This manual documents how to install and use the Multiple Precision Floating-Point Reliable Library, version 3.1.5.


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MPFR Copying Conditions

The GNU MPFR library (or MPFR for short) is free; this means that everyone is free to use it and free to redistribute it on a free basis. The library is not in the public domain; it is copyrighted and there are restrictions on its distribution, but these restrictions are designed to permit everything that a good cooperating citizen would want to do. What is not allowed is to try to prevent others from further sharing any version of this library that they might get from you.

Specifically, we want to make sure that you have the right to give away copies of the library, that you receive source code or else can get it if you want it, that you can change this library or use pieces of it in new free programs, and that you know you can do these things.

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The precise conditions of the license for the GNU MPFR library are found in the Lesser General Public License that accompanies the source code. See the file COPYING.LESSER.
1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to provide a class of floating-point numbers with precise semantics. The main characteristics of MPFR, which make it differ from most arbitrary precision floating-point software tools, are:

- the MPFR code is portable, i.e., the result of any operation does not depend on the machine word size \texttt{mp_bits_per_limb} (64 on most current processors);
- the precision in bits can be set \textit{exactly} to any valid value for each variable (including very small precision);
- MPFR provides the four rounding modes from the IEEE 754-1985 standard, plus away-from-zero, as well as for basic operations as for other mathematical functions.

In particular, with a precision of 53 bits, MPFR is able to exactly reproduce all computations with double-precision machine floating-point numbers (e.g., \texttt{double} type in C, with a C implementation that rigorously follows Annex F of the ISO C99 standard and \texttt{FP_CONTRACT} pragma set to \texttt{OFF}) on the four arithmetic operations and the square root, except the default exponent range is much wider and subnormal numbers are not implemented (but can be emulated).

This version of MPFR is released under the GNU Lesser General Public License, version 3 or any later version. It is permitted to link MPFR to most non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

1.1 How to Use This Manual

Everyone should read Chapter 4 [MPFR Basics], page 6. If you need to install the library yourself, you need to read Chapter 2 [Installing MPFR], page 3, too. To use the library you will need to refer to Chapter 5 [MPFR Interface], page 11.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.
2 Installing MPFR

The MPFR library is already installed on some GNU/Linux distributions, but the development files necessary to the compilation such as mpfr.h are not always present. To check that MPFR is fully installed on your computer, you can check the presence of the file mpfr.h in /usr/include, or try to compile a small program having #include <mpfr.h> (since mpfr.h may be installed somewhere else). For instance, you can try to compile:

```c
#include <stdio.h>
#include <mpfr.h>
int main (void)
{
    printf ("MPFR library: %-12s\nMPFR header: %s (based on %d.%d.%d)\n", mpfr_get_version (), MPFR_VERSION_STRING, MPFR_VERSION_MAJOR, MPFR_VERSION_MINOR, MPFR_VERSION_PATCHLEVEL);
    return 0;
}
```

with

```bash
c -o version version.c -lmpfr -lgmp
```

and if you get errors whose first line looks like

```
version.c:2:19: error: mpfr.h: No such file or directory
```

then MPFR is probably not installed. Running this program will give you the MPFR version.

If MPFR is not installed on your computer, or if you want to install a different version, please follow the steps below.

2.1 How to Install

Here are the steps needed to install the library on Unix systems (more details are provided in the INSTALL file):

1. To build MPFR, you first have to install GNU MP (version 4.1 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need the standard Unix `make' command, plus some other standard Unix utility commands.

Then, in the MPFR build directory, type the following commands.

2. `./configure'

This will prepare the build and setup the options according to your system. You can give options to specify the install directories (instead of the default /usr/local), threading support, and so on. See the INSTALL file and/or the output of `./configure --help' for more information, in particular if you get error messages.

3. `make'

This will compile MPFR, and create a library archive file libmpfr.a. On most platforms, a dynamic library will be produced too.

4. `make check'

This will make sure that MPFR was built correctly. If any test fails, information about this failure can be found in the tests/test-suite.log file. If you want the contents of this file to be automatically output in case of failure, you can set the `VERBOSE' environment variable to 1 before running `make check', for instance by typing:

`VERBOSE=1 make check'
In case of failure, you may want to check whether the problem is already known. If not, please report this failure to the MPFR mailing-list 'mpfr@inria.fr'. For details, See Chapter 3 [Reporting Bugs], page 5.

5. `make install`

This will copy the files mpfr.h and mpf2mpfr.h to the directory /usr/local/include, the library files (libmpfr.a and possibly others) to the directory /usr/local/lib, the file mpfr.info to the directory /usr/local/share/info, and some other documentation files to the directory /usr/local/share/doc/mpfr (or if you passed the `--prefix` option to configure, using the prefix directory given as argument to `--prefix` instead of /usr/local).

2.2 Other `make` Targets

There are some other useful make targets:

- `mpfr.info` or `info`
  Create or update an info version of the manual, in mpfr.info.
  This file is already provided in the MPFR archives.
- `mpfr.pdf` or `pdf`
  Create a PDF version of the manual, in mpfr.pdf.
- `mpfr.dvi` or `dvi`
  Create a DVI version of the manual, in mpfr.dvi.
- `mpfr.ps` or `ps`
  Create a Postscript version of the manual, in mpfr.ps.
- `mpfr.html` or `html`
  Create a HTML version of the manual, in several pages in the directory doc/mpfr.html; if you want only one output HTML file, then type `makeinfo --html --no-split mpfr.texi` from the 'doc' directory instead.
- `clean`
  Delete all object files and archive files, but not the configuration files.
- `distclean`
  Delete all generated files not included in the distribution.
- `uninstall`
  Delete all files copied by `make install`.

2.3 Build Problems

In case of problem, please read the INSTALL file carefully before reporting a bug, in particular section “In case of problem”. Some problems are due to bad configuration on the user side (not specific to MPFR). Problems are also mentioned in the FAQ http://www.mpfr.org/faq.html.

Please report problems to the MPFR mailing-list 'mpfr@inria.fr'. See Chapter 3 [Reporting Bugs], page 5. Some bug fixes are available on the MPFR 3.1.5 web page http://www.mpfr.org/mpfr-3.1.5/.

2.4 Getting the Latest Version of MPFR

3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR 3.1.5 web page http://www.mpfr.org/mpfr-3.1.5/ and the FAQ http://www.mpfr.org/faq.html: perhaps this bug is already known, in which case you may find there a workaround for it. You might also look in the archives of the MPFR mailing-list: https://sympa.inria.fr/sympa/arc/mpfr. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug, i.e., a small self-contained program, using no other library than MPFR. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results you get are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using ‘cc -V’ on some machines, or, if you’re using GCC, ‘gcc -v’. Also, include the output from ‘uname -a’ and the MPFR version (the GMP version may be useful too). If you get a failure while running ‘make’ or ‘make check’, please include the config.log file in your bug report, and in case of test failure, the tests/test-suite.log file too.

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we will not do anything about it (aside of chiding you to send better bug reports).

Send your bug report to the MPFR mailing-list ‘mpfr@inria.fr’.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.
4 MPFR Basics

4.1 Headers and Libraries

All declarations needed to use MPFR are collected in the include file mpfr.h. It is designed to work with both C and C++ compilers. You should include that file in any program using the MPFR library:

```c
#include <mpfr.h>
```

Note however that prototypes for MPFR functions with FILE * parameters are provided only if `<stdio.h>` is included too (before mpfr.h):

```c
#include <stdio.h>
#include <mpfr.h>
```

Likewise `<stdarg.h>` (or `<varargs.h>`) is required for prototypes with va_list parameters, such as mpfr_vprintf.

And for any functions using intmax_t, you must include `<stdint.h>` or `<inttypes.h>` before mpfr.h, to allow mpfr.h to define prototypes for these functions. Moreover, users of C++ compilers under some platforms may need to define `MPFR_USE_INTMAX_T` (and should do it for portability) before mpfr.h has been included; of course, it is possible to do that on the command line, e.g., with `-DMPFR_USE_INTMAX_T`.

Note: If mpfr.h and/or gmp.h (used by mpfr.h) are included several times (possibly from another header file), `<stdio.h>` and/or `<stdarg.h>` (or `<varargs.h>`) should be included before the first inclusion of mpfr.h or gmp.h. Alternatively, you can define `MPFR_USE_FILE` (for MPFR I/O functions) and/or `MPFR_USE_VA_LIST` (for MPFR functions with va_list parameters) anywhere before the last inclusion of mpfr.h. As a consequence, if your file is a public header that includes mpfr.h, you need to use the latter method.

When calling a MPFR macro, it is not allowed to have previously defined a macro with the same name as some keywords (currently `do`, `while` and `sizeof`).

You can avoid the use of MPFR macros encapsulating functions by defining the `MPFR_USE_NO_MACRO` macro before mpfr.h is included. In general this should not be necessary, but this can be useful when debugging user code: with some macros, the compiler may emit spurious warnings with some warning options, and macros can prevent some prototype checking.

All programs using MPFR must link against both `libmpfr` and `libgmp` libraries. On a typical Unix-like system this can be done with `'-lmpfr -lgmp'` (in that order), for example:

```
gcc myprogram.c -lmpfr -lgmp
```

MPFR is built using Libtool and an application can use that to link if desired, see GNU Libtool.

If MPFR has been installed to a non-standard location, then it may be necessary to set up environment variables such as `C_INCLUDE_PATH` and `LIBRARY_PATH`, or use `'-I'` and `'-L'` compiler options, in order to point to the right directories. For a shared library, it may also be necessary to set up some sort of run-time library path (e.g., `LD_LIBRARY_PATH`) on some systems. Please read the INSTALL file for additional information.

4.2 Nomenclature and Types

A **floating-point number**, or **float** for short, is an arbitrary precision significand (also called mantissa) with a limited precision exponent. The C data type for such objects is `mpfr_t`
(internally defined as a one-element array of a structure, and \texttt{mpfr\_ptr} is the C data type representing a pointer to this structure). A floating-point number can have three special values: Not-a-Number (NaN) or plus or minus Infinity. NaN represents an uninitialized object, the result of an invalid operation (like 0 divided by 0), or a value that cannot be determined (like +Infinity minus +Infinity). Moreover, like in the IEEE 754 standard, zero is signed, i.e., there are both +0 and −0; the behavior is the same as in the IEEE 754 standard and it is generalized to the other functions supported by MPFR. Unless documented otherwise, the sign bit of a NaN is unspecified.

The precision is the number of bits used to represent the significand of a floating-point number; the corresponding C data type is \texttt{mpfr\_prec\_t}. The precision can be any integer between \texttt{MPFR\_PREC\_MIN} and \texttt{MPFR\_PREC\_MAX}. In the current implementation, \texttt{MPFR\_PREC\_MIN} is equal to 2.

Warning! MPFR needs to increase the precision internally, in order to provide accurate results (and in particular, correct rounding). Do not attempt to set the precision to any value near \texttt{MPFR\_PREC\_MAX}, otherwise MPFR will abort due to an assertion failure. Moreover, you may reach some memory limit on your platform, in which case the program may abort, crash or have undefined behavior (depending on your C implementation).

The rounding mode specifies the way to round the result of a floating-point operation, in case the exact result can not be represented exactly in the destination significand; the corresponding C data type is \texttt{mpfr\_rnd\_t}.

### 4.3 MPFR Variable Conventions

Before you can assign to an MPFR variable, you need to initialize it by calling one of the special initialization functions. When you’re done with a variable, you need to clear it out, using one of the functions for that purpose. A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned to any number of times. For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited. You do not need to be concerned about allocating additional space for MPFR variables, since any variable has a significand of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator. MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication, \texttt{mpfr\_mul}, can be used like this: \texttt{mpfr\_mul (x, x, x, rnd)}. This computes the square of \( x \) with rounding mode \( \texttt{rnd} \) and puts the result back in \( x \).

### 4.4 Rounding Modes

The following five rounding modes are supported:

- \texttt{MPFR\_RNDN}: round to nearest (roundTiesToEven in IEEE 754-2008),
- \texttt{MPFR\_RNDZ}: round toward zero (roundTowardZero in IEEE 754-2008),
- \texttt{MPFR\_RNDU}: round toward plus infinity (roundTowardPositive in IEEE 754-2008),
- \texttt{MPFR\_RNDD}: round toward minus infinity (roundTowardNegative in IEEE 754-2008),
- \texttt{MPFR\_RNDA}: round away from zero.

The ‘round to nearest’ mode works as in the IEEE 754 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 2.5, which is represented by (10.1) in
binary, is rounded to \( (10.0)=2 \) with a precision of two bits, and not to \( (11.0)=3 \). This rule avoids the \textit{drift} phenomenon mentioned by Knuth in volume 2 of \textit{The Art of Computer Programming} (Section 4.2.2).

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value of type \texttt{int}, called the \textit{ternary value}. The value stored in the destination variable is correctly rounded, i.e., MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

As a consequence, in case of a non-zero real rounded result, the error on the result is less or equal to 1/2 ulp (unit in the last place) of that result in the rounding to nearest mode, and less than 1 ulp of that result in the directed rounding modes (a ulp is the weight of the least significant represented bit of the result after rounding).

Unless documented otherwise, functions returning an \texttt{int} return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the MPFR\_RNDU rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an \texttt{int}.

Unless documented otherwise, functions returning as result the value 1 (or any other value specified in this manual) for special cases (like \texttt{acos(0)}) yield an overflow or an underflow if that value is not representable in the current exponent range.

### 4.5 Floating-Point Values on Special Numbers

This section specifies the floating-point values (of type \texttt{mpfr_t}) returned by MPFR functions (where by “returned” we mean here the modified value of the destination object, which should not be mixed with the ternary return value of type \texttt{int} of those functions). For functions returning several values (like \texttt{mpfr_sin_cos}), the rules apply to each result separately.

Functions can have one or several input arguments. An input point is a mapping from these input arguments to the set of the MPFR numbers. When none of its components are NaN, an input point can also be seen as a tuple in the extended real numbers (the set of the real numbers with both infinities).

When the input point is in the domain of the mathematical function, the result is rounded as described in Section “Rounding Modes” (but see below for the specification of the sign of an exact zero). Otherwise the general rules from this section apply unless stated otherwise in the description of the MPFR function (Chapter 5 [MPFR Interface], page 11).

When the input point is not in the domain of the mathematical function but is in its closure in the extended real numbers and the function can be extended by continuity, the result is the obtained limit. Examples: \texttt{mpfr_hypot} on \((\text{+Inf},0)\) gives \texttt{+Inf}. But \texttt{mpfr_pow} cannot be defined on \((1,\text{+Inf})\) using this rule, as one can find sequences \((x_n,y_n)\) such that \(x_n\) goes to 1, \(y_n\) goes to \texttt{+Inf} and \((x_n)^{y_n}\) goes to any positive value when \(n\) goes to the infinity.

