided it contains nothing but endorsements of your Modified Version by various parties—for example, statements of peer review or that the text has been approved by an organization as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

## 5. COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section 4 above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled “History” in the various original documents, forming one section Entitled “History”; likewise combine any sections Entitled “Acknowledgements”, and any sections Entitled “Dedications”. You must delete all sections Entitled “Endorsements.”

## 6. COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.

## 7. AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an “aggregate” if the copyright resulting from the compilation is not used to limit the legal rights of the compilation’s users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section 3 is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document’s Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

## 8. TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section 4. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled “Acknowledgements”, “Dedications”, or “History”, the requirement (section 4) to Preserve its Title (section 1) will typically require changing the actual title.

## 9. TERMINATION

You may not copy, modify, sublicense, or distribute the Document except as expressly provided for under this License. Any other attempt to copy, modify, sublicense or distribute the Document is void, and will automatically terminate your rights under this License. However, parties who have received copies, or rights, from you under this License will not have their licenses terminated so long as such parties remain in full compliance.

## 10. FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See <http://www.gnu.org/copyleft/>.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License “or any later version” applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation.

## A.1 ADDENDUM: How to Use This License For Your Documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

```
Copyright (C)  year  your name.
Permission is granted to copy, distribute and/or modify this document
under the terms of the GNU Free Documentation License, Version 1.2
or any later version published by the Free Software Foundation;
with no Invariant Sections, no Front-Cover Texts, and no Back-Cover
Texts.  A copy of the license is included in the section entitled ‘‘GNU
Free Documentation License’’.
```

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the “with...Texts.” line with this:

```
with the Invariant Sections being list their titles, with
the Front-Cover Texts being list, and with the Back-Cover Texts
being list.
```

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.



# Concept Index

## A

Accuracy .....	11
Arithmetic functions .....	18
Assignment functions .....	13

## B

Basic arithmetic functions .....	18
----------------------------------	----

## C

Combined initialization and assignment functions .....	15
Comparison functions .....	20
Compatibility with MPF .....	37
Conditions for copying MPFR .....	1
Conversion functions .....	16
Copying conditions .....	1
Custom interface .....	37

## E

Exception related functions .....	34
-----------------------------------	----

## F

Float arithmetic functions .....	18
Float comparisons functions .....	20
Float functions .....	11
Float input and output functions .....	26
Float output functions .....	26
Floating-point functions .....	11
Floating-point number .....	6

## G

GNU Free Documentation License .....	47
--------------------------------------	----

## I

I/O functions .....	26
Initialization functions .....	11
Input functions .....	26
Installation .....	3
Integer related functions .....	29

Internals .....	39
<code>intmax_t</code> .....	6
<code>inttypes.h</code> .....	6

## L

<code>libmpfr</code> .....	6
Libraries .....	6
Libtool .....	6
Limb .....	39
Linking .....	6

## M

Miscellaneous float functions .....	32
<code>mpfr.h</code> .....	6

## O

Output functions .....	26
------------------------	----

## P

Precision .....	7, 11
-----------------	-------

## R

Reporting bugs .....	5
Rounding mode related functions .....	31
Rounding Modes .....	7

## S

Special functions .....	22
<code>stdarg.h</code> .....	6
<code>stdint.h</code> .....	6
<code>stdio.h</code> .....	6

