This manual documents how to install and use the Multiple Precision Floating-Point Reliable Library, version 2.3.2.


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MPFR Copying Conditions

This library is free; this means that everyone is free to use it and free to redistribute it on a free basis. The library is not in the public domain; it is copyrighted and there are restrictions on its distribution, but these restrictions are designed to permit everything that a good cooperating citizen would want to do. What is not allowed is to try to prevent others from further sharing any version of this library that they might get from you.

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The precise conditions of the license for the MPFR library are found in the Lesser General Public License that accompanies the source code. See the file COPYING.LIB.
1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to extend the class of floating-point numbers provided by the GNU MP library by a precise semantics. The main differences with the mpfr class from GNU MP are:

- the MPFR code is portable, i.e. the result of any operation does not depend (or should not) on the machine word size mp_bits_per_limb (32 or 64 on most machines);
- the precision in bits can be set exactly to any valid value for each variable (including very small precision);
- MPFR provides the four rounding modes from the IEEE 754-1985 standard.

In particular, with a precision of 53 bits, MPFR should be able to exactly reproduce all computations with double-precision machine floating-point numbers (e.g., double type in C, with a C implementation that rigorously follows Annex F of the ISO C99 standard and FP_CONTRACT pragma set to OFF) on the four arithmetic operations and the square root, except the default exponent range is much wider and subnormal numbers are not implemented (but can be emulated).

This version of MPFR is released under the GNU Lesser General Public License, Version 2.1 or any later version. It is permitted to link MPFR to most non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

1.1 How to Use This Manual

Everyone should read Chapter 4 [MPFR Basics], page 6. If you need to install the library yourself, you need to read Chapter 2 [Installing MPFR], page 3, too.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.
2 Installing MPFR

2.1 How to Install

Here are the steps needed to install the library on Unix systems (more details are provided in the ‘INSTALL’ file):

1. To build MPFR, you first have to install GNU MP (version 4.1 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need a standard Unix ‘make’ program, plus some other standard Unix utility programs.

2. In the MPFR build directory, type ‘./configure’
   This will prepare the build and setup the options according to your system. If you get error messages, you might check that you use the same compiler and compile options as for GNU MP (see the ‘INSTALL’ file).

3. ‘make’
   This will compile MPFR, and create a library archive file ‘libmpfr.a’. A dynamic library may be produced too (see configure).

4. ‘make check’
   This will make sure MPFR was built correctly. If you get error messages, please report this to ‘mpfr@loria.fr’. (See Chapter 3 [Reporting Bugs], page 5, for information on what to include in useful bug reports.)

5. ‘make install’
   This will copy the files ‘mpfr.h’ and ‘mpf2mpfr.h’ to the directory ‘/usr/local/include’, the file ‘libmpfr.a’ to the directory ‘/usr/local/lib’, and the file ‘mpfr.info’ to the directory ‘/usr/local/share/info’ (or if you passed the ‘--prefix’ option to ‘configure’, using the prefix directory given as argument to ‘--prefix’ instead of ‘/usr/local’).

2.2 Other ‘make’ Targets

There are some other useful make targets:

- ‘mpfr.info’ or ‘info’
  Create an info version of the manual, in ‘mpfr.info’.

- ‘mpfr.pdf’ or ‘pdf’
  Create a PDF version of the manual, in ‘mpfr.pdf’.

- ‘mpfr.dvi’ or ‘dvi’
  Create a DVI version of the manual, in ‘mpfr.dvi’.

- ‘mpfr.ps’ or ‘ps’
  Create a Postscript version of the manual, in ‘mpfr.ps’.

- ‘mpfr.html’ or ‘html’
  Create a HTML version of the manual, in several pages in the directory ‘mpfr.html’; if you want only one output HTML file, then type ‘makeinfo --html --no-split mpfr.texi’ instead.

- ‘clean’
  Delete all object files and archive files, but not the configuration files.

- ‘distclean’
  Delete all files not included in the distribution.

- ‘uninstall’
  Delete all files copied by ‘make install’. 
2.3 Build Problems

In case of problem, please read the ‘INSTALL’ file carefully before reporting a bug, in particular section “In case of problem”. Some problems are due to bad configuration on the user side (not specific to MPFR). Problems are also mentioned in the FAQ http://www.mpfr.org/faq.html. Please report problems to ‘mpfr@loria.fr’. See Chapter 3 [Reporting Bugs], page 5. Some bug fixes are available on the MPFR 2.3.2 web page http://www.mpfr.org/mpfr-2.3.2/.