When the input point is in the closure of the domain of the mathematical function and an input argument is \texttt{+0} (resp. \texttt{-0}), one considers the limit when the corresponding argument approaches 0 from above (resp. below), if possible. If the limit is not defined (e.g., \texttt{mpfr_sqrt} and \texttt{mpfr_}
\( \log \) on \(-0\)), the behavior is specified in the description of the MPFR function, but must be consistent with the rule from the above paragraph (e.g., \( \text{mpfr\_log} \) on \( \pm 0 \) gives \(-\infty\)).

When the result is equal to 0, its sign is determined by considering the limit as if the input point were not in the domain: If one approaches 0 from above (resp. below), the result is \(+0\) (resp. \(-0\)); for example, \( \text{mpfr\_sin} \) on \(-0\) gives \(-0\) and \( \text{mpfr\_acos} \) on 1 gives \(+0\) (in all rounding modes). In the other cases, the sign is specified in the description of the MPFR function; for example \( \text{mpfr\_max} \) on \(-0\) and \(+0\) gives \(+0\).

When the input point is not in the closure of the domain of the function, the result is NaN. Example: \( \text{mpfr\_sqrt} \) on \(-17\) gives NaN.

When an input argument is NaN, the result is NaN, possibly except when a partial function is constant on the finite floating-point numbers; such a case is always explicitly specified in Chapter 5 [MPFR Interface], page 11. Example: \( \text{mpfr\_hypot} \) on (NaN,0) gives NaN, but \( \text{mpfr\_hypot} \) on (NaN,\(+\infty\)) gives \(+\infty\) (as specified in Section 5.7 [Special Functions], page 22), since for any finite or infinite input \( x \), \( \text{mpfr\_hypot} \) on \((x,\,+\infty)\) gives \(+\infty\).

### 4.6 Exceptions

MPFR supports 6 exception types:

- **Underflow**: An underflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent smaller than the minimum value of the current exponent range. (In the round-to-nearest mode, the halfway case is rounded toward zero.)

  Note: This is not the single possible definition of the underflow. MPFR chooses to consider the underflow after rounding. The underflow before rounding can also be defined. For instance, consider a function that has the exact result \( 7 \times 2^{-4} \), where \( e \) is the smallest exponent (for a significand between 1/2 and 1), with a 2-bit target precision and rounding toward plus infinity. The exact result has the exponent \( e-1 \). With the underflow before rounding, such a function call would yield an underflow, as \( e-1 \) is outside the current exponent range. However, MPFR first considers the rounded result assuming an unbounded exponent range. The exact result cannot be represented exactly in precision 2, and here, it is rounded to \( 0.5 \times 2^e \), which is representable in the current exponent range. As a consequence, this will not yield an underflow in MPFR.

- **Overflow**: An overflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent larger than the maximum value of the current exponent range. In the round-to-nearest mode, the result is infinite. Note: unlike the underflow case, there is only one possible definition of overflow here.

- **Divide-by-zero**: An exact infinite result is obtained from finite inputs.

- **NaN**: A NaN exception occurs when the result of a function is NaN.

- **Inexact**: An inexact exception occurs when the result of a function cannot be represented exactly and must be rounded.

- **Range error**: A range exception occurs when a function that does not return a MPFR number (such as comparisons and conversions to an integer) has an invalid result (e.g., an argument is NaN in \( \text{mpfr\_cmp} \), or a conversion to an integer cannot be represented in the target type).

MPFR has a global flag for each exception, which can be cleared, set or tested by functions described in Section 5.13 [Exception Related Functions], page 35.

Differences with the ISO C99 standard:
• In C, only quiet NaNs are specified, and a NaN propagation does not raise an invalid exception. Unless explicitly stated otherwise, MPFR sets the NaN flag whenever a NaN is generated, even when a NaN is propagated (e.g., in NaN + NaN), as if all NaNs were signaling.

• An invalid exception in C corresponds to either a NaN exception or a range error in MPFR.

4.7 Memory Handling

MPFR functions may create caches, e.g., when computing constants such as π, either because the user has called a function like mpfr_const_pi directly or because such a function was called internally by the MPFR library itself to compute some other function.

At any time, the user can free the various caches with mpfr_free_cache. It is strongly advised to do that before terminating a thread, or before exiting when using tools like ‘valgrind’ (to avoid memory leaks being reported).

MPFR internal data such as flags, the exponent range, the default precision and rounding mode, and caches (i.e., data that are not accessed via parameters) are either global (if MPFR has not been compiled as thread safe) or per-thread (thread local storage, TLS). The initial values of TLS data after a thread is created entirely depend on the compiler and thread implementation (MPFR simply does a conventional variable initialization, the variables being declared with an implementation-defined TLS specifier).
5 MPFR Interface

The floating-point functions expect arguments of type `mpfr_t`.

The MPFR floating-point functions have an interface that is similar to the GNU MP functions. The function prefix for floating-point operations is `mpfr_`.

The user has to specify the precision of each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend on the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with “infinite accuracy”), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754 arithmetic. The results obtained on a given computer are identical to those obtained on a computer with a different word size, or with a different compiler or operating system.

MPFR does not keep track of the accuracy of a computation. This is left to the user or to a higher layer (for example the MPFI library for interval arithmetic). As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with large precision, then MPFR will still compute the result with full precision.

The value of the standard C macro `errno` may be set to non-zero by any MPFR function or macro, whether or not there is an error.

5.1 Initialization Functions

An `mpfr_t` object must be initialized before storing the first value in it. The functions `mpfr_init` and `mpfr_init2` are used for that purpose.

```c
void mpfr_init2 (mpfr_t x, mpfr_prec_t prec) /* Function */
Initialize x, set its precision to be exactly prec bits and its value to NaN. (Warning: the corresponding MPF function initializes to zero instead.)
```

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. To change the precision of a variable which has already been initialized, use `mpfr_set_prec`. The precision `prec` must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined).

```c
void mpfr_inits2 (mpfr_prec_t prec, mpfr_t x, ...) /* Function */
Initialize all the `mpfr_t` variables of the given variable argument `va_list`, set their precision to be exactly `prec` bits and their value to NaN. See `mpfr_init2` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from `x`, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).
```

```c
void mpfr_clear (mpfr_t x) /* Function */
Free the space occupied by the significand of `x`. Make sure to call this function for all `mpfr_t` variables when you are done with them.
```

```c
void mpfr_clears (mpfr_t x, ...) /* Function */
Free the space occupied by all the `mpfr_t` variables of the given `va_list`. See `mpfr_clear` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from `x`, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).
```
Here is an example of how to use multiple initialization functions (since NULL is not necessarily defined in this context, we use (mpfr_ptr) 0 instead, but (mpfr_ptr) NULL is also correct).

```
{
    mpfr_t x, y, z, t;
    mpfr_inits2 (256, x, y, z, t, (mpfr_ptr) 0);
    ...
    mpfr_clears (x, y, z, t, (mpfr_ptr) 0);
}
```

**void mpfr_init (mpfr_t x)**

Initialize x, set its precision to the default precision, and set its value to NaN. The default precision can be changed by a call to `mpfr_set_default_prec`.

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use `mpfr_init2`.

**void mpfr_inits (mpfr_t x, ...)**

Initialize all the mpfr_t variables of the given va_list, set their precision to the default precision and their value to NaN. See `mpfr_init` for more details. The va_list is assumed to be composed only of type mpfr_t (or equivalently mpfr_ptr). It begins from x, and ends when it encounters a null pointer (whose type must also be mpfr_ptr).

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use `mpfr_inits2`.

**MPFR_DECL_INIT (name, prec)**

This macro declares name as an automatic variable of type mpfr_t, initializes it and sets its precision to be exactly prec bits and its value to NaN. name must be a valid identifier.

You must use this macro in the declaration section. This macro is much faster than using `mpfr_init2` but has some drawbacks:

- You must not call `mpfr_clear` with variables created with this macro (the storage is allocated at the point of declaration and deallocated when the brace-level is exited).
- You cannot change their precision.
- You should not create variables with huge precision with this macro.
- Your compiler must support ‘Non-Constant Initializers’ (standard in C++ and ISO C99) and ‘Token Pasting’ (standard in ISO C89). If prec is not a constant expression, your compiler must support ‘variable-length automatic arrays’ (standard in ISO C99). GCC 2.95.3 and above supports all these features. If you compile your program with GCC in C89 mode and with ‘-pedantic’, you may want to define the MPFR_USE_EXTENSION macro to avoid warnings due to the MPFR_DECL_INIT implementation.

**void mpfr_set_default_prec (mpfr_prec_t prec)**

Set the default precision to be exactly prec bits, where prec can be any integer between MPFR_PREC_MIN and MPFR_PREC_MAX. The precision of a variable means the number of bits used to store its significand. All subsequent calls to mpfr_init or mpfr_inits will use this precision, but previously initialized variables are unaffected. The default precision is set to 53 bits initially.

Note: when MPFR is built with the --enable-thread-safe configure option, the default precision is local to each thread. See Section 4.7 [Memory Handling], page 10, for more information.
mpfr_prec_t mpfr_get_default_prec (void) [Function]
Return the current default MPFR precision in bits. See the documentation of mpfr_set_default_prec.

Here is an example on how to initialize floating-point variables:

```c
{  mpfr_t x, y;
  mpfr_init (x); /* use default precision */
  mpfr_init2 (y, 256); /* precision exactly 256 bits */
  ...
  /* When the program is about to exit, do ... */
  mpfr_clear (x);
  mpfr_clear (y);
  mpfr_free_cache (); /* free the cache for constants like pi */
}
```

The following functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

void mpfr_set_prec (mpfr_t x, mpfr_prec_t prec) [Function]
Reset the precision of x to be exactly prec bits, and set its value to NaN. The previous value stored in x is lost. It is equivalent to a call to mpfr_clear(x) followed by a call to mpfr_init2(x, prec), but more efficient as no allocation is done in case the current allocated space for the significand of x is enough. The precision prec can be any integer between MPFR_PREC_MIN and MPFR_PREC_MAX. In case you want to keep the previous value stored in x, use mpfr_prec_round instead.

Warning! You must not use this function if x was initialized with MPFR_DECL_INIT or with mpfr_custom_init_set (see Section 5.15 [Custom Interface], page 38).

mpfr_prec_t mpfr_get_prec (mpfr_t x) [Function]
Return the precision of x, i.e., the number of bits used to store its significand.

5.2 Assignment Functions
These functions assign new values to already initialized floats (see Section 5.1 [Initialization Functions], page 11).

int mpfr_set (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_set_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd) [Function]
int mpfr_set_si (mpfr_t rop, long int op, mpfr_rnd_t rnd) [Function]
int mpfr_set_uj (mpfr_t rop, uintmax_t op, mpfr_rnd_t rnd) [Function]
int mpfr_set_sj (mpfr_t rop, intmax_t op, mpfr_rnd_t rnd) [Function]
int mpfr_set_flt (mpfr_t rop, float op, mpfr_rnd_t rnd) [Function]
int mpfr_set_d (mpfr_t rop, double op, mpfr_rnd_t rnd) [Function]
int mpfr_set_ld (mpfr_t rop, long double op, mpfr_rnd_t rnd) [Function]
int mpfr_set_decimal64 (mpfr_t rop, _Decimal64 op, mpfr_rnd_t rnd) [Function]
int mpfr_set_z (mpfr_t rop, mpz_t op, mpfr_rnd_t rnd) [Function]
int mpfr_set_q (mpfr_t rop, mpq_t op, mpfr_rnd_t rnd) [Function]
int mpfr_set_f (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]

Set the value of rop from op, rounded toward the given direction rnd. Note that the input 0 is converted to +0 by mpfr_set_ui, mpfr_set_si, mpfr_set_uj, mpfr_set_sj,
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mpfr_set_z, mpfr_set_q and mpfr_set_f, regardless of the rounding mode. If the system does not support the IEEE 754 standard, mpfr_set_flt, mpfr_set_d, mpfr_set_ld and mpfr_set_decimal64 might not preserve the signed zeros. The mpfr_set_decimal64 function is built only with the configure option ‘--enable-decimal-float’, which also requires ‘--with-gmp-build’, and when the compiler or system provides the '_Decimal64' data type (recent versions of GCC support this data type); to use mpfr_set_decimal64, one should define the macro MPFR_WANT_DECIMAL_FLOATS before including mpfr.h. mpfr_set_q might fail if the numerator (or the denominator) can not be represented as a mpfr_t.

Note: If you want to store a floating-point constant to a mpfr_t, you should use mpfr_set_str (or one of the MPFR constant functions, such as mpfr_const_pi for π) instead of mpfr_set_flt, mpfr_set_d, mpfr_set_ld or mpfr_set_decimal64. Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary (or decimal, for mpfr_set_decimal64) number before MPFR can work with it.

int mpfr_set_ui_2exp (mpfr_t rop, unsigned long int op, mpfr_exp_t e, mpfr_rnd_t rnd)  
int mpfr_set_si_2exp (mpfr_t rop, long int op, mpfr_exp_t e, mpfr_rnd_t rnd)  
int mpfr_set_uj_2exp (mpfr_t rop, uintmax_t op, intmax_t e, mpfr_rnd_t rnd)  
int mpfr_set_sj_2exp (mpfr_t rop, intmax_t op, intmax_t e, mpfr_rnd_t rnd)  
int mpfr_set_z_2exp (mpfr_t rop, mpz_t op, mpfr_exp_t e, mpfr_rnd_t rnd)

Set the value of rop from op × 2^e, rounded toward the given direction rnd. Note that the input 0 is converted to +0.

int mpfr_set_str (mpfr_t rop, const char *s, int base, mpfr_rnd_t rnd)  
int mpfr_strtofr (mpfr_t rop, const char *nptr, char **endptr, int base, mpfr_rnd_t rnd)

Set rop to the value of the string s in base base, rounded in the direction rnd. See the documentation of mpfr_strtofr for a detailed description of the valid string formats. Contrary to mpfr_strtofr, mpfr_set_str requires the whole string to represent a valid floating-point number.

The meaning of the return value differs from other MPFR functions: it is 0 if the entire string up to the final null character is a valid number in base base; otherwise it is −1, and rop may have changed (users interested in the [ternary value], page 8 should use mpfr_strtofr instead).

Note: it is preferable to use mpfr_strtofr if one wants to distinguish between an infinite rop value coming from an infinite s or from an overflow.

int mpfr_strtofr (mpfr_t rop, const char *nptr, char **endptr, int base, mpfr_rnd_t rnd)

Read a floating-point number from a string nptr in base base, rounded in the direction rnd; base must be either 0 (to detect the base, as described below) or a number from 2 to 62 (otherwise the behavior is undefined). If nptr starts with valid data, the result is stored in rop and *endptr points to the character just after the valid data (if endptr is not a null pointer); otherwise rop is set to zero (for consistency with strtod) and the value of nptr is stored in the location referenced by endptr (if endptr is not a null pointer). The usual ternary value is returned.