## T

Ternary value .....	8
---------------------	---

## U

<code>uintmax_t</code> .....	6
------------------------------	---

## Function and Type Index

<code>mpfr_abs</code>	20	<code>mpfr_custom_init_set</code>	38
<code>mpfr_acos</code>	22	<code>mpfr_custom_move</code>	39
<code>mpfr_acosh</code>	24	<code>mpfr_d_div</code>	19
<code>mpfr_add</code>	18	<code>mpfr_d_sub</code>	18
<code>mpfr_add_d</code>	18	<code>MPFR_DECL_INIT</code>	12
<code>mpfr_add_q</code>	18	<code>mpfr_digamma</code>	24
<code>mpfr_add_si</code>	18	<code>mpfr_dim</code>	20
<code>mpfr_add_ui</code>	18	<code>mpfr_div</code>	19
<code>mpfr_add_z</code>	18	<code>mpfr_div_2exp</code>	37
<code>mpfr_agm</code>	25	<code>mpfr_div_2si</code>	20
<code>mpfr_ai</code>	25	<code>mpfr_div_2ui</code>	20
<code>mpfr_asin</code>	22	<code>mpfr_div_d</code>	19
<code>mpfr_asinh</code>	24	<code>mpfr_div_q</code>	19
<code>mpfr_asprintf</code>	29	<code>mpfr_div_si</code>	19
<code>mpfr_atan</code>	22	<code>mpfr_div_ui</code>	19
<code>mpfr_atan2</code>	23	<code>mpfr_div_z</code>	19
<code>mpfr_atanh</code>	24	<code>mpfr_divby0_p</code>	36
<code>mpfr_buildopt_decimal_p</code>	34	<code>mpfr_eint</code>	24
<code>mpfr_buildopt_gmpinternals_p</code>	34	<code>mpfr_eq</code>	37
<code>mpfr_buildopt_tls_p</code>	34	<code>mpfr_equal_p</code>	21
<code>mpfr_buildopt_tune_case</code>	34	<code>mpfr_erangeflag_p</code>	37
<code>mpfr_can_round</code>	31	<code>mpfr_ erf</code>	24
<code>mpfr_cbrt</code>	19	<code>mpfr_erfc</code>	24
<code>mpfr_ceil</code>	29	<code>mpfr_exp</code>	22
<code>mpfr_check_range</code>	35	<code>mpfr_exp10</code>	22
<code>mpfr_clear</code>	11	<code>mpfr_exp2</code>	22
<code>mpfr_clear_divby0</code>	36	<code>mpfr_expm1</code>	24
<code>mpfr_clear_erangeflag</code>	36	<code>mpfr_fac_ui</code>	24
<code>mpfr_clear_flags</code>	36	<code>mpfr_fits_intmax_p</code>	18
<code>mpfr_clear_inexflag</code>	36	<code>mpfr_fits_sint_p</code>	18
<code>mpfr_clear_nanflag</code>	36	<code>mpfr_fits_slong_p</code>	18
<code>mpfr_clear_overflow</code>	36	<code>mpfr_fits_ushort_p</code>	18
<code>mpfr_clear_underflow</code>	36	<code>mpfr_fits_uint_p</code>	18
<code>mpfr_clears</code>	11	<code>mpfr_fits_uintmax_p</code>	18
<code>mpfr_cmp</code>	21	<code>mpfr_fits_ulong_p</code>	18
<code>mpfr_cmp_d</code>	21	<code>mpfr_fits_ushort_p</code>	18
<code>mpfr_cmp_f</code>	21	<code>mpfr_floor</code>	29
<code>mpfr_cmp_ld</code>	21	<code>mpfr_fma</code>	25
<code>mpfr_cmp_q</code>	21	<code>mpfr_fmod</code>	30
<code>mpfr_cmp_si</code>	21	<code>mpfr_fms</code>	25
<code>mpfr_cmp_si_2exp</code>	21	<code>mpfr_fprintf</code>	29
<code>mpfr_cmp_ui</code>	21	<code>mpfr_frac</code>	30
<code>mpfr_cmp_ui_2exp</code>	21	<code>mpfr_free_cache</code>	25
<code>mpfr_cmp_z</code>	21	<code>mpfr_free_str</code>	18
<code>mpfr_cmpabs</code>	21	<code>mpfr_frexp</code>	16
<code>mpfr_const_catalan</code>	25	<code>mpfr_gamma</code>	24
<code>mpfr_const_euler</code>	25	<code>mpfr_get_d</code>	16
<code>mpfr_const_log2</code>	25	<code>mpfr_get_d_2exp</code>	16
<code>mpfr_const_pi</code>	25	<code>mpfr_get_decimal64</code>	16
<code>mpfr_copysign</code>	33	<code>mpfr_get_default_prec</code>	13
<code>mpfr_cos</code>	22	<code>mpfr_get_default_rounding_mode</code>	31
<code>mpfr_cosh</code>	23	<code>mpfr_get_emax</code>	35
<code>mpfr_cot</code>	22	<code>mpfr_get_emax_max</code>	35
<code>mpfr_coth</code>	22	<code>mpfr_get_emax_min</code>	35
<code>mpfr_csc</code>	23	<code>mpfr_get_emin</code>	35
<code>mpfr_csch</code>	23	<code>mpfr_get_emin_max</code>	35
<code>mpfr_custom_get_exp</code>	39	<code>mpfr_get_emin_min</code>	35
<code>mpfr_custom_get_kind</code>	38	<code>mpfr_get_exp</code>	33
<code>mpfr_custom_get_significand</code>	38	<code>mpfr_get_f</code>	17
<code>mpfr_custom_get_size</code>	38	<code>mpfr_getflt</code>	16
<code>mpfr_custom_init</code>	38	<code>mpfr_get_ld</code>	16