2.4 Getting the Latest Version of MPFR

The latest version of MPFR is available from http://www.mpfr.org/.
3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR 2.3.2 web page http://www.mpfr.org/mpfr-2.3.2/ and the FAQ http://www.mpfr.org/faq.html: perhaps this bug is already known, in which case you may find there a workaround for it. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results printed are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using ‘cc -V’ on some machines, or, if you’re using gcc, ‘gcc -v’. Also, include the output from ‘uname -a’ and the MPFR version (the GMP version may be useful too).

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we won’t do anything about it (aside of chiding you to send better bug reports).

Send your bug report to: ‘mpfr@loria.fr’.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.
4 MPFR Basics

4.1 Headers and Libraries

All declarations needed to use MPFR are collected in the include file ‘mpfr.h’. It is designed
to work with both C and C++ compilers. You should include that file in any program using the
MPFR library:

```c
#include <mpfr.h>
```

Note however that prototypes for MPFR functions with FILE * parameters are provided only if
<stdio.h> is included too (before ‘mpfr.h’).

```c
#include <stdio.h>
#include <mpfr.h>
```

You can avoid the use of MPFR macros encapsulating functions by defining the
‘MPFR_USE_NO_MACRO’ macro before ‘mpfr.h’ is included. In general this should not be
necessary, but this can be useful when debugging user code: with some macros, the compiler
may emit spurious warnings with some warning options, and macros can prevent some
prototype checking.

All programs using MPFR must link against both ‘libmpfr’ and ‘libgmp’ libraries. On a typical
Unix-like system this can be done with ‘-lmpfr -lgmp’ (in that order), for example

```c
gcc myprogram.c -lmpfr -lgmp
```

MPFR is built using Libtool and an application can use that to link if desired, see GNU Libtool.

If MPFR has been installed to a non-standard location, then it may be necessary to set up envi-
ronment variables such as ‘C_INCLUDE_PATH’ and ‘LIBRARY_PATH’, or use ‘-I’ and ‘-L’ compiler
options, in order to point to the right directories. For a shared library, it may also be necessary
to set up some sort of run-time library path (e.g., ‘LD_LIBRARY_PATH’) on some systems. Please
read the ‘INSTALL’ file for additional information.

4.2 Nomenclature and Types

A floating-point number or float for short, is an arbitrary precision significand (also called
mantissa) with a limited precision exponent. The C data type for such objects is mpfr_t
(internally defined as a one-element array of a structure, and mpfr_ptr is the C data type
representing a pointer to this structure). A floating-point number can have three special values:
Not-a-Number (NaN) or plus or minus Infinity. NaN represents an uninitialized object, the
result of an invalid operation (like 0 divided by 0), or a value that cannot be determined (like
+Infinity minus +Infinity). Moreover, like in the IEEE 754-1985 standard, zero is signed, i.e.
there are both +0 and −0; the behavior is the same as in the IEEE 754-1985 standard and it is
generalized to the other functions supported by MPFR.

The precision is the number of bits used to represent the significand of a floating-point number;
the corresponding C data type is mp_prec_t. The precision can be any integer between MPFR_
PREC_MIN and MPFR_PREC_MAX. In the current implementation, MPFR_PREC_MIN is equal to 2.

Warning! MPFR needs to increase the precision internally, in order to provide accurate results
(and in particular, correct rounding). Do not attempt to set the precision to any value near
MPFR_PREC_MAX, otherwise MPFR will abort due to an assertion failure. Moreover, you may
reach some memory limit on your platform, in which case the program may abort, crash or have
undefined behavior (depending on your C implementation).
The *rounding mode* specifies the way to round the result of a floating-point operation, in case the exact result cannot be represented exactly in the destination significand; the corresponding C data type is `mp_rnd_t`.

A *limb* means the part of a multi-precision number that fits in a single word. (We chose this word because a limb of the human body is analogous to a digit, only larger, and containing several digits.) Normally a limb contains 32 or 64 bits. The C data type for a limb is `mp_limb_t`.