Parsing follows the standard C strtod function with some extensions. After optional leading whitespace, one has a subject sequence consisting of an optional sign (+ or −), and either nu-
meric data or special data. The subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-whitespace character, that is of the expected form.

The form of numeric data is a non-empty sequence of significand digits with an optional decimal point, and an optional exponent consisting of an exponent prefix followed by an optional sign and a non-empty sequence of decimal digits. A significand digit is either a decimal digit or a Latin letter (62 possible characters), with \( A = 10, B = 11, \ldots, Z = 35 \); case is ignored in bases less or equal to 36, in bases larger than 36, \( a = 36, b = 37, \ldots, z = 61 \). The value of a significand digit must be strictly less than the base. The decimal point can be either the one defined by the current locale or the period (the first one is accepted for consistency with the C standard and the practice, the second one is accepted to allow the programmer to provide MPFR numbers from strings in a way that does not depend on the current locale). The exponent prefix can be \( e \) or \( E \) for bases up to 10, or \( @ \) in any base; it indicates a multiplication by a power of the base. In bases 2 and 16, the exponent prefix can also be \( p \) or \( P \), in which case the exponent, called binary exponent, indicates a multiplication by a power of 2 instead of the base (there is a difference only for base 16); in base 16 for example \( 1p2 \) represents 4 whereas \( 1@2 \) represents 256. The value of an exponent is always written in base 10.

If the argument \( base \) is 0, then the base is automatically detected as follows. If the significand starts with \( 0b \) or \( 0B \), base 2 is assumed. If the significand starts with \( 0x \) or \( 0X \), base 16 is assumed. Otherwise base 10 is assumed.

Note: The exponent (if present) must contain at least a digit. Otherwise the possible exponent prefix and sign are not part of the number (which ends with the significand). Similarly, if \( 0b, 0B, 0x \) or \( 0X \) is not followed by a binary/hexadecimal digit, then the subject sequence stops at the character 0, thus 0 is read.

Special data (for infinities and NaN) can be \( @inf@ \) or \( @nan@(n-char-sequence-opt) \), and if \( base \leq 16 \), it can also be \( infinity \), \( inf \) or \( nan \) or \( nan(n-char-sequence-opt) \), all case insensitive. A \( n-char-sequence-opt \) is a possibly empty string containing only digits, Latin letters and the underscore (0, 1, 2, \ldots, 9, a, b, \ldots, z, A, B, \ldots, Z, _). Note: one has an optional sign for all data, even NaN. For example, \(-@nAn@(This_Is_Not_17)\) is a valid representation for NaN in base 17.

```c
void mpfr_set_nan (mpfr_t x) [Function]
void mpfr_set_inf (mpfr_t x, int sign) [Function]
void mpfr_set_zero (mpfr_t x, int sign) [Function]
```

Set the variable \( x \) to NaN (Not-a-Number), infinity or zero respectively. In `mpfr_set_inf` or `mpfr_set_zero`, \( x \) is set to plus infinity or plus zero iff \( sign \) is nonnegative; in `mpfr_set_nan`, the sign bit of the result is unspecified.

```c
void mpfr_swap (mpfr_t x, mpfr_t y) [Function]
```

Swap the structures pointed to by \( x \) and \( y \). In particular, the values are exchanged without rounding (this may be different from three `mpfr_set` calls using a third auxiliary variable).

Warning! Since the precisions are exchanged, this will affect future assignments. Moreover, since the significand pointers are also exchanged, you must not use this function if the allocation method used for \( x \) and/or \( y \) does not permit it. This is the case when \( x \) and/or \( y \) were declared and initialized with `MPFR_DECL_INIT`, and possibly with `mpfr_custom_init_set` (see Section 5.15 [Custom Interface], page 38).
5.3 Combined Initialization and Assignment Functions

```c
int mpfr_init_set (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_si (mpfr_t rop, long int op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_d (mpfr_t rop, double op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_ld (mpfr_t rop, long double op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_z (mpfr_t rop, mpz_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_q (mpfr_t rop, mpq_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_f (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Macro]
```

Initialize rop and set its value from op, rounded in the direction rnd. The precision of rop will be taken from the active default precision, as set by mpfr_set_default_prec.

```c
int mpfr_init_set_str (mpfr_t x, const char *s, int base, mpfr_rnd_t rnd) [Function]
```

Initialize x and set its value from the string s in base base, rounded in the direction rnd. See mpfr_set_str.

5.4 Conversion Functions

```c
float mpfr_get_flt (mpfr_t op, mpfr_rnd_t rnd) [Function]
double mpfr_get_d (mpfr_t op, mpfr_rnd_t rnd) [Function]
long double mpfr_get_ld (mpfr_t op, mpfr_rnd_t rnd) [Function]
_DECIMAL64 mpfr_get_decimal64 (mpfr_t op, mpfr_rnd_t rnd) [Function]
```

Convert op to a float (respectively double, long double or _Decimal64), using the rounding mode rnd. If op is NaN, some fixed NaN (either quiet or signaling) or the result of 0.0/0.0 is returned. If op is ±Inf, an infinity of the same sign or the result of ±1.0/0.0 is returned. If op is zero, these functions return a zero, trying to preserve its sign, if possible. The mpfr_get_decimal64 function is built only under some conditions: see the documentation of mpfr_set_decimal64.

```c
long mpfr_get_si (mpfr_t op, mpfr_rnd_t rnd) [Function]
unsigned long mpfr_get_ui (mpfr_t op, mpfr_rnd_t rnd) [Function]
intmax_t mpfr_get_sj (mpfr_t op, mpfr_rnd_t rnd) [Function]
uintmax_t mpfr_get_uj (mpfr_t op, mpfr_rnd_t rnd) [Function]
```

Convert op to a long, an unsigned long, an intmax_t or an uintmax_t (respectively) after rounding it with respect to rnd. If op is NaN, 0 is returned and the erange flag is set. If op is too big for the return type, the function returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow; the erange flag is set too. See also mpfr_fits_slong_p, mpfr_fits_ulong_p, mpfr_fits_intmax_p and mpfr_fits_uintmax_p.

```c
double mpfr_get_d_2exp (long *exp, mpfr_t op, mpfr_rnd_t rnd) [Function]
long double mpfr_get_ld_2exp (long *exp, mpfr_t op, mpfr_rnd_t rnd) [Function]
```

Return d and set exp (formally, the value pointed to by exp) such that 0.5 ≤ |d| < 1 and \(d \times 2^{-exp}\) equals op rounded to double (resp. long double) precision, using the given rounding mode. If op is zero, then a zero of the same sign (or an unsigned zero, if the implementation does not have signed zeros) is returned, and exp is set to 0. If op is NaN or an infinity, then the corresponding double precision (resp. long-double precision) value is returned, and exp is undefined.

```c
int mpfr_frexp (mpfr_t *exp, mpfr_t y, mpfr_t x, mpfr_rnd_t rnd) [Function]
```

Set exp (formally, the value pointed to by exp) and y such that 0.5 ≤ |y| < 1 and \(y \times 2^{-exp}\) equals x rounded to the precision of y, using the given rounding mode. If x is zero, then y
is set to a zero of the same sign and \( \exp \) is set to 0. If \( x \) is NaN or an infinity, then \( y \) is set to the same value and \( \exp \) is undefined.

\[
\text{mpfr} \_\text{exp} \_t \ 	ext{mpfr} \_\text{get} \_z \_2\text{exp} \ (\text{mpz} \_t \ \text{rop}, \ \text{mpfr} \_t \ \text{op})
\]

Put the scaled significand of \( \text{op} \) (regarded as an integer, with the precision of \( \text{op} \)) into \( \text{rop} \), and return the exponent \( \exp \) (which may be outside the current exponent range) such that \( \text{op} \) exactly equals \( \text{rop} \times 2^{\exp} \). If \( \text{op} \) is zero, the minimal exponent \( \text{emin} \) is returned. If \( \text{op} \) is NaN or an infinity, the \( \text{erange} \) flag is set, \( \text{rop} \) is set to 0, and the the minimal exponent \( \text{emin} \) is returned. The returned exponent may be less than the minimal exponent \( \text{emin} \) of MPFR numbers in the current exponent range; in case the exponent is not representable in the \text{mpfr} \_\text{exp} \_t \) type, the \( \text{erange} \) flag is set and the minimal value of the \text{mpfr} \_\text{exp} \_t \) type is returned.

\[
\text{int} \ 	ext{mpfr} \_\text{get} \_z \ (\text{mpz} \_t \ \text{rop}, \ \text{mpfr} \_t \ \text{op}, \ \text{mpfr} \_\text{rnd} \_t \ \text{rnd})
\]

Convert \( \text{op} \) to a \text{mpz} \_t, after rounding it with respect to \( \text{rnd} \). If \( \text{op} \) is NaN or an infinity, the \( \text{erange} \) flag is set, \( \text{rop} \) is set to 0, and 0 is returned.

\[
\text{int} \ 	ext{mpfr} \_\text{get} \_f \ (\text{mpf} \_t \ \text{rop}, \ \text{mpfr} \_t \ \text{op}, \ \text{mpfr} \_\text{rnd} \_t \ \text{rnd})
\]

Convert \( \text{op} \) to a \text{mpf} \_t, after rounding it with respect to \( \text{rnd} \). The \( \text{erange} \) flag is set if \( \text{op} \) is NaN or an infinity, which do not exist in MPF. If \( \text{op} \) is NaN, then \( \text{rop} \) is undefined. If \( \text{op} \) is +\( \text{Inf} \) (resp. −\( \text{Inf} \)), then \( \text{rop} \) is set to the maximum (resp. minimum) value in the precision of the MPF number; if a future MPF version supports infinities, this behavior will be considered incorrect and will change (portable programs should assume that \( \text{rop} \) is set either to this finite number or to an infinite number). Note that since MPFR currently has the same exponent type as MPF (but not with the same radix), the range of values is much larger in MPF than in MPFR, so that an overflow or underflow is not possible.

\[
\text{char} \* \ 	ext{mpfr} \_\text{get} \_\text{str} \ (\text{char} \ * \ \text{str}, \ \text{mpfr} \_\text{exp} \_t \ * \ \text{expptr}, \ \text{int} \ b, \ \text{size} \_t \ n, \ \text{mpfr} \_t \ \text{op}, \ \text{mpfr} \_\text{rnd} \_t \ \text{rnd})
\]

Convert \( \text{op} \) to a string of digits in base \( b \), with rounding in the direction \( \text{rnd} \), where \( n \) is either zero (see below) or the number of significant digits output in the string; in the latter case, \( n \) must be greater or equal to 2. The base may vary from 2 to 62; otherwise the function does nothing and immediately returns a null pointer. If the input number is an ordinary number, the exponent is written through the pointer \( \text{expptr} \) (for input 0, the current minimal exponent is written); the type \text{mpfr} \_\text{exp} \_t \) is large enough to hold the exponent in all cases.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number \( -3.1416 \) would be returned as \( ^\prime-31416^\prime \) in the string and 1 written at \( \text{expptr} \). If \( \text{rnd} \) is to nearest, and \( \text{op} \) is exactly in the middle of two consecutive possible outputs, the one with an even significand is chosen, where both significands are considered with the exponent of \( \text{op} \). Note that for an odd base, this may not correspond to an even last digit: for example with 2 digits in base 7, (14) and a half is rounded to (15) which is 12 in decimal, (16) and a half is rounded to (20) which is 14 in decimal, and (26) and a half is rounded to (26) which is 20 in decimal.

If \( n \) is zero, the number of digits of the significand is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding to nearest, will recover the original value of \( \text{op} \). More precisely, in most cases, the chosen precision of \( \text{str} \) is the minimal precision \( m \) depending only on \( p = \text{PREC}(\text{op}) \) and \( b \) that satisfies the above property, i.e., \( m = 1 + \lceil \frac{\log p}{\log b} \rceil \), with \( p \) replaced by \( p-1 \) if \( b \) is a power of 2, but in some very rare cases, it might be \( m + 1 \) (the smallest case for bases up to 62 is when \( p \) equals 186564318007 for bases 7 and 49).
If \( str \) is a null pointer, space for the significand is allocated using the current allocation function and a pointer to the string is returned (unless the base is invalid). To free the returned string, you must use \texttt{mpfr_free_str}.

If \( str \) is not a null pointer, it should point to a block of storage large enough for the significand, i.e., at least \( \max(n + 2, 7) \). The extra two bytes are for a possible minus sign, and for the terminating null character, and the value 7 accounts for \( \sim0\text{Inf}0 \) plus the terminating null character. The pointer to the string \( str \) is returned (unless the base is invalid).

Note: The NaN and inexact flags are currently not set when need be; this will be fixed in future versions. Programmers should currently assume that whether the flags are set by this function is unspecified.

\begin{verbatim}
void mpfr_free_str (char *str)
  Free a string allocated by mpfr_get_str using the current unallocation function. The block
  is assumed to be strlen(str)\(+1 \) bytes. For more information about how it is done: see
  Section “Custom Allocation” in GNU MP.
\end{verbatim}

\begin{verbatim}
int mpfr_fits_ulong_p (mpfr_t op, mpfr_rnd_t rnd)
  [Function]
int mpfr_fits_slong_p (mpfr_t op, mpfr_rnd_t rnd)
  [Function]
int mpfr_fits_uint_p (mpfr_t op, mpfr_rnd_t rnd)
  [Function]
int mpfr_fits_sint_p (mpfr_t op, mpfr_rnd_t rnd)
  [Function]
int mpfr_fits_uintmax_p (mpfr_t op, mpfr_rnd_t rnd)
  [Function]
int mpfr_fits_intmax_p (mpfr_t op, mpfr_rnd_t rnd)
  [Function]

  Return non-zero if \( op \) would fit in the respective C data type, respectively unsigned long,
  long, unsigned int, int, unsigned short, short, uintmax_t, intmax_t, when rounded to
  an integer in the direction \( rnd \).
\end{verbatim}

\section{5.5 Basic Arithmetic Functions}

\begin{verbatim}
int mpfr_add (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)
  [Function]
int mpfr_add_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2,
    mpfr_rnd_t rnd)
int mpfr_add_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)
int mpfr_add_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)
int mpfr_add_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)
int mpfr_add_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)

  Set \( rop \) to \( op1 + op2 \) rounded in the direction \( \text{rnd} \). The IEEE-754 rules are used,
  in particular for signed zeros. But for types having no signed zeros, 0 is considered unsigned
  (i.e., \(+0\) \( + 0 = (+0) \) and \((-0) + 0 = (-0)) \). The \texttt{mpfr_add_d} function assumes
  that the radix of the double type is a power of 2, with a precision at most that declared by
  the C implementation (macro IEEE_DBL_MANT_DIG, and if not defined 53 bits).
\end{verbatim}

\begin{verbatim}
int mpfr_sub (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)
  [Function]
int mpfr_ui_sub (mpfr_t rop, unsigned long int op1, mpfr_t op2,
    mpfr_rnd_t rnd)
int mpfr_sub_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2,
    mpfr_rnd_t rnd)
int mpfr_si_sub (mpfr_t rop, long int op1, mpfr_t op2, mpfr_rnd_t rnd)
int mpfr_sub_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)
int mpfr_d_sub (mpfr_t rop, double op1, mpfr_t op2, mpfr_rnd_t rnd)
int mpfr_sub_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)
\end{verbatim}
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int mpfr_z_sub (mpfr_t rop, mpz_t op1, mpfr_t op2, mpfr_rnd_t rnd)  [Function]
int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)  [Function]
int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)  [Function]