<code>mpfr_get_ld_2exp</code>	16	<code>mpfr_out_str</code>	26
<code>mpfr_get_patches</code>	34	<code>mpfr_overflow_p</code>	36
<code>mpfr_get_prec</code>	13	<code>mpfr_pow</code>	19
<code>mpfr_get_si</code>	16	<code>mpfr_pow_si</code>	19
<code>mpfr_get_sj</code>	16	<code>mpfr_pow_ui</code>	19
<code>mpfr_get_str</code>	17	<code>mpfr_pow_z</code>	20
<code>mpfr_get_ui</code>	16	<code>mpfr_prec_round</code>	31
<code>mpfr_get_uj</code>	16	<code>mpfr_prec_t</code>	7
<code>mpfr_get_version</code>	34	<code>mpfr_print_rnd_mode</code>	32
<code>mpfr_get_z</code>	17	<code>mpfr_printf</code>	29
<code>mpfr_get_z_2exp</code>	17	<code>mpfr_rec_sqrt</code>	19
<code>mpfr_grandom</code>	33	<code>mpfr_regular_p</code>	21
<code>mpfr_greater_p</code>	21	<code>mpfr_reldiff</code>	37
<code>mpfr_greaterequal_p</code>	21	<code>mpfr_remainder</code>	30
<code>mpfr_hypot</code>	25	<code>mpfr_remquo</code>	30
<code>mpfr_inexflag_p</code>	37	<code>mpfr rint</code>	29
<code>mpfr_inf_p</code>	21	<code>mpfr_rint_ceil</code>	30
<code>mpfr_init</code>	12	<code>mpfr_rint_floor</code>	30
<code>mpfr_init_set</code>	16	<code>mpfr_rint_round</code>	30
<code>mpfr_init_set_d</code>	16	<code>mpfr_rint_trunc</code>	30
<code>mpfr_init_set_f</code>	16	<code>mpfr_rnd_t</code>	7
<code>mpfr_init_set_ld</code>	16	<code>mpfr_root</code>	19
<code>mpfr_init_set_q</code>	16	<code>mpfr_round</code>	29
<code>mpfr_init_set_si</code>	16	<code>mpfr_sec</code>	22
<code>mpfr_init_set_str</code>	16	<code>mpfr_sech</code>	23
<code>mpfr_init_set_ui</code>	16	<code>mpfr_set</code>	13
<code>mpfr_init_set_z</code>	16	<code>mpfr_set_d</code>	13
<code>mpfr_init2</code>	11	<code>mpfr_set_decimal64</code>	13
<code>mpfr_inits</code>	12	<code>mpfr_set_default_prec</code>	12
<code>mpfr_inits2</code>	11	<code>mpfr_set_default_rounding_mode</code>	31
<code>mpfr_inp_str</code>	26	<code>mpfr_set_divby0</code>	36
<code>mpfr_integer_p</code>	31	<code>mpfr_set_emax</code>	35
<code>mpfr_j0</code>	25	<code>mpfr_set_emin</code>	35
<code>mpfr_j1</code>	25	<code>mpfr_set_erangeflag</code>	36
<code>mpfr_jn</code>	25	<code>mpfr_set_exp</code>	33
<code>mpfr_less_p</code>	21	<code>mpfr_set_f</code>	13
<code>mpfr_lessequal_p</code>	21	<code>mpfr_setflt</code>	13
<code>mpfr_lessgreater_p</code>	22	<code>mpfr_set_inexflag</code>	36
<code>mpfr_lgamma</code>	24	<code>mpfr_set_inf</code>	15
<code>mpfr_li2</code>	24	<code>mpfr_set_ld</code>	13
<code>mpfr_lngamma</code>	24	<code>mpfr_set_nan</code>	15
<code>mpfr_log</code>	22	<code>mpfr_set_nanflag</code>	36
<code>mpfr_log10</code>	22	<code>mpfr_set_overflow</code>	36
<code>mpfr_log1p</code>	24	<code>mpfr_set_prec</code>	13
<code>mpfr_log2</code>	22	<code>mpfr_set_prec_raw</code>	37
<code>mpfr_max</code>	32	<code>mpfr_set_q</code>	13
<code>mpfr_min</code>	32	<code>mpfr_set_si</code>	13
<code>mpfr_min_prec</code>	32	<code>mpfr_set_si_2exp</code>	14
<code>mpfr_modf</code>	30	<code>mpfr_set_sj</code>	13
<code>mpfr_mul</code>	19	<code>mpfr_set_sj_2exp</code>	14
<code>mpfr_mul_2exp</code>	37	<code>mpfr_set_str</code>	14
<code>mpfr_mul_2si</code>	20	<code>mpfr_set_ui</code>	13
<code>mpfr_mul_2ui</code>	20	<code>mpfr_set_ui_2exp</code>	14
<code>mpfr_mul_d</code>	19	<code>mpfr_set_uj</code>	13
<code>mpfr_mul_q</code>	19	<code>mpfr_set_uj_2exp</code>	14
<code>mpfr_mul_si</code>	19	<code>mpfr_set_underflow</code>	36
<code>mpfr_mul_ui</code>	19	<code>mpfr_set_z</code>	13
<code>mpfr_mul_z</code>	19	<code>mpfr_set_z_2exp</code>	14
<code>mpfr_nan_p</code>	21	<code>mpfr_set_zero</code>	15
<code>mpfr_nanflag_p</code>	37	<code>mpfr_setsign</code>	33
<code>mpfr_neg</code>	20	<code>mpfr_sgn</code>	21
<code>mpfr_nextabove</code>	32	<code>mpfr_si_div</code>	19
<code>mpfr_nextbelow</code>	32	<code>mpfr_si_sub</code>	18
<code>mpfr_nexttoward</code>	32	<code>mpfr_signbit</code>	33
<code>mpfr_number_p</code>	21	<code>mpfr_sin</code>	22