### 4.3 Function Classes

There is only one class of functions in the MPFR library:

1. Functions for floating-point arithmetic, with names beginning with `mpfr_`. The associated type is `mpfr_t`.

### 4.4 MPFR Variable Conventions

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator.

MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication, `mpfr_mul`, can be used like this: `mpfr_mul (x, x, x, rnd_mode)`. This computes the square of `x` with rounding mode `rnd_mode` and puts the result back in `x`.

Before you can assign to an MPFR variable, you need to initialize it by calling one of the special initialization functions. When you’re done with a variable, you need to clear it out, using one of the functions for that purpose.

A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned to any number of times.

For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited.

You don’t need to be concerned about allocating additional space for MPFR variables, since any variable has a significand of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

### 4.5 Rounding Modes

The following four rounding modes are supported:

- `GMP_RNDN`: round to nearest
- `GMP_RNDZ`: round toward zero
- `GMP_RNDU`: round toward plus infinity
- `GMP_RNDD`: round toward minus infinity

The ‘*round to nearest*’ mode works as in the IEEE 754-1985 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 5/2, which is represented by (10.1) in binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the *drift* phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value.
of type \texttt{int}, called the \textit{ternary value}. The value stored in the destination variable is correctly rounded, i.e. MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

As a consequence, in case of a non-zero real rounded result, the error on the result is less or equal to 1/2 ulp (unit in the last place) of the target in the rounding to nearest mode, and less than 1 ulp of the target in the directed rounding modes (a ulp is the weight of the least significant represented bit of the target after rounding).

Unless documented otherwise, functions returning an \texttt{int} return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the \texttt{GMP_RNDU} rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an \texttt{int}.

Unless documented otherwise, functions returning a 1 (or any other value specified in this manual) for special cases (like \texttt{acos(0)}) should return an overflow or an underflow if 1 is not representable in the current exponent range.

### 4.6 Floating-Point Values on Special Numbers

This section specifies the floating-point values (of type \texttt{mpfr_t}) returned by MPFR functions. For functions returning several values (like \texttt{mpfr_sin_cos}), the rules apply to each result separately.

Functions can have one or several input arguments. An input point is a mapping from these input arguments to the set of the MPFR numbers. When none of its components are NaN, an input point can also be seen as a tuple in the extended real numbers (the set of the real numbers with both infinities).

When the input point is in the domain of the mathematical function, the result is rounded as described in Section “Rounding Modes” (but see below for the specification of the sign of an exact zero). Otherwise the general rules from this section apply unless stated otherwise in the description of the MPFR function (Chapter 5 [MPFR Interface], page 11).

When the input point is not in the domain of the mathematical function but is in its closure in the extended real numbers and the function can be extended by continuity, the result is the obtained limit. Examples: \texttt{mpfr_hypot} on (+Inf,0) gives +Inf. But \texttt{mpfr_pow} cannot be defined on (1,+Inf) using this rule, as one can find sequences \((x_n,y_n)\) such that \(x_n\) goes to 1, \(y_n\) goes to +Inf and \((x_n)^{y_n}\) goes to any positive value when \(n\) goes to the infinity.

When the input point is in the closure of the domain of the mathematical function and an input argument is +0 (resp. −0), one considers the limit when the corresponding argument approaches 0 from above (resp. below). If the limit is not defined (e.g., \texttt{mpfr_log} on −0), the behavior must be specified in the description of the MPFR function.

When the result is equal to 0, its sign is determined by considering the limit as if the input point were not in the domain: If one approaches 0 from above (resp. below), the result is +0 (resp. −0). In the other cases, the sign must be specified in the description of the MPFR function. Example: \texttt{mpfr_sin} on +0 gives +0.
When the input point is not in the closure of the domain of the function, the result is NaN. Example: \texttt{mpfr\_sqrt} on \(-17\) gives NaN.

When an input argument is NaN, the result is NaN, possibly except when a partial function is constant on the finite floating-point numbers; such a case is always explicitly specified in Chapter 5 [MPFR Interface], page 11. Example: \texttt{mpfr\_hypot} on (NaN,0) gives NaN, but \texttt{mpfr\_hypot} on (NaN,\(+\infty\)) gives \(+\infty\) (as specified in Section 5.7 [Special Functions], page 20), since for any finite input \(x\), \texttt{mpfr\_hypot} on \((x,\infty)\) gives \(+\infty\).