Set rop to op1 – op2 rounded in the direction rnd. The IEEE-754 rules are used, in particular for signed zeros. But for types having no signed zeros, 0 is considered unsigned (i.e., (+0) – 0 = (+0), (−0) – 0 = (−0), 0 – (+0) = (−0) and 0 – (−0) = (+0)). The same restrictions than for mpfr_add_d apply to mpfr_d_sub and mpfr_sub_d.

int mpfr_mul (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)  [Function]
int mpfr_mul_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)  [Function]
int mpfr_mul_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)  [Function]
int mpfr_mul_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)  [Function]
int mpfr_mul_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)  [Function]

Set rop to op1 × op2 rounded in the direction rnd. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zeros, 0 is considered positive). The same restrictions than for mpfr_add_d apply to mpfr_mul_d.

int mpfr_sqr (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]

Set rop to op2 rounded in the direction rnd.

int mpfr_div (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)  [Function]
int mpfr_ui_div (mpfr_t rop, unsigned long int op1, mpfr_t op2, mpfr_rnd_t rnd)  [Function]
int mpfr_div_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)  [Function]
int mpfr_si_div (mpfr_t rop, long int op1, mpfr_t op2, mpfr_rnd_t rnd)  [Function]
int mpfr_div_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)  [Function]
int mpfr_div_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)  [Function]
int mpfr_div_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)  [Function]
int mpfr_div_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)  [Function]

Set rop to op1/op2 rounded in the direction rnd. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zeros, 0 is considered positive). The same restrictions than for mpfr_add_d apply to mpfr_d_div and mpfr_div_d.

int mpfr_sqrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_sqrtr (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)  [Function]

Set rop to \sqrt{op} rounded in the direction rnd. Set rop to −0 if op is −0, to be consistent with the IEEE 754 standard. Set rop to NaN if op is negative.

int mpfr_recsqrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]

Set rop to \sqrt[1]2 \sqrt{op} rounded in the direction rnd. Set rop to +Inf if op is ±0, +0 if op is ±Inf, and NaN if op is negative. Warning! Therefore the result on −0 is different from the one of the rSqrt function recommended by the IEEE 754-2008 standard (Section 9.2.1), which is −Inf instead of +Inf.

int mpfr_cbrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_root (mpfr_t rop, mpfr_t op, unsigned long int k, mpfr_rnd_t rnd)  [Function]

Set rop to the cubic root (resp. the kth root) of op rounded in the direction rnd. For k odd (resp. even) and op negative (including −Inf), set rop to a negative number (resp. NaN). The kth root of −0 is defined to be −0, whatever the parity of k.
int mpfr_pow (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_pow_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_pow_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_pow_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_ui_pow (mpfr_t rop, unsigned long int op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_ui_pow_ui (mpfr_t rop, unsigned long int op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]

Set rop to $op1^{op2}$, rounded in the direction rnd. Special values are handled as described in the ISO C99 and IEEE 754-2008 standards for the pow function:

- $\text{pow}(\pm 0, y)$ returns plus or minus infinity for $y$ a negative odd integer.
- $\text{pow}(\pm 0, y)$ returns plus infinity for $y$ negative and not an odd integer.
- $\text{pow}(\pm 0, y)$ returns plus or minus zero for $y$ a positive odd integer.
- $\text{pow}(\pm 0, y)$ returns plus zero for $y$ positive and not an odd integer.
- $\text{pow}(-1, \pm \text{Inf})$ returns 1.
- $\text{pow}(+1, y)$ returns 1 for any $y$, even a NaN.
- $\text{pow}(x, \pm 0)$ returns 1 for any $x$, even a NaN.
- $\text{pow}(x, y)$ returns NaN for finite negative $x$ and finite non-integer $y$.
- $\text{pow}(x, -\text{Inf})$ returns plus infinity for $0 < |x| < 1$, and plus zero for $|x| > 1$.
- $\text{pow}(x, +\text{Inf})$ returns plus zero for $0 < |x| < 1$, and plus infinity for $|x| > 1$.
- $\text{pow}(-\text{Inf}, y)$ returns minus zero for $y$ a negative odd integer.
- $\text{pow}(-\text{Inf}, y)$ returns plus zero for $y$ negative and not an odd integer.
- $\text{pow}(-\text{Inf}, y)$ returns minus infinity for $y$ a positive odd integer.
- $\text{pow}(-\text{Inf}, y)$ returns plus infinity for $y$ positive and not an odd integer.
- $\text{pow}(+\text{Inf}, y)$ returns plus zero for $y$ negative, and plus infinity for $y$ positive.

int mpfr_neg (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_abs (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]

Set rop to $-op$ and the absolute value of $op$ respectively, rounded in the direction rnd. Just changes or adjusts the sign if rop and op are the same variable, otherwise a rounding might occur if the precision of rop is less than that of op.

int mpfr_dim (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]

Set rop to the positive difference of op1 and op2, i.e., $op1 - op2$ rounded in the direction rnd if op1 > op2, +0 if op1 ≤ op2, and NaN if op1 or op2 is NaN.

int mpfr_mul_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_mul_2si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd) [Function]

Set rop to $op1 \times 2^{op2}$ rounded in the direction rnd. Just increases the exponent by op2 when rop and op1 are identical.

int mpfr_div_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_div_2si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd) [Function]

Set rop to $op1 / 2^{op2}$ rounded in the direction rnd. Just decreases the exponent by op2 when rop and op1 are identical.
5.6 Comparison Functions

```c
int mpfr_cmp (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_cmp_ui (mpfr_t op1, unsigned long int op2) [Function]
int mpfr_cmp_si (mpfr_t op1, long int op2) [Function]
int mpfr_cmp_d (mpfr_t op1, double op2) [Function]
int mpfr_cmp_ld (mpfr_t op1, long double op2) [Function]
int mpfr_cmp_z (mpfr_t op1, mpz_t op2) [Function]
int mpfr_cmp_q (mpfr_t op1, mpq_t op2) [Function]
int mpfr_cmp_f (mpfr_t op1, mpfr_t op2) [Function]
```

Compare `op1` and `op2`. Return a positive value if `op1 > op2`, zero if `op1 = op2`, and a negative value if `op1 < op2`. Both `op1` and `op2` are considered to their full own precision, which may differ. If one of the operands is NaN, set the `erange` flag and return zero.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., `mpfr_equal_p` for the equality) described below; they behave like the IEEE 754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).

```c
int mpfr_cmp_ui_2exp (mpfr_t op1, unsigned long int op2, mpfr_exp_t e) [Function]
int mpfr_cmp_si_2exp (mpfr_t op1, long int op2, mpfr_exp_t e) [Function]
```

Compare `op1` and `op2 × 2^e`. Similar as above.

```c
int mpfr_cmpabs (mpfr_t op1, mpfr_t op2) [Function]
```

Compare `|op1|` and `|op2|`. Return a positive value if `|op1| > |op2|`, zero if `|op1| = |op2|`, and a negative value if `|op1| < |op2|`. If one of the operands is NaN, set the `erange` flag and return zero.

```c
int mpfr_nan_p (mpfr_t op) [Function]
int mpfr_inf_p (mpfr_t op) [Function]
int mpfr_number_p (mpfr_t op) [Function]
int mpfr_zero_p (mpfr_t op) [Function]
int mpfr_regular_p (mpfr_t op) [Function]
```

Return non-zero if `op` is respectively NaN, an infinity, an ordinary number (i.e., neither NaN nor an infinity), zero, or a regular number (i.e., neither NaN, nor an infinity nor zero). Return zero otherwise.

```c
int mpfr_sgn (mpfr_t op) [Macro]
```

Return a positive value if `op > 0`, zero if `op = 0`, and a negative value if `op < 0`. If the operand is NaN, set the `erange` flag and return zero. This is equivalent to `mpfr_cmp_ui (op, 0)`, but more efficient.

```c
int mpfr_greater_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_greaterequal_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_less_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_lessequal_p (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_equal_p (mpfr_t op1, mpfr_t op2) [Function]
```

Return non-zero if `op1 > op2`, `op1 ≥ op2`, `op1 < op2`, `op1 ≤ op2`, `op1 = op2` respectively, and zero otherwise. Those functions return zero whenever `op1` and/or `op2` is NaN.
5.7 Special Functions

All those functions, except explicitly stated (for example mpfr_sin_cos), return a [ternary value], page 8, i.e., zero for an exact return value, a positive value for a return value larger than the exact result, and a negative value otherwise.

Important note: in some domains, computing special functions (either with correct or incorrect rounding) is expensive, even for small precision, for example the trigonometric and Bessel functions for large argument.

```c
int mpfr_lessgreater_p (mpfr_t rop, mpfr_t op1, mpfr_t op2) [Function]
Return non-zero if op1 < op2 or op1 > op2 (i.e., neither op1, nor op2 is NaN, and op1 ≠ op2), zero otherwise (i.e., op1 and/or op2 is NaN, or op1 = op2).

int mpfr_unordered_p (mpfr_t op1, mpfr_t op2) [Function]
Return non-zero if op1 or op2 is a NaN (i.e., they cannot be compared), zero otherwise.
```

```c
int mpfr_log (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_log2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_log10 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the natural logarithm of op, log2 op or log10 op, respectively, rounded in the direction rnd. Set rop to +0 if op is 1 (in all rounding modes), for consistency with the ISO C99 and IEEE 754-2008 standards. Set rop to −Inf if op is ±0 (i.e., the sign of the zero has no influence on the result).

int mpfr_exp (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_exp2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_exp10 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the exponential of op, to 2^op or to 10^op, respectively, rounded in the direction rnd.
```

```c
int mpfr_cos (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_sin (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_tan (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the cosine of op, sine of op, tangent of op, rounded in the direction rnd.

int mpfr_sin_cos (mpfr_t sop, mpfr_t cop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set simultaneously sop to the sine of op and cop to the cosine of op, rounded in the direction rnd with the corresponding precisions of sop and cop, which must be different variables. Return 0 iff both results are exact, more precisely it returns s + 4c where s = 0 if sop is exact, s = 1 if sop is larger than the sine of op, s = 2 if sop is smaller than the sine of op, and similarly for c and the cosine of op.
```

```c
int mpfr_sec (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_csc (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_cot (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the secant of op, cosecant of op, cotangent of op, rounded in the direction rnd.

int mpfr_acos (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_asin (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_atan (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the arc-cosine, arc-sine or arc-tangent of op, rounded in the direction rnd. Note that since acos(-1) returns the floating-point number closest to π according to the given rounding mode, this number might not be in the output range 0 ≤ rop < π of the arc-cosine
function; still, the result lies in the image of the output range by the rounding function. The same holds for \( \text{asin}(-1), \text{asin}(1), \text{atan}(-\infty), \text{atan}(+\infty) \) or for \( \text{atan}(\text{op}) \) with large \( \text{op} \) and small precision of \( \text{rop} \).

```c
int mpfr_atan2 (mpfr_t rop, mpfr_t y, mpfr_t x, mpfr_rnd_t rnd)
```

Set \( \text{rop} \) to the arc-tangent of \( y \) and \( x \), rounded in the direction \( \text{rnd} \): if \( x > 0 \), \( \text{atan2}(y, x) = \text{atan}(y/x); \) if \( x < 0 \), \( \text{atan2}(y, x) = \text{sign}(y) \ast (\text{Pi} - \text{atan}(|y/x|)) \), thus a number from \( -\pi \) to \( \pi \). As for \( \text{atan} \), in case the exact mathematical result is \( +\pi \) or \( -\pi \), its rounded result might be outside the function output range.

\( \text{atan2}(y, 0) \) does not raise any floating-point exception. Special values are handled as described in the ISO C99 and IEEE 754-2008 standards for the \( \text{atan2} \) function:

- \( \text{atan2}(+0, -0) \) returns \( +\pi \).
- \( \text{atan2}(-0, -0) \) returns \( -\pi \).
- \( \text{atan2}(+0, +0) \) returns \( +0 \).
- \( \text{atan2}(-0, +0) \) returns \( -0 \).
- \( \text{atan2}(+0, x) \) returns \( +\pi \) for \( x < 0 \).
- \( \text{atan2}(-0, x) \) returns \( -\pi \) for \( x < 0 \).
- \( \text{atan2}(+0, x) \) returns \( +0 \) for \( x > 0 \).
- \( \text{atan2}(-0, x) \) returns \( 0 \) for \( x > 0 \).
- \( \text{atan2}(y, 0) \) returns \( -\pi/2 \) for \( y < 0 \).
- \( \text{atan2}(y, 0) \) returns \( +\pi/2 \) for \( y > 0 \).
- \( \text{atan2}(+\infty, -\infty) \) returns \( +3\pi/4 \).
- \( \text{atan2}(-\infty, -\infty) \) returns \( -3\pi/4 \).
- \( \text{atan2}(+\infty, +\infty) \) returns \( +\pi/4 \).
- \( \text{atan2}(-\infty, +\infty) \) returns \( -\pi/4 \).
- \( \text{atan2}(+\infty, -\infty) \) returns \( +\pi/2 \) for finite \( x \).
- \( \text{atan2}(-\infty, -\infty) \) returns \( -\pi/2 \) for finite \( x \).
- \( \text{atan2}(y, -\infty) \) returns \( +\pi \) for finite \( y > 0 \).
- \( \text{atan2}(y, -\infty) \) returns \( -\pi \) for finite \( y < 0 \).
- \( \text{atan2}(y, +\infty) \) returns \( +0 \) for finite \( y > 0 \).
- \( \text{atan2}(y, +\infty) \) returns \( -0 \) for finite \( y < 0 \).

```c
int mpfr_cosh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
int mpfr_sinh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
int mpfr_tanh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
```

Set \( \text{rop} \) to the hyperbolic cosine, sine or tangent of \( \text{op} \), rounded in the direction \( \text{rnd} \).

```c
int mpfr_sinh_cosh (mpfr_t sop, mpfr_t cop, mpfr_t op, mpfr_rnd_t rnd)
```

Set simultaneously \( \text{sop} \) to the hyperbolic sine of \( \text{op} \) and \( \text{cop} \) to the hyperbolic cosine of \( \text{op} \), rounded in the direction \( \text{rnd} \) with the corresponding precision of \( \text{sop} \) and \( \text{cop} \), which must be different variables. Return 0 iff both results are exact (see mpfr_sin_cos for a more detailed description of the return value).