mpfr_sin_cos .....	22	mpfr_ui_pow_ui .....	20
mpfr_sinh .....	23	mpfr_ui_sub .....	18
mpfr_sinh_cosh .....	23	mpfr_underflow_p .....	36
mpfr_snprintf .....	29	mpfr_unordered_p .....	22
mpfr_sprintf .....	29	mpfr_urandom .....	33
mpfr_sqr .....	19	mpfr_urandomb .....	32
mpfr_sqrt .....	19	mpfr_vasprintf .....	29
mpfr_sqrt_ui .....	19	MPFR_VERSION .....	34
mpfr_strtofr .....	14	MPFR_VERSION_MAJOR .....	34
mpfr_sub .....	18	MPFR_VERSION_MINOR .....	34
mpfr_sub_d .....	18	MPFR_VERSION_NUM .....	34
mpfr_sub_q .....	19	MPFR_VERSION_PATCHLEVEL .....	34
mpfr_sub_si .....	18	MPFR_VERSION_STRING .....	34
mpfr_sub_ui .....	18	mpfr_vfprintf .....	29
mpfr_sub_z .....	18	mpfr_vprintf .....	29
mpfr_subnormalize .....	35	mpfr_vsnprintf .....	29
mpfr_sum .....	26	mpfr_vsprintf .....	29
mpfr_swap .....	15	mpfr_y0 .....	25
mpfr_t .....	6	mpfr_y1 .....	25
mpfr_tan .....	22	mpfr_yn .....	25
mpfr_tanh .....	23	mpfr_z_sub .....	18
mpfr_trunc .....	29	mpfr_zero_p .....	21
mpfr_ui_div .....	19	mpfr_zeta .....	24
mpfr_ui_pow .....	20	mpfr_zeta_ui .....	24