### 4.7 Exceptions

MPFR supports 5 exception types:

- **Underflow**: An underflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent smaller than the minimum exponent of the current range. In the round-to-nearest mode, the halfway case is rounded toward zero.

  Note: This is not the single definition of the underflow. MPFR chooses to consider the underflow after rounding. The underflow before rounding can also be defined. For instance, consider a function that has the exact result \(7 \times 2^{-4}\), where \(e\) is the smallest exponent (for a significand between 1/2 and 1) in the current range, with a 2-bit target precision and rounding toward plus infinity. The exact result has the exponent \(e-1\). With the underflow before rounding, such a function call would yield an underflow, as \(e-1\) is outside the current exponent range. However, MPFR first considers the rounded result assuming an unbounded exponent range. The exact result cannot be represented exactly in precision 2, and here, it is rounded to \(0.5 \times 2^e\), which is representable in the current exponent range. As a consequence, this will not yield an underflow in MPFR.

- **Overflow**: An overflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent larger than the maximum exponent of the current range. In the round-to-nearest mode, the result is infinite.

- **NaN**: A NaN exception occurs when the result of a function is a NaN.

- **Inexact**: An inexact exception occurs when the result of a function cannot be represented exactly and must be rounded.

- **Range error**: A range exception occurs when a function that does not return a MPFR number (such as comparisons and conversions to an integer) has an invalid result (e.g. an argument is NaN in \texttt{mpfr\_cmp} or in a conversion to an integer).

MPFR has a global flag for each exception, which can be cleared, set or tested by functions described in Section 5.12 [Exception Related Functions], page 27.

Differences with the ISO C99 standard:

- In C, only quiet NaNs are specified, and a NaN propagation does not raise an invalid exception. Unless explicitly stated otherwise, MPFR sets the NaN flag whenever a NaN is generated, even when a NaN is propagated (e.g. in NaN + NaN), as if all NaNs were signaling.

- An invalid exception in C corresponds to either a NaN exception or a range error in MPFR.

### 4.8 Memory Handling

MPFR functions may create caches, e.g. when computing constants such as \(\pi\), either because the user has called a function like \texttt{mpfr\_const\_pi} directly or because such a function was called internally by the MPFR library itself to compute some other function.
At any time, the user can free the various caches with `mpfr_free_cache`. It is strongly advised to do that before terminating a thread, or before exiting when using tools like ‘valgrind’ (to avoid memory leaks being reported).

MPFR internal data such as flags, the exponent range, the default precision and rounding mode, and caches (i.e., data that are not accessed via parameters) are either global (if MPFR has not been compiled as thread safe) or per-thread (thread local storage).
5 MPFR Interface

The floating-point functions expect arguments of type `mpfr_t`.

The MPFR floating-point functions have an interface that is similar to the GNU MP integer functions. The function prefix for floating-point operations is `mpfr_`.

There is one significant characteristic of floating-point numbers that has motivated a difference between this function class and other GNU MP function classes: the inherent inexactness of floating-point arithmetic. The user has to specify the precision for each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend from the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with “infinite accuracy”), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754-1985 arithmetic. The results obtained on one computer should not differ from the results obtained on a computer with a different word size.

MPFR does not keep track of the accuracy of a computation. This is left to the user or to a higher layer. As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with large precision, then MPFR will still compute the result with full precision.

The value of the standard C macro `errno` may be set to non-zero by any MPFR function or macro, whether or not there is an error.

5.1 Initialization Functions

An `mpfr_t` object must be initialized before storing the first value in it. The functions `mpfr_init` and `mpfr_init2` are used for that purpose.

```c
void mpfr_init2 (mpfr_t x, mp_prec_t prec)                      [Function]
Initialize x, set its precision to be exactly prec bits and its value to NaN. (Warning: the corresponding `mpf` functions initialize to zero instead.)
```

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. To change the precision of a variable which has already been initialized, use `mpfr_set_prec`. The precision `prec` must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined).

```c
void mpfr_clear (mpfr_t x)                                    [Function]
Free the space occupied by x. Make sure to call this function for all `mpfr_t` variables when you are done with them.
```

```c
void mpfr_init (mpfr_t x)                                     [Function]
Initialize x and set its value to NaN.
```