```c
int mpfr_sech (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
int mpfr_csch (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
int mpfr_coth (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
```

Set \( \text{rop} \) to the hyperbolic secant of \( \text{op} \), cosecant of \( \text{op} \), cotangent of \( \text{op} \), rounded in the direction \( \text{rnd} \).
int mpfr_acosh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the inverse hyperbolic cosine of op, rounded in the direction rnd.

int mpfr_asinh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the inverse hyperbolic sine of op, rounded in the direction rnd.

int mpfr_atanh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the inverse hyperbolic tangent of op, rounded in the direction rnd.

int mpfr_floor (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the floor of op, rounded in the direction rnd.

int mpfr_hexp2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the base 2 exponential of op, rounded in the direction rnd.

int mpfr_hexp1 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to e op - 1, rounded in the direction rnd.

int mpfr_expm1 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to e op - 1, rounded in the direction rnd.

int mpfr_eint (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the exponential integral of op, rounded in the direction rnd. For positive op, the exponential integral is the sum of Euler’s constant, of the logarithm of op, and of the sum for k from 1 to infinity of op k / (k · k!). For negative op, rop is set to NaN (this definition for negative argument follows formula 5.1.2 from the Handbook of Mathematical Functions from Abramowitz and Stegun, a future version might use another definition).

int mpfr_lgamma (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the logarithm of the Gamma function on op, rounded in the direction rnd.

int mpfr_lngamma (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the value of the logarithm of the Gamma function on op, rounded in the direction rnd. When op is a negative integer, rop is set to NaN.

int mpfr_lngamma (mpfr_t rop, int *signp, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the value of the logarithm of the absolute value of the Gamma function on op, rounded in the direction rnd. The sign (1 or -1) of Gamma(op) is returned in the object pointed to by signp. When op is an infinity or a nonpositive integer, set rop to +Inf, following the general rules on special values. When -2k - 1 < op < -2k, k being a nonnegative integer, set rop to NaN. See also mpfr_lgamma.

int mpfr_log1p (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the logarithm of one plus op, rounded in the direction rnd.

int mpfr_log1p (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the logarithm of one plus op, rounded in the direction rnd.

int mpfr_zeta (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
Set rop to the value of the Riemann Zeta function on op, rounded in the direction rnd.

int mpfr_zeta_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd) [Function]
Set rop to the value of the Riemann Zeta function on op, rounded in the direction rnd.
int mpfr_erf (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_erfc (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
  Set rop to the value of the error function on op (resp. the complementary error function on op) rounded in the direction rnd.

int mpfr_j0 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_j1 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_jn (mpfr_t rop, long n, mpfr_t op, mpfr_rnd_t rnd)  [Function]
  Set rop to the value of the first kind Bessel function of order 0, (resp. 1 and n) on op, rounded in the direction rnd. When op is NaN, rop is always set to NaN. When op is plus or minus Infinity, rop is set to +0. When op is zero, and n is not zero, rop is set to +0 or −0 depending on the parity and sign of n, and the sign of op.

int mpfr_y0 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_y1 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_yn (mpfr_t rop, long n, mpfr_t op, mpfr_rnd_t rnd)  [Function]
  Set rop to the value of the second kind Bessel function of order 0 (resp. 1 and n) on op, rounded in the direction rnd. When op is NaN or negative, rop is always set to NaN. When op is +Inf, rop is set to +0. When op is zero, rop is set to +Inf or −Inf depending on the parity and sign of n.

int mpfr_fma (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_rnd_t rnd)  [Function]
int mpfr_fms (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_rnd_t rnd)  [Function]
  Set rop to (op1 × op2) + op3 (resp. (op1 × op2)− op3) rounded in the direction rnd. Concerning special values (signed zeros, infinities, NaN), these functions behave like a multiplication followed by a separate addition or subtraction. That is, the fused operation matters only for rounding.

int mpfr_agm (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)  [Function]
  Set rop to the arithmetic-geometric mean of op1 and op2, rounded in the direction rnd. The arithmetic-geometric mean is the common limit of the sequences \( u_n \) and \( v_n \), where \( u_0=op1, v_0=op2 \), \( u_{n+1} \) is the arithmetic mean of \( u_n \) and \( v_n \), and \( v_{n+1} \) is the geometric mean of \( u_n \) and \( v_n \). If any operand is negative, set rop to NaN.

int mpfr_hypot (mpfr_t rop, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  [Function]
  Set rop to the Euclidean norm of x and y, i.e., \( \sqrt{x^2 + y^2} \), rounded in the direction rnd. Special values are handled as described in the ISO C99 (Section F.9.4.3) and IEEE 754-2008 (Section 9.2.1) standards: If x or y is an infinity, then +Inf is returned in rop, even if the other number is NaN.

int mpfr_ai (mpfr_t rop, mpfr_t x, mpfr_rnd_t rnd)  [Function]
  Set rop to the value of the Airy function Ai on x, rounded in the direction rnd. When x is NaN, rop is always set to NaN. When x is +Inf or −Inf, rop is +0. The current implementation is not intended to be used with large arguments. It works with —x— typically smaller than 500. For larger arguments, other methods should be used and will be implemented in a future version.

int mpfr_const_log2 (mpfr_t rop, mpfr_rnd_t rnd)  [Function]
int mpfr_const_pi (mpfr_t rop, mpfr_rnd_t rnd)  [Function]
int mpfr_const_euler (mpfr_t rop, mpfr_rnd_t rnd)  [Function]
int mpfr_const_catalan (mpfr_t rop, mpfr_rnd_t rnd) [Function]
Set rop to the logarithm of 2, the value of π, of Euler’s constant 0.577..., of Catalan’s constant 0.915..., respectively, rounded in the direction rnd. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use mpfr_free_cache.

void mpfr_free_cache (void) [Function]
Free various caches used by MPFR internally, in particular the caches used by the functions computing constants (mpfr_const_log2, mpfr_const_pi, mpfr_const_euler and mpfr_const_catalan). You should call this function before terminating a thread, even if you did not call these functions directly (they could have been called internally).

int mpfr_sum (mpfr_t rop, mpfr_ptr const tab[], unsigned long int n, mpfr_rnd_t rnd) [Function]
Set rop to the sum of all elements of tab, whose size is n, rounded in the direction rnd. Warning: for efficiency reasons, tab is an array of pointers to mpfr_t, not an array of mpfr_t. If the returned int value is zero, rop is guaranteed to be the exact sum; otherwise rop might be smaller than, equal to, or larger than the exact sum (in accordance to the rounding mode). However, mpfr_sum does guarantee the result is correctly rounded.

5.8 Input and Output Functions
This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a stream to any of these functions will make them read from stdin and write to stdout, respectively.

When using any of these functions, you must include the <stdio.h> standard header before mpfr.h, to allow mpfr.h to define prototypes for these functions.

size_t mpfr_out_str (FILE *stream, int base, size_t n, mpfr_t op, mpfr_rnd_t rnd) [Function]
Output op on stream stream, as a string of digits in base base, rounded in the direction rnd. The base may vary from 2 to 62. Print n significant digits exactly, or if n is 0, enough digits so that op can be read back exactly (see mpfr_get_str).

In addition to the significant digits, a decimal point (defined by the current locale) at the right of the first digit and a trailing exponent in base 10, in the form ‘eNNN’, are printed. If base is greater than 10, ‘@’ will be used instead of ‘e’ as exponent delimiter.

Return the number of characters written, or if an error occurred, return 0.

size_t mpfr_inp_str (mpfr_t rop, FILE *stream, int base, mpfr_rnd_t rnd) [Function]
Input a string in base base from stream stream, rounded in the direction rnd, and put the read float in rop.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using mpfr_set_str. See the documentation of mpfr_strtofr for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

5.9 Formatted Output Functions
5.9.1 Requirements

The class of `mpfr_printf` functions provides formatted output in a similar manner as the standard C `printf`. These functions are defined only if your system supports ISO C variadic functions and the corresponding argument access macros.

When using any of these functions, you must include the `<stdio.h>` standard header before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions.

5.9.2 Format String

The format specification accepted by `mpfr_printf` is an extension of the `printf` one. The conversion specification is of the form:

\[
\% \ [\text{flags}] \ [\text{width}] \ [.\ [\text{precision}]] \ [\text{type}] \ [\text{rounding}] \ \text{conv}
\]

‘flags’, ‘width’, and ‘precision’ have the same meaning as for the standard `printf` (in particular, notice that the ‘precision’ is related to the number of digits displayed in the base chosen by ‘conv’ and not related to the internal precision of the `mpfr_t` variable). `mpfr_printf` accepts the same ‘type’ specifiers as GMP (except the non-standard and deprecated ‘q’, use ‘ll’ instead), namely the length modifiers defined in the C standard:

- ‘h’ short
- ‘hh’ char
- ‘j’ intmax_t or uintmax_t
- ‘l’ long or wchar_t
- ‘ll’ long long
- ‘L’ long double
- ‘t’ ptrdiff_t
- ‘z’ size_t

and the ‘type’ specifiers defined in GMP plus ‘R’ and ‘P’ specific to MPFR (the second column in the table below shows the type of the argument read in the argument list and the kind of ‘conv’ specifier to use after the ‘type’ specifier):

- ‘F’ `mpfr_t`, float conversions
- ‘Q’ `mpq_t`, integer conversions
- ‘M’ `mp_limb_t`, integer conversions
- ‘N’ `mp_limb_t` array, integer conversions
- ‘Z’ `mpz_t`, integer conversions
- ‘P’ `mpfr_prec_t`, integer conversions
- ‘R’ `mpfr_t`, float conversions

The ‘type’ specifiers have the same restrictions as those mentioned in the GMP documentation: see Section “Formatted Output Strings” in GNU MP. In particular, the ‘type’ specifiers (except ‘R’ and ‘P’) are supported only if they are supported by `gmp_printf` in your GMP build; this implies that the standard specifiers, such as ‘t’, must also be supported by your C library if you want to use them.

The ‘rounding’ field is specific to `mpfr_t` arguments and should not be used with other types.

With conversion specification not involving ‘P’ and ‘R’ types, `mpfr_printf` behaves exactly as `gmp_printf`.

The ‘P’ type specifies that a following ‘d’, ‘i’, ‘o’, ‘u’, ‘x’, or ‘X’ conversion specifier applies to a `mpfr_prec_t` argument. It is needed because the `mpfr_prec_t` type does not necessarily
correspond to an int or any fixed standard type. The ‘precision’ field specifies the minimum number of digits to appear. The default ‘precision’ is 1. For example:

```c
mpfr_t x;
mpfr_prec_t p;
mpfr_init (x);
...
```
```c
p = mpfr_get_prec (x);
mpfr_printf ("variable x with %Pu bits", p);
```

The ‘R’ type specifies that a following ‘a’, ‘A’, ‘b’, ‘e’, ‘E’, ‘f’, ‘F’, ‘g’, ‘G’, or ‘n’ conversion specifier applies to a mpfr_t argument. The ‘R’ type can be followed by a ‘rounding’ specifier denoted by one of the following characters:

- ‘U’ round toward plus infinity
- ‘D’ round toward minus infinity
- ‘Y’ round away from zero
- ‘Z’ round toward zero
- ‘N’ round to nearest (with ties to even)
- ‘*’ rounding mode indicated by the mpfr_rnd_t argument just before the corresponding mpfr_t variable.

The default rounding mode is rounding to nearest. The following three examples are equivalent:

```c
mpfr_t x;
mpfr_init (x);
...
```
```c
mpfr_printf ("%.128Rf", x);
mpfr_printf ("%.128RNf", x);
mpfr_printf ("%.128R*f", MPFR_RNDN, x);
```

Note that the rounding away from zero mode is specified with ‘Y’ because ISO C reserves the ‘A’ specifier for hexadecimal output (see below).

The output ‘conv’ specifiers allowed with mpfr_t parameter are:

- ‘a’ ‘A’ hex float, C99 style
- ‘b’ binary output
- ‘e’ ‘E’ scientific format float
- ‘f’ ‘F’ fixed point float
- ‘g’ ‘G’ fixed or scientific float

The conversion specifier ‘b’ which displays the argument in binary is specific to mpfr_t arguments and should not be used with other types. Other conversion specifiers have the same meaning as for a double argument.

In case of non-decimal output, only the significand is written in the specified base, the exponent is always displayed in decimal. Special values are always displayed as nan, -inf, and inf for ‘a’, ‘b’, ‘e’, ‘f’, and ‘g’ specifiers and NaN, -INF, and INF for ‘A’, ‘E’, ‘F’, and ‘G’ specifiers.

If the ‘precision’ field is not empty, the mpfr_t number is rounded to the given precision in the direction specified by the rounding mode. If the precision is zero with rounding to nearest mode and one of the following ‘conv’ specifiers: ‘a’, ‘A’, ‘b’, ‘e’, ‘E’, tie case is rounded to even when it lies between two consecutive values at the wanted precision which have the same exponent, otherwise, it is rounded away from zero. For instance, 85 is displayed as "8e+1" and 95 is displayed as "1e+2" with the format specification "%.0RNe". This also applies when the
‘g’ (resp. ‘G’) conversion specifier uses the ‘e’ (resp. ‘E’) style. If the precision is set to a value greater than the maximum value for an int, it will be silently reduced down to INT_MAX.

If the ‘precision’ field is empty (as in %Re or %.RE) with ‘conv’ specifier ‘e’ and ‘E’, the number is displayed with enough digits so that it can be read back exactly, assuming that the input and output variables have the same precision and that the input and output rounding modes are both rounding to nearest (as for mpfr_get_str). The default precision for an empty ‘precision’ field with ‘conv’ specifiers ‘f’, ‘F’, ‘g’, and ‘G’ is 6.

5.9.3 Functions

For all the following functions, if the number of characters which ought to be written appears to exceed the maximum limit for an int, nothing is written in the stream (resp. to stdout, to buf, to str), the function returns −1, sets the errno flag, and (in POSIX system only) errno is set to EOVERFLOW.

**int mpfr_fprintf (FILE *stream, const char *template, ...)**

Print to the stream stream the optional arguments under the control of the template string template. Return the number of characters written or a negative value if an error occurred.

**int mpfr_vfprintf (FILE *stream, const char *template, va_list ap)**

Print to the stream stream the optional arguments under the control of the template string template. Return the number of characters written or a negative value if an error occurred.

**int mpfr_printf (const char *template, ...)**

Print to stdout the optional arguments under the control of the template string template. Return the number of characters written or a negative value if an error occurred.

**int mpfr_vprintf (const char *template, va_list ap)**

Form a null-terminated string corresponding to the optional arguments under the control of the template string template, and print it in buf. No overlap is permitted between buf and the other arguments. Return the number of characters written in the array buf not counting the terminating null character or a negative value if an error occurred.

**int mpfr_snprintf (char *buf, size_t n, const char *template, ...)**

Form a null-terminated string corresponding to the optional arguments under the control of the template string template, and print it in buf. If n is zero, nothing is written and buf may be a null pointer, otherwise, the n−1 first characters are written in buf and the n-th is a null character. Return the number of characters that would have been written had n be sufficiently large, not counting the terminating null character, or a negative value if an error occurred.

**int mpfr_asprintf (char **str, const char *template, ...)**

Write their output as a null terminated string in a block of memory allocated using the current allocation function. A pointer to the block is stored in str. The block of memory must be freed using mpfr_free_str. The return value is the number of characters written in the string, excluding the null-terminator, or a negative value if an error occurred.

5.10 Integer and Remainder Related Functions

**int mpfr_rint (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)**

**int mpfr_ceil (mpfr_t rop, mpfr_t op)**

**int mpfr_floor (mpfr_t rop, mpfr_t op)**
int mpfr_round (mpfr_t rop, mpfr_t op)         [Function]
int mpfr_trunc (mpfr_t rop, mpfr_t op)        [Function]

Set rop to op rounded to an integer. mpfr_round rounds to the nearest representable integer in
the given direction rnd, mpfr ceil rounds to the next higher or equal representable integer,
mpfr_floor to the next lower or equal representable integer, mpfr_round to the nearest
representable integer, rounding halfway cases away from zero (as in the roundTiesToAway
mode of IEEE 754-2008), and mpfr_trunc to the next representable integer toward zero.

The returned value is zero when the result is exact, positive when it is greater than the
original value of op, and negative when it is smaller. More precisely, the returned value is
0 when op is an integer representable in rop, 1 or −1 when op is an integer that is not
representable in rop, 2 or −2 when op is not an integer.

When op is NaN, the NaN flag is set as usual. In the other cases, the inexact flag is set
when rop differs from op, following the ISO C99 rule for the rint function. If you want
the behavior to be more like IEEE 754 / ISO TS 18661-1, i.e., the usual behavior where the
round-to-integer function is regarded as any other mathematical function, you should use one
the mpfr_rint_* functions instead (however it is not possible to round to nearest with the
even rounding rule yet).

Note that mpfr_round is different from mpfr_round called with the rounding to nearest mode
(where halfway cases are rounded to an even integer or significand). Note also that no double
rounding is performed; for instance, 10.5 (1010.1 in binary) is rounded by mpfr_round with
rounding to nearest to 12 (1100 in binary) in 2-bit precision, because the two enclosing
numbers representable on two bits are 8 and 12, and the closest is 12. (If one first rounded
to an integer, one would round 10.5 to 10 with even rounding, and then 10 would be rounded
to 8 again with even rounding.)

int mpfr_rint ceil (mpfr_t rop, mpfr_t op, mpfrRnd_t rnd)        [Function]
int mpfr_rint_floor (mpfr_t rop, mpfr_t op, mpfrRnd_t rnd)        [Function]
int mpfr_rint_round (mpfr_t rop, mpfr_t op, mpfrRnd_t rnd)        [Function]
int mpfr_rint_trunc (mpfr_t rop, mpfr_t op, mpfrRnd_t rnd)        [Function]

Set rop to op rounded to an integer. mpfr_rint ceil rounds to the next higher or equal
integer, mpfr_rint_floor to the next lower or equal integer, mpfr_rint_round to the nearest
integer, rounding halfway cases away from zero, and mpfr_rint_trunc to the next integer
toward zero. If the result is not representable, it is rounded in the direction rnd. The returned
value is the ternary value associated with the considered round-to-integer function (regarded
in the same way as any other mathematical function).

Contrary to mpfr_rint, those functions do perform a double rounding: first op is rounded
to the nearest integer in the direction given by the function name, then this nearest integer
(if not representable) is rounded in the given direction rnd. Thus these round-to-integer
functions behave more like the other mathematical functions, i.e., the returned result is the
correct rounding of the exact result of the function in the real numbers.

For example, mpfr_rint_round with rounding to nearest and a precision of two bits rounds
6.5 to 7 (halfway cases away from zero), then 7 is rounded to 8 by the round-even rule, despite
the fact that 6 is also representable on two bits, and is closer to 6.5 than 8.

int mpfr_frac (mpfr_t rop, mpfr_t op, mpfrRnd_t rnd)        [Function]
Set rop to the fractional part of op, having the same sign as op, rounded in the direction rnd
(unlike in mpfr_rint, rnd affects only how the exact fractional part is rounded, not how the
fractional part is generated).
int mpfr_modf (mpfr_t iop, mpfr_t fop, mpfr_t op, mpfr_rnd_t rnd)  
Set simultaneously iop to the integral part of op and fop to the fractional part of op, rounded in the direction rnd with the corresponding precision of iop and fop (equivalent to mpfr_trunc(iop, op, rnd) and mpfr_frac(fop, op, rnd)). The variables iop and fop must be different. Return 0 iff both results are exact (see mpfr_sin_cos for a more detailed description of the return value).

int mpfr_fmod (mpfr_t r, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  
int mpfr_remainder (mpfr_t r, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  
int mpfr_remquo (mpfr_t r, long* q, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  
Set r to the value of x − ny, rounded according to the direction rnd, where n is the integer quotient of x divided by y, defined as follows: n is rounded toward zero for mpfr_fmod, and to the nearest integer (ties rounded to even) for mpfr_remainder and mpfr_remquo.

Special values are handled as described in Section F.9.7.1 of the ISO C99 standard: If x is infinite or y is zero, r is NaN. If y is infinite and x is finite, r is x rounded to the precision of r. If r is zero, it has the sign of x. The return value is the ternary value corresponding to r.

Additionally, mpfr_remquo stores the low significant bits from the quotient n in *q (more precisely the number of bits in a long minus one), with the sign of x divided by y (except if those low bits are all zero, in which case zero is returned). Note that x may be so large in magnitude relative to y that an exact representation of the quotient is not practical. The mpfr_remainder and mpfr_remquo functions are useful for additive argument reduction.

int mpfr_integer_p (mpfr_t op)  
Return non-zero iff op is an integer.

5.11 Rounding Related Functions

void mpfr_set_default_rounding_mode (mpfr_rnd_t rnd)  
Set the default rounding mode to rnd. The default rounding mode is to nearest initially.

mpfr_rnd_t mpfr_get_default_rounding_mode (void)  
Get the default rounding mode.

int mpfr_prec_round (mpfr_t x, mpfr_prec_t prec, mpfr_rnd_t rnd)  
Round x according to rnd with precision prec, which must be an integer between MPFR_PREC_MIN and MPFR_PREC_MAX (otherwise the behavior is undefined). If prec is greater or equal to the precision of x, then new space is allocated for the significand, and it is filled with zeros. Otherwise, the significand is rounded to precision prec with the given direction. In both cases, the precision of x is changed to prec.

Here is an example of how to use mpfr_prec_round to implement Newton’s algorithm to compute the inverse of a, assuming x is already an approximation to n bits:

```c
mpfr_set_prec (t, 2 * n);
mpfr_set (t, a, MPFR_RNDN); /* round a to 2n bits */
mpfr_mul (t, t, x, MPFR_RNDN); /* t is correct to 2n bits */
mpfr_ui_sub (t, 1, t, MPFR_RNDN); /* high n bits cancel with 1 */
mpfr_prec_round (t, n, MPFR_RNDN); /* t is correct to n bits */
mpfr_mul (t, t, x, MPFR_RNDN); /* t is correct to n bits */
mpfr_prec_round (x, 2 * n, MPFR_RNDN); /* exact */
mpfr_add (x, x, t, MPFR_RNDN); /* x is correct to 2n bits */
```
Warning! You must not use this function if \( x \) was initialized with MPFR_DECL_INIT or with mpfr_custom_init_set (see Section 5.15 [Custom Interface], page 38).

### Function

**int mpfr_can_round (mpfr_t b, mpfr_exp_t err, mpfr_rnd_t rnd1, mpfr_rnd_t rnd2, mpfr_prec_t prec)**

Assuming \( b \) is an approximation of an unknown number \( x \) in the direction \( rnd1 \) with error at most two to the power \( E(b) - \text{err} \) where \( E(b) \) is the exponent of \( b \), return a non-zero value if one is able to round correctly \( x \) to precision \( prec \) with the direction \( rnd2 \), and 0 otherwise (including for NaN and Inf). This function does not modify its arguments.

If \( rnd1 \) is MPFR_RNDN, then the sign of the error is unknown, but its absolute value is the same, so that the possible range is twice as large as with a directed rounding for \( rnd1 \).

Note: if one wants to also determine the correct ternary value, page 8 when rounding \( b \) to precision \( prec \) with rounding mode \( rnd \), a useful trick is the following:

```c
if (mpfr_can_round (b, err, MPFR_RNDN, MPFR_RNDZ, prec + (rnd == MPFR_RNDN)))
    ...
```

Indeed, if \( rnd \) is MPFR_RNDN, this will check if one can round to \( prec+1 \) bits with a directed rounding: if so, one can surely round to nearest to \( prec \) bits, and in addition one can determine the correct ternary value, which would not be the case when \( b \) is near from a value exactly representable on \( prec \) bits.

**mpfr_prec_t mpfr_min_prec (mpfr_t x)**

Return the minimal number of bits required to store the significand of \( x \), and 0 for special values, including 0. (Warning: the returned value can be less than MPFR_PREC_MIN.)

The function name is subject to change.

**const char * mpfr_print_rnd_mode (mpfr_rnd_t rnd)**

Return a string ("MPFR_RNDD", "MPFR_RNDU", "MPFR_RNDN", "MPFR_RNDZ", "MPFR_RNDA") corresponding to the rounding mode \( rnd \), or a null pointer if \( rnd \) is an invalid rounding mode.

### 5.12 Miscellaneous Functions

**void mpfr_nexttoward (mpfr_t x, mpfr_t y)**

If \( x \) or \( y \) is NaN, set \( x \) to NaN. If \( x \) and \( y \) are equal, \( x \) is unchanged. Otherwise, if \( x \) is different from \( y \), replace \( x \) by the next floating-point number (with the precision of \( x \) and the current exponent range) in the direction of \( y \) (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow or overflow is generated.

**void mpfr_nextabove (mpfr_t x)**

**void mpfr_nextbelow (mpfr_t x)**

Equivalent to mpfr_nexttoward where \( y \) is plus infinity (resp. minus infinity).

**int mpfr_min (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)**

**int mpfr_max (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)**

Set \( rop \) to the minimum (resp. maximum) of \( op1 \) and \( op2 \). If \( op1 \) and \( op2 \) are both NaN, then \( rop \) is set to NaN. If \( op1 \) or \( op2 \) is NaN, then \( rop \) is set to the numeric value. If \( op1 \) and \( op2 \) are zeros of different signs, then \( rop \) is set to \( -0 \) (resp. \(+0\)).
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int mpfr_urandomb (mpfr_t rop, gmp_randstate_t state)  [Function]
Generate a uniformly distributed random float in the interval $0 \leq \text{rop} < 1$. More precisely, the number can be seen as a float with a random non-normalized significand and exponent 0, which is then normalized (thus if $e$ denotes the exponent after normalization, then the least $-e$ significant bits of the significand are always 0).

Return 0, unless the exponent is not in the current exponent range, in which case rop is set to NaN and a non-zero value is returned (this should never happen in practice, except in very specific cases). The second argument is a gmp_randstate_t structure which should be created using the GMP gmp_randinit function (see the GMP manual).

Note: for a given version of MPFR, the returned value of rop and the new value of state (which controls further random values) do not depend on the machine word size.

int mpfr_urandom (mpfr_t rop, gmp_randstate_t state, mpfr_rnd_t rnd)  [Function]
Generate a uniformly distributed random float. The floating-point number rop can be seen as if a random real number is generated according to the continuous uniform distribution on the interval [0, 1] and then rounded in the direction rnd.

The second argument is a gmp_randstate_t structure which should be created using the GMP gmp_randinit function (see the GMP manual).

Note: the note for mpfr_urandomb holds too. In addition, the exponent range and the rounding mode might have a side effect on the next random state.

int mpfr_grandom (mpfr_t rop1, mpfr_t rop2, gmp_randstate_t state, mpfr_rnd_t rnd)  [Function]
Generate two random floats according to a standard normal gaussian distribution. If rop2 is a null pointer, then only one value is generated and stored in rop1.

The floating-point number rop1 (and rop2) can be seen as if a random real number were generated according to the standard normal gaussian distribution and then rounded in the direction rnd.

The third argument is a gmp_randstate_t structure, which should be created using the GMP gmp_randinit function (see the GMP manual).

The combination of the ternary values is returned like with mpfr_sin_cos. If rop2 is a null pointer, the second ternary value is assumed to be 0 (note that the encoding of the only ternary value is not the same as the usual encoding for functions that return only one result). Otherwise the ternary value of a random number is always non-zero.

Note: the note for mpfr_urandomb holds too. In addition, the exponent range and the rounding mode might have a side effect on the next random state.

mpfr_exp_t mpfr_get_exp (mpfr_t x)  [Function]
Return the exponent of x, assuming that x is a non-zero ordinary number and the significand is considered in [1/2,1). The behavior for NaN, infinity or zero is undefined.

int mpfr_set_exp (mpfr_t x, mpfr_exp_t e)  [Function]
Set the exponent of x if e is in the current exponent range, and return 0 (even if x is not a non-zero ordinary number); otherwise, return a non-zero value. The significand is assumed to be in [1/2,1).
int mpfr_signbit (mpfr_t op)  
  Return a non-zero value iff op has its sign bit set (i.e., if it is negative, −0, or a NaN whose  
  representation has its sign bit set).

int mpfr_setsign (mpfr_t rop, mpfr_t op, int s, mpfr_rnd_t rnd)  
  Set the value of rop from op, rounded toward the given direction rnd, then set (resp. clear)  
  its sign bit if s is non-zero (resp. zero), even when op is a NaN.

int mpfr_copysign (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)  
  Set the value of rop from op1, rounded toward the given direction rnd, then set its sign bit to  
  that of op2 (even when op1 or op2 is a NaN). This function is equivalent to mpfr_setsign  
  (rop, op1, mpfr_signbit (op2), rnd).

const char * mpfr_get_version (void)  
  Return the MPFR version, as a null-terminated string.

MPFR_VERSION  
MPFR_VERSION_MAJOR  
MPFR_VERSION_MINOR  
MPFR_VERSION_PATCHLEVEL  
MPFR_VERSION_STRING  

MPFR_VERSION is the version of MPFR as a preprocessing constant. MPFR_VERSION_MAJOR,  
MPFR_VERSION_MINOR and MPFR_VERSION_PATCHLEVEL are respectively the major, minor and  
patch level of MPFR version, as preprocessing constants. MPFR_VERSION_STRING is the version  
(with an optional suffix, used in development and pre-release versions) as a string constant,  
which can be compared to the result of mpfr_get_version to check at run time the  
header file and library used match:

    if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))  
      fprintf (stderr, "Warning: header and library do not match\n");

Note: Obtaining different strings is not necessarily an error, as in general, a program compiled  
with some old MPFR version can be dynamically linked with a newer MPFR library version  
(if allowed by the library versioning system).

long MPFR_VERSION_NUM (major, minor, patchlevel)  
  Create an integer in the same format as used by MPFR_VERSION from the given major, minor  
  and patchlevel. Here is an example of how to check the MPFR version at compile time:

    #if (!defined(MPFR_VERSION) || (MPFR_VERSION<MPFR_VERSION_NUM(3,0,0)))
    # error "Wrong MPFR version."
    #endif

const char * mpfr_get_patches (void)  
  Return a null-terminated string containing the ids of the patches applied to the MPFR library  
  (contents of the PATCHES file), separated by spaces. Note: If the program has been compiled  
  with an older MPFR version and is dynamically linked with a new MPFR library version, the  
  identifiers of the patches applied to the old (compile-time) MPFR version are not available  
  (however this information should not have much interest in general).

int mpfr_buildopt_tls_p (void)  
  Return a non-zero value if MPFR was compiled as thread safe using compiler-level Thread  
  Local Storage (that is, MPFR was built with the --enable-thread-safe configure option,  
  see INSTALL file), return zero otherwise.
int mpfr_buildopt_decimal_p (void) [Function]
Return a non-zero value if MPFR was compiled with decimal float support (that is, MPFR was built with the --enable-decimal-float configure option), return zero otherwise.

int mpfr_buildopt_gmpinternals_p (void) [Function]
Return a non-zero value if MPFR was compiled with GMP internals (that is, MPFR was built with either --with-gmp-build or --enable-gmp-internals configure option), return zero otherwise.

const char * mpfr_buildopt_tune_case (void) [Function]
Return a string saying which thresholds file has been used at compile time. This file is normally selected from the processor type.

5.13 Exception Related Functions

mpfr_exp_t mpfr_get_emin (void) [Function]
mpfr_exp_t mpfr_get_emax (void) [Function]
Return the (current) smallest and largest exponents allowed for a floating-point variable. The smallest positive value of a floating-point variable is $1/2 \times 2^{\text{emin}}$ and the largest value has the form $(1 - \varepsilon) \times 2^{\text{emax}}$, where $\varepsilon$ depends on the precision of the considered variable.

int mpfr_set_emin (mpfr_exp_t exp) [Function]
int mpfr_set_emax (mpfr_exp_t exp) [Function]
Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero value when exp is not in the range accepted by the implementation (in that case the smallest or largest exponent is not changed), and zero otherwise. If the user changes the exponent range, it is her/his responsibility to check that all current floating-point variables are in the new allowed range (for example using mpfr_check_range), otherwise the subsequent behavior will be undefined, in the sense of the ISO C standard.

mpfr_exp_t mpfr_get_emin_min (void) [Function]
mpfr_exp_t mpfr_get_emin_max (void) [Function]
mpfr_exp_t mpfr_get_emax_min (void) [Function]
mpfr_exp_t mpfr_get_emax_max (void) [Function]
Return the minimum and maximum of the exponents allowed for mpfr_set_emin and mpfr_set_emax respectively. These values are implementation dependent, thus a program using mpfr_set_emax(mpfr_get_emax_max()) or mpfr_set_emin(mpfr_get_emin_min()) may not be portable.

int mpfr_check_range (mpfr_t x, int t, mpfr_rnd_t rnd) [Function]
This function assumes that x is the correctly-rounded value of some real value y in the direction rnd and some extended exponent range, and that t is the corresponding ternary value, page 8. For example, one performed $t = \text{mpfr_log} (x, u, \text{rnd})$, and y is the exact logarithm of u. Thus t is negative if x is smaller than y, positive if x is larger than y, and zero if x equals y. This function modifies x if needed to be in the current range of acceptable values: It generates an underflow or an overflow if the exponent of x is outside the current allowed range; the value of t may be used to avoid a double rounding. This function returns zero if the new value of x equals the exact one y, a positive value if that new value is larger than y, and a negative value if it is smaller than y. Note that unlike most functions, the new result x is compared to the (unknown) exact one y, not the input value x, i.e., the ternary value is propagated.
Note: If \( x \) is an infinity and \( t \) is different from zero (i.e., if the rounded result is an inexact infinity), then the overflow flag is set. This is useful because mpfr_check_range is typically called (at least in MPFR functions) after restoring the flags that could have been set due to internal computations.