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. The precision of `x` is the default precision, which can be changed by a call to `mpfr_set_default_prec`.

```c
void mpfr_set_default_prec (mp_prec_t prec)                  [Function]
Set the default precision to be exactly prec bits. The precision of a variable means the number of bits used to store its significand. All subsequent calls to `mpfr_init` will use this precision,
```
but previously initialized variables are unaffected. This default precision is set to 53 bits initially. The precision can be any integer between \texttt{MPFR\_PREC\_MIN} and \texttt{MPFR\_PREC\_MAX}.

\begin{verbatim}
mp\_prec\_t mpfr\_get\_default\_prec (void)       \ [Function]
    \text{Return the default MPFR precision in bits.}
\end{verbatim}

Here is an example on how to initialize floating-point variables:
\begin{verbatim}
    {
        mpfr\_t x, y;
        mpfr\_init (x);       /* use default precision */
        mpfr\_init2 (y, 256); /* precision exactly 256 bits */
        ...
        /* When the program is about to exit, do ... */
        mpfr\_clear (x);
        mpfr\_clear (y);
        mpfr\_free\_cache ()
    }
\end{verbatim}

The following functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

\begin{verbatim}
void mpfr\_set\_prec (mpfr\_t x, mp\_prec\_t prec)     \ [Function]
    \text{Reset the precision of } x \text{ to be exactly } prec \text{ bits, and set its value to NaN. The previous value stored in } x \text{ is lost. It is equivalent to a call to } mpfr\_clear(x) \text{ followed by a call to } mpfr\_init2(x, prec), \text{ but more efficient as no allocation is done in case the current allocated space for the significand of } x \text{ is enough. The precision } prec \text{ can be any integer between } MPFR\_PREC\_MIN \text{ and } MPFR\_PREC\_MAX.}

In case you want to keep the previous value stored in } x \text{, use } mpfr\_prec\_round \text{ instead.}

mp\_prec\_t mpfr\_get\_prec (mpfr\_t x)            \ [Function]
    \text{Return the precision actually used for assignments of } x, \text{i.e. the number of bits used to store its significand.}
\end{verbatim}

5.2 Assignment Functions

These functions assign new values to already initialized floats (see Section 5.1 \textit{Initialization Functions}, page 11). When using any functions using \texttt{intmax\_t}, you must include \texttt{<stdint.h>} or \texttt{<inttypes.h>} before \texttt{`mpfr.h`}, to allow \texttt{`mpfr.h`} to define prototypes for these functions.

\begin{verbatim}
int mpfr\_set (mpfr\_t rop, mpfr\_t op, mp\_rnd\_t rnd)    \ [Function]
int mpfr\_set\_ui (mpfr\_t rop, unsigned long int op, mp\_rnd\_t rnd) \ [Function]
int mpfr\_set\_si (mpfr\_t rop, long int op, mp\_rnd\_t rnd)    \ [Function]
int mpfr\_set\_uj (mpfr\_t rop, uintmax\_t op, mp\_rnd\_t rnd) \ [Function]
int mpfr\_set\_sj (mpfr\_t rop, intmax\_t op, mp\_rnd\_t rnd)    \ [Function]
int mpfr\_set\_d (mpfr\_t rop, double op, mp\_rnd\_t rnd) \ [Function]
int mpfr\_set\_ld (mpfr\_t rop, long double op, mp\_rnd\_t rnd) \ [Function]
int mpfr\_set\_decimal64 (mpfr\_t rop, _Decimal64 op, mp\_rnd\_t rnd) \ [Function]
int mpfr\_set\_z (mpfr\_t rop, mpz\_t op, mp\_rnd\_t rnd) \ [Function]
int mpfr\_set\_q (mpfr\_t rop, mpq\_t op, mp\_rnd\_t rnd) \ [Function]
\end{verbatim}


```c
int mpfr_set_f (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)  
```  
Set the value of `rop` from `op`, rounded toward the given direction `rnd`. Note that the input 0 is converted to +0 by `mpfr_set_ui`, `mpfr_set_si`, `mpfr_set_sj`, `mpfr_set_uj`, `mpfr_set_z`, `mpfr_set_q` and `mpfr_set_f`, regardless of the rounding mode. If the system doesn’t support the IEEE-754 standard, `mpfr_set_d`, `mpfr_set_ld` and `mpfr_set_decimal64` might not preserve the signed zeros. The `mpfr_set_decimal64` function is built only with the configure option `--enable-decimal-float`, which also requires `--with-gmp-build`, and when the compiler or system provides the `_Decimal64` data type (GCC version 4.2.0 is known to support this data type, but only when configured with `--enable-decimal-float` too). `mpfr_set_q` might not be able to work if the numerator (or the denominator) can not be representable as a `mpfr_t`.