```c
int mpfr_subnormalize (mpfr_t x, int t, mpfr_rnd_t rnd)
```

This function rounds \( x \) emulating subnormal number arithmetic: if \( x \) is outside the subnormal exponent range, it just propagates the [ternary value], page 8 \( t \); otherwise, it rounds \( x \) to precision \( \text{EXP}(x) - \text{emin} + 1 \) according to rounding mode \( \text{rnd} \) and previous ternary value \( t \), avoiding double rounding problems. More precisely in the subnormal domain, denoting by \( e \) the value of \( \text{emin} \), \( x \) is rounded in fixed-point arithmetic to an integer multiple of \( 2^{e-1} \); as a consequence, \( 1.5 \times 2^{e-1} \) when \( t \) is zero is rounded to \( 2^e \) with rounding to nearest.

\( \text{PREC}(x) \) is not modified by this function. \( \text{rnd} \) and \( t \) must be the rounding mode and the returned ternary value used when computing \( x \) (as in mpfr_check_range). The subnormal exponent range is from \( \text{emin} \) to \( \text{emin} + \text{PREC}(x) - 1 \). If the result cannot be represented in the current exponent range (due to a too small \( \text{emax} \)), the behavior is undefined. Note that unlike most functions, the result is compared to the exact one, not the input value \( x \), i.e., the ternary value is propagated.

As usual, if the returned ternary value is non zero, the inexact flag is set. Moreover, if a second rounding occurred (because the input \( x \) was in the subnormal range), the underflow flag is set.

This is an example of how to emulate binary double IEEE 754 arithmetic (binary64 in IEEE 754-2008) using MPFR:

```c
{
    mpfr_t xa, xb; int i; volatile double a, b;

    mpfr_set_default_prec (53);
    mpfr_set_emin (-1073); mpfr_set_emax (1024);
    mpfr_init (xa); mpfr_init (xb);

    b = 34.3; mpfr_set_d (xb, b, MPFR_RNDN);
    a = 0x1.1235P-1021; mpfr_set_d (xa, a, MPFR_RNDN);

    a /= b;
    i = mpfr_div (xa, xa, xb, MPFR_RNDN);
    i = mpfr_subnormalize (xa, i, MPFR_RNDN); /* new ternary value */

    mpfr_clear (xa); mpfr_clear (xb);
}
```

Warning: this emulates a double IEEE 754 arithmetic with correct rounding in the subnormal range, which may not be the case for your hardware.

```c
void mpfr_clear_underflow (void) [Function]
void mpfr_clear_overflow (void) [Function]
void mpfr_clear_divby0 (void) [Function]
void mpfr_clear_nanflag (void) [Function]
void mpfr_clear_inexflag (void) [Function]
```
void mpfr_clear_erangeflag (void)  
Clear the underflow, overflow, divide-by-zero, invalid, inexact and erreange flags.

void mpfr_set_underflow (void)  
void mpfr_set_overflow (void)  
void mpfr_set_divby0 (void)  
void mpfr_set_nanflag (void)  
void mpfr_set_inexflag (void)  
void mpfr_set_erangeflag (void)  
Set the underflow, overflow, divide-by-zero, invalid, inexact and erreange flags.

void mpfr_clear_flags (void)  
Clear all global flags (underflow, overflow, divide-by-zero, invalid, inexact, erreange).

int mpfr_underflow_p (void)  
int mpfr_overflow_p (void)  
int mpfr_divby0_p (void)  
int mpfr_nanflag_p (void)  
int mpfr_inexflag_p (void)  
int mpfr_erangeflag_p (void)  
Return the corresponding (underflow, overflow, divide-by-zero, invalid, inexact, erreange) flag,  
which is non-zero iff the flag is set.

5.14 Compatibility With MPF
A header file mpf2mpfr.h is included in the distribution of MPFR for compatibility with the  
GNU MP class MPF. By inserting the following two lines after the #include <gmp.h> line,

#include <mpfr.h>
#include <mpf2mpfr.h>

any program written for MPF can be compiled directly with MPFR without any changes (except  
the gmp_printf functions will not work for arguments of type mpfr_t). All operations are then  
performed with the default MPFR rounding mode, which can be reset with mpfr_set_default_  
rounding_mode.

Warning: the mpf_init and mpf_init2 functions initialize to zero, whereas the corresponding  
MPFR functions initialize to NaN: this is useful to detect uninitialized values, but is slightly  
incompatible with MPF.

void mpfr_set_prec_raw (mpfr_t x, mpfr_prec_t prec)  
Reset the precision of x to be exactly prec bits. The only difference with mpfr_set_prec is  
that prec is assumed to be small enough so that the significand fits into the current allocated  
memory space for x. Otherwise the behavior is undefined.

int mpfr_eq (mpfr_t op1, mpfr_t op2, unsigned long int op3)  
Return non-zero if op1 and op2 are both non-zero ordinary numbers with the same exponent  
and the same first op3 bits, both zero, or both infinities of the same sign. Return zero  
otherwise. This function is defined for compatibility with MPF, we do not recommend to  
use it otherwise. Do not use it either if you want to know whether two numbers are close to  
each other; for instance, 1.011111 and 1.100000 are regarded as different for any value of op3  
larger than 1.
void mpfr_reldiff (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
Compute the relative difference between op1 and op2 and store the result in rop. This function does not guarantee the correct rounding on the relative difference; it just computes $|op1 - op2|/op1$, using the precision of rop and the rounding mode rnd for all operations.

int mpfr_mul_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_div_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
These functions are identical to mpfr_mul_2ui and mpfr_div_2ui respectively. These functions are only kept for compatibility with MPF, one should prefer mpfr_mul_2ui and mpfr_div_2ui otherwise.

5.15 Custom Interface

Some applications use a stack to handle the memory and their objects. However, the MPFR memory design is not well suited for such a thing. So that such applications are able to use MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows one to use MPFR in two ways:

- Either directly store a floating-point number as a mpfr_t on the stack.
- Either store its own representation on the stack and construct a new temporary mpfr_t each time it is needed.

Nothing has to be done to destroy the floating-point numbers except garbageing the used memory: all the memory management (allocating, destroying, garbageing) is left to the application.

Each function in this interface is also implemented as a macro for efficiency reasons: for example mpfr_custom_init (s, p) uses the macro, while (mpfr_custom_init) (s, p) uses the function.

Note 1: MPFR functions may still initialize temporary floating-point numbers using mpfr_init and similar functions. See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (mpfr_const_pi for example), even if they are not explicitly called. You have to call mpfr_free_cache each time you garbage the memory iff mpfr_init, through GMP Custom Allocation, allocates its memory on the application stack.

size_t mpfr_custom_get_size (mpfr_prec_t prec) [Function]
Return the needed size in bytes to store the significand of a floating-point number of precision prec.

void mpfr_custom_init (void *significand, mpfr_prec_t prec) [Function]
Initialize a significand of precision prec, where significand must be an area of mpfr_custom_get_size (prec) bytes at least and be suitably aligned for an array of mp_limb_t (GMP type, see Section 5.16 [Internals], page 39).

void mpfr_custom_init_set (mpfr_t x, int kind, mpfr_exp_t exp, mpfr_prec_t prec, void *significand) [Function]
Perform a dummy initialization of a mpfr_t and set it to:

- if ABS(kind) == MPFR_NAN_KIND, x is set to NaN;
- if ABS(kind) == MPFR_INF_KIND, x is set to the infinity of sign sign(kind);
• if \( \text{ABS}(\text{kind}) == \text{MPFR\_ZERO\_KIND} \), \( x \) is set to the zero of sign \( \text{sign(kind)} \);
• if \( \text{ABS}(\text{kind}) == \text{MPFR\_REGULAR\_KIND} \), \( x \) is set to a regular number: \( x = \text{sign(kind)}*\text{significand}*2^\text{exp} \).

In all cases, it uses \textit{significand} directly for further computing involving \( x \). It will not allocate anything. A floating-point number initialized with this function cannot be resized using \texttt{mpfr\_set\_prec} or \texttt{mpfr\_prec\_round}, or cleared using \texttt{mpfr\_clear}! The \textit{significand} must have been initialized with \texttt{mpfr\_custom\_init} using the same precision \texttt{prec}.

\[
\text{int mpfr\_custom\_get\_kind (mpfr\_t x)} \quad \text{[Function]}
\]
Return the current kind of a mpfr\_t as created by \texttt{mpfr\_custom\_init\_set}. The behavior of this function for any mpfr\_t not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

\[
\text{void \* mpfr\_custom\_get\_significand (mpfr\_t x)} \quad \text{[Function]}
\]
Return a pointer to the significand used by a mpfr\_t initialized with \texttt{mpfr\_custom\_init\_set}. The behavior of this function for any mpfr\_t not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

\[
\text{mpfr\_exp\_t mpfr\_custom\_get\_exp (mpfr\_t x)} \quad \text{[Function]}
\]
Return the exponent of \( x \), assuming that \( x \) is a non-zero ordinary number. The return value for NaN, Infinity or zero is unspecified but does not produce any trap. The behavior of this function for any mpfr\_t not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

\[
\text{void mpfr\_custom\_move (mpfr\_t x, void \*new\_position)} \quad \text{[Function]}
\]
Inform MPFR that the significand of \( x \) has moved due to a garbage collect and update its new position to \texttt{new\_position}. However the application has to move the significand and the mpfr\_t itself. The behavior of this function for any mpfr\_t not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

5.16 Internals

A \textit{limb} means the part of a multi-precision number that fits in a single word. Usually a limb contains 32 or 64 bits. The C data type for a limb is \texttt{mp\_limb\_t}.

The mpfr\_t type is internally defined as a one-element array of a structure, and mpfr\_ptr is the C data type representing a pointer to this structure. The mpfr\_t type consists of four fields:

• The \_mpfr\_prec field is used to store the precision of the variable (in bits); this is not less than \texttt{MPFR\_PREC\_MIN}.
• The \_mpfr\_sign field is used to store the sign of the variable.
• The \_mpfr\_exp field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values \( n \) are a multiplier \( 2^n \) relative to that point. A NaN, an infinity and a zero are indicated by special values of the exponent field.
• Finally, the \_mpfr\_d field is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by \_mpfr\_prec, namely ceil(\_mpfr\_prec/mp\_bits\_per\_limb). Non-singular (i.e., different from NaN, Infinity or zero) values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zeros.
6 API Compatibility

The goal of this section is to describe some API changes that occurred from one version of MPFR to another, and how to write code that can be compiled and run with older MPFR versions. The minimum MPFR version that is considered here is 2.2.0 (released on 20 September 2005).

API changes can only occur between major or minor versions. Thus the patchlevel (the third number in the MPFR version) will be ignored in the following. If a program does not use MPFR internals, changes in the behavior between two versions differing only by the patchlevel should only result from what was regarded as a bug or unspecified behavior.

As a general rule, a program written for some MPFR version should work with later versions, possibly except at a new major version, where some features (described as obsolete for some time) can be removed. In such a case, a failure should occur during compilation or linking. If a result becomes incorrect because of such a change, please look at the various changes below (they are minimal, and most software should be unaffected), at the FAQ and at the MPFR web page for your version (a bug could have been introduced and be already fixed); and if the problem is not mentioned, please send us a bug report (see Chapter 3 [Reporting Bugs], page 5).

However, a program written for the current MPFR version (as documented by this manual) may not necessarily work with previous versions of MPFR. This section should help developers to write portable code.

Note: Information given here may be incomplete. API changes are also described in the NEWS file (for each version, instead of being classified like here), together with other changes.

6.1 Type and Macro Changes

The official type for exponent values changed from \texttt{mp\_exp\_t} to \texttt{mpfr\_exp\_t} in MPFR 3.0. The type \texttt{mp\_exp\_t} will remain available as it comes from GMP (with a different meaning). These types are currently the same (\texttt{mpfr\_exp\_t} is defined as \texttt{mp\_exp\_t} with \texttt{typedef}), so that programs can still use \texttt{mp\_exp\_t}; but this may change in the future. Alternatively, using the following code after including \texttt{mpfr.h} will work with official MPFR versions, as \texttt{mpfr\_exp\_t} was never defined in MPFR 2.x:

\begin{verbatim}
#if MPFR_VERSION_MAJOR < 3
typedef mp\_exp\_t mpfr\_exp\_t;
#endif
\end{verbatim}

The official types for precision values and for rounding modes respectively changed from \texttt{mp\_prec\_t} and \texttt{mp\_rnd\_t} to \texttt{mpfr\_prec\_t} and \texttt{mpfr\_rnd\_t} in MPFR 3.0. This change was actually done a long time ago in MPFR, at least since MPFR 2.2.0, with the following code in \texttt{mpfr.h}:

\begin{verbatim}
#ifndef mp\_rnd\_t
#define mp\_rnd\_t mpfr\_rnd\_t
#endif
#ifndef mp\_prec\_t
#define mp\_prec\_t mpfr\_prec\_t
#endif
\end{verbatim}

This means that it is safe to use the new official types \texttt{mpfr\_prec\_t} and \texttt{mpfr\_rnd\_t} in your programs. The types \texttt{mp\_prec\_t} and \texttt{mp\_rnd\_t} (defined in MPFR only) may be removed in the future, as the prefix \texttt{mp\_} is reserved by GMP.

The precision type \texttt{mpfr\_prec\_t} (\texttt{mp\_prec\_t}) was unsigned before MPFR 3.0; it is now signed. \texttt{MPFR\_PREC\_MAX} has not changed, though. Indeed the MPFR code requires that \texttt{MPFR\_PREC\_MAX}
be representable in the exponent type, which may have the same size as \texttt{mpfr\_prec\_t} but has always been signed. The consequence is that valid code that does not assume anything about the signedness of \texttt{mpfr\_prec\_t} should work with past and new MPFR versions. This change was useful as the use of unsigned types tends to convert signed values to unsigned ones in expressions due to the usual arithmetic conversions, which can yield incorrect results if a negative value is converted in such a way. Warning! A program assuming (intentionally or not) that \texttt{mpfr\_prec\_t} is signed may be affected by this problem when it is built and run against MPFR 2.x.

The rounding modes \texttt{GMP\_RNDx} were renamed to \texttt{MPFR\_RNDx} in MPFR 3.0. However the old names \texttt{GMP\_RNDx} have been kept for compatibility (this might change in future versions), using:

\begin{verbatim}
#define GMP_RNDN MPFR_RNDN
#define GMP_RNDZ MPFR_RNDZ
#define GMP_RNDU MPFR_RNDU
#define GMP_RNDD MPFR_RNDD
\end{verbatim}

The rounding mode “round away from zero” (\texttt{MPFR\_RNDA}) was added in MPFR 3.0 (however no rounding mode \texttt{GMP\_RNDA} exists).

### 6.2 Added Functions

We give here in alphabetical order the functions that were added after MPFR 2.2, and in which MPFR version.

- \texttt{mpfr\_add\_d} in MPFR 2.4.
- \texttt{mpfr\_ai} in MPFR 3.0 (incomplete, experimental).
- \texttt{mpfr\_asprintf} in MPFR 2.4.
- \texttt{mpfr\_buildopt\_decimal\_p} and \texttt{mpfr\_buildopt\_tls\_p} in MPFR 3.0.
- \texttt{mpfr\_buildopt\_gmpinternals\_p} and \texttt{mpfr\_buildopt\_tune\_case} in MPFR 3.1.
- \texttt{mpfr\_clear\_divby0} in MPFR 3.1 (new divide-by-zero exception).
- \texttt{mpfr\_copysign} in MPFR 2.3. Note: MPFR 2.2 had a \texttt{mpfr\_copysign} function that was available, but not documented, and with a slight difference in the semantics (when the second input operand is a NaN).
- \texttt{mpfr\_custom\_get\_significand} in MPFR 3.0. This function was named \texttt{mpfr\_custom\_get\_mantissa} in previous versions; \texttt{mpfr\_custom\_get\_mantissa} is still available via a macro in \texttt{mpfr.h}:

\begin{verbatim}
#define mpfr_custom_get_mantissa mpfr_custom_get_significand
\end{verbatim}

Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use \texttt{mpfr\_custom\_get\_mantissa}.

- \texttt{mpfr\_d\_div} and \texttt{mpfr\_d\_sub} in MPFR 2.4.
- \texttt{mpfr\_digamma} in MPFR 3.0.
- \texttt{mpfr\_divby0\_p} in MPFR 3.1 (new divide-by-zero exception).
- \texttt{mpfr\_div\_d} in MPFR 2.4.
- \texttt{mpfr\_fmod} in MPFR 2.4.
- \texttt{mpfr\_fms} in MPFR 2.3.
- \texttt{mpfr\_fprintf} in MPFR 2.4.
- \texttt{mpfr\_frexp} in MPFR 3.1.
- \texttt{mpfr\_get\_flt} in MPFR 3.0.
- \texttt{mpfr\_get\_patches} in MPFR 2.3.
- \texttt{mpfr\_get\_z\_2exp} in MPFR 3.0. This function was named \texttt{mpfr\_get\_z\_exp} in previous versions; \texttt{mpfr\_get\_z\_exp} is still available via a macro in \texttt{mpfr.h}:
#define mpfr_get_z_exp mpfr_get_z_2exp
Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use mpfr_get_z_exp.

- mpfr_grandom in MPFR 3.1.
- mpfr_j0, mpfr_j1 and mpfr_jn in MPFR 2.3.
- mpfr_lgamma in MPFR 2.3.
- mpfr_li2 in MPFR 2.4.
- mpfr_min_prec in MPFR 3.0.
- mpfr_modf in MPFR 2.4.
- mpfr_mul_d in MPFR 2.4.
- mpfr_printf in MPFR 2.4.
- mpfr_rec_sqrt in MPFR 2.4.
- mpfr_regular_p in MPFR 3.0.
- mpfr_remainder and mpfr_remquo in MPFR 2.3.
- mpfr_set_divby0 in MPFR 3.1 (new divide-by-zero exception).
- mpfr_set_flt in MPFR 3.0.
- mpfr_set_z_2exp in MPFR 3.0.
- mpfr_set_zero in MPFR 3.0.
- mpfr_setsign in MPFR 2.3.
- mpfr_signbit in MPFR 2.3.
- mpfr_sinh_cosh in MPFR 2.4.
- mpfr_snprintf and mpfr_sprintf in MPFR 2.4.
- mpfr_sub_d in MPFR 2.4.
- mpfr_urandom in MPFR 3.0.
- mpfr_vasprintf, mpfr_vfprintf, mpfr_vprintf, mpfr_vsprintf and mpfr_vsnprintf in MPFR 2.4.
- mpfr_y0, mpfr_y1 and mpfr_yn in MPFR 2.3.
- mpfr_z_sub in MPFR 3.1.

### 6.3 Changed Functions

The following functions have changed after MPFR 2.2. Changes can affect the behavior of code written for some MPFR version when built and run against another MPFR version (older or newer), as described below.

- mpfr_check_range changed in MPFR 2.3.2 and MPFR 2.4. If the value is an inexact infinity, the overflow flag is now set (in case it was lost), while it was previously left unchanged. This is really what is expected in practice (and what the MPFR code was expecting), so that the previous behavior was regarded as a bug. Hence the change in MPFR 2.3.2.
- mpfr_get_f changed in MPFR 3.0. This function was returning zero, except for NaN and Inf, which do not exist in MPF. The erange flag is now set in these cases, and mpfr_get_f now returns the usual ternary value.
- mpfr_get_si, mpfr_get_sj, mpfr_get_ui and mpfr_get_uj changed in MPFR 3.0. In previous MPFR versions, the cases where the erange flag is set were unspecified.
- mpfr_get_z changed in MPFR 3.0. The return type was void; it is now int, and the usual ternary value is returned. Thus programs that need to work with both MPFR 2.x and 3.x must not use the return value. Even in this case, C code using mpfr_get_z as the second
or third term of a conditional operator may also be affected. For instance, the following is
correct with MPFR 3.0, but not with MPFR 2.x:

```c
bool ? mpfr_get_z(...) : mpfr_add(...);
```

On the other hand, the following is correct with MPFR 2.x, but not with MPFR 3.0:

```c
bool ? mpfr_get_z(...) : (void) mpfr_add(...);
```

Portable code should cast `mpfr_get_z(...)` to `void` to use the type `void` for both terms
of the conditional operator, as in:

```c
bool ? (void) mpfr_get_z(...) : (void) mpfr_add(...);
```

Alternatively, if ... else can be used instead of the conditional operator.

Moreover the cases where the `erange` flag is set were unspecified in MPFR 2.x.

- `mpfr_get_z_exp` changed in MPFR 3.0. In previous MPFR versions, the cases where the `erange`
  flag is set were unspecified. Note: this function has been renamed to `mpfr_get_z_2exp` in MPFR 3.0,
  but `mpfr_get_z_exp` is still available for compatibility reasons.

- `mpfr_strtofr` changed in MPFR 2.3.1 and MPFR 2.4. This was actually a bug fix since
  the code and the documentation did not match. But both were changed in order to have
  a more consistent and useful behavior. The main changes in the code are as follows. The
  binary exponent is now accepted even without the `0b` or `0x` prefix. Data corresponding to
  NaN can now have an optional sign (such data were previously invalid).

- `mpfr_strtofr` changed in MPFR 3.0. This function now accepts bases from 37 to 62 (no
  changes for the other bases). Note: if an unsupported base is provided to this function, the
  behavior is undefined; more precisely, in MPFR 2.3.1 and later, providing an unsupported
  base yields an assertion failure (this behavior may change in the future).

- `mpfr_subnormalize` changed in MPFR 3.1. This was actually regarded as a bug fix. The
  `mpfr_subnormalize` implementation up to MPFR 3.0.0 did not change the flags. In partic-
  ular, it did not follow the generic rule concerning the inexact flag (and no special behavior
  was specified). The case of the underflow flag was more a lack of specification.

- `mpfr_urandom` and `mpfr_urandomb` changed in MPFR 3.1. Their behavior no longer de-
  pends on the platform (assuming this is also true for GMP’s random generator, which is not
  the case between GMP 4.1 and 4.2 if `gmp_randinit_default` is used). As a consequence,
  the returned values can be different between MPFR 3.1 and previous MPFR versions. Note:
  as the reproducibility of these functions was not specified before MPFR 3.1, the MPFR 3.1
  behavior is not regarded as backward incompatible with previous versions.

### 6.4 Removed Functions

Functions `mpfr_random` and `mpfr_random2` have been removed in MPFR 3.0 (this only affects
old code built against MPFR 3.0 or later). (The function `mpfr_random` had been deprecated
since at least MPFR 2.2.0, and `mpfr_random2` since MPFR 2.4.0.)

### 6.5 Other Changes

For users of a C++ compiler, the way how the availability of `intmax_t` is detected has changed
in MPFR 3.0. In MPFR 2.x, if a macro `INTMAX_C` or `UINTMAX_C` was defined (e.g. when the
`__STDC_CONSTANT_MACROS` macro had been defined before `<stdint.h>` or `<inttypes.h>` has
been included), `intmax_t` was assumed to be defined. However this was not always the case
(more precisely, `intmax_t` can be defined only in the namespace std, as with Boost), so that
compilations could fail. Thus the check for `INTMAX_C` or `UINTMAX_C` is now disabled for C++
compilers, with the following consequences:

- Programs written for MPFR 2.x that need `intmax_t` may no longer be compiled against
  MPFR 3.0: a `#define MPFR_USE_INTMAX_T` may be necessary before `mpfr.h` is included.
• The compilation of programs that work with MPFR 3.0 may fail with MPFR 2.x due to the problem described above. Workarounds are possible, such as defining `intmax_t` and `uintmax_t` in the global namespace, though this is not clean.

The divide-by-zero exception is new in MPFR 3.1. However it should not introduce incompatible changes for programs that strictly follow the MPFR API since the exception can only be seen via new functions.

As of MPFR 3.1, the `mpfr.h` header can be included several times, while still supporting optional functions (see Section 4.1 [Headers and Libraries], page 6).
Contributors

The main developers of MPFR are Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier, Philippe Théveny and Paul Zimmermann.

Sylvie Boldo from ENS-Lyon, France, contributed the functions mpfr_agm and mpfr_log. Sylvain Chevillard contributed the mpfr_ai function. David Daney contributed the hyperbolic and inverse hyperbolic functions, the base-2 exponential, and the factorial function. Alain Delplanque contributed the new version of the mpfr_get_str function. Mathieu Dutour contributed the functions mpfr_acos, mpfr_asin and mpfr_atan, and a previous version of mpfr_gamma. Laurent Fousse contributed the mpfr_sum function. Emmanuel Jeandel, from ENS-Lyon too, contributed the generic hypergeometric code, as well as the internal function mpfr_exp3, a first implementation of the sine and cosine, and improved versions of mpfr_const_log2 and mpfr_const_pi. Ludovic Meunier helped in the design of the mpfr_erf code. Jean-Luc Rémy contributed the mpfr_zeta code. Fabrice Rouillier contributed the mpfr_xxx_z and mpfr_xxx_q functions, and helped to the Microsoft Windows porting. Damien Stehlé contributed the mpfr_get_ld_2exp function.

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- Richard Brent and Paul Zimmermann, "Modern Computer Arithmetic", Cambridge University Press (to appear), also available from the authors' web pages.
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