Note: If you want to store a floating-point constant to a `mpfr_t`, you should use `mpfr_set_str` (or one of the MPFR constant functions, such as `mpfr_const_pi` for π) instead of `mpfr_set_d`, `mpfr_set_ld` or `mpfr_set_decimal64`. Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary number before MPFR can work with it.

```c
int mpfr_set_ui_2exp (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)  
```  
```c
int mpfr_set_si_2exp (mpfr_t rop, long int op, mp_rnd_t rnd)  
```  
```c
int mpfr_set_uj_2exp (mpfr_t rop, uintmax_t op, intmax_t e, mp_rnd_t rnd)  
```  
```c
int mpfr_set_sj_2exp (mpfr_t rop, intmax_t op, intmax_t e, mp_rnd_t rnd)  
```  
Set the value of `rop` from `op × 2^e`, rounded toward the given direction `rnd`. Note that the input 0 is converted to +0.

```c
int mpfr_set_str (mpfr_t rop, const char *s, int base, mp_rnd_t rnd)  
```  
Set `rop` to the value of the whole string `s` in base `base`, rounded in the direction `rnd`. See the documentation of `mpfr_strtofr` for a detailed description of the valid string formats. This function returns 0 if the entire string up to the final null character is a valid number in base `base`; otherwise it returns −1, and `rop` may have changed.

```c
int mpfr_strtofr (mpfr_t rop, const char *nptr, char **endptr, int base, mp_rnd_t rnd)  
```  
Read a floating-point number from a string `nptr` in base `base`, rounded in the direction `rnd`; `base` must be either 0 (to detect the base, as described below) or a number from 2 to 36 (otherwise the behavior is undefined). If `nptr` starts with valid data, the result is stored in `rop` and `*endptr` points to the character just after the valid data (if `endptr` is not a null pointer); otherwise `rop` is set to zero and the value of `nptr` is stored in the location referenced by `endptr` (if `endptr` is not a null pointer). The usual ternary value is returned.

Parsing follows the standard C `strtod` function with some extensions. Case is ignored. After optional leading whitespace, one has a subject sequence consisting of an optional sign (+ or −), and either numeric data or special data. The subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-whitespace character, that is of the expected form.

The form of numeric data is a non-empty sequence of significand digits with an optional decimal point, and an optional exponent consisting of an exponent prefix followed by an optional sign and a non-empty sequence of decimal digits. A significand digit is either a decimal digit or a Latin letter (62 possible characters), with a = 10, b = 11, . . . , z = 36; its
value must be strictly less than the base. The decimal point can be either the one defined by
the current locale or the period (the first one is accepted for consistency with the C standard
and the practice, the second one is accepted to allow the programmer to provide MPFR
numbers from strings in a way that does not depend on the current locale). The exponent
prefix can be e or E for bases up to 10, or @ in any base; it indicates a multiplication by
a power of the base. In bases 2 and 16, the exponent prefix can also be p or P, in which
case it introduces a binary exponent: it indicates a multiplication by a power of 2 (there is a
difference only for base 16). The value of an exponent is always written in base 10. In base
2, the significand can start with 0b or 0B, and in base 16, it can start with 0x or 0X.

If the argument base is 0, then the base is automatically detected as follows. If the significand
starts with 0b or 0B, base 2 is assumed. If the significand starts with 0x or 0X, base 16 is
assumed. Otherwise base 10 is assumed.

Note: The exponent must contain at least a digit. Otherwise the possible exponent prefix
and sign are not part of the number (which ends with the significand). Similarly, if 0b, 0B,
0x or 0X is not followed by a binary/hexadecimal digit, then the subject sequence stops at
the character 0.

Special data (for infinities and NaN) can be @inf@ or @nan@(n-char-sequence), and if
base ≤ 16, it can also be infinity, inf, nan or nan(n-char-sequence), all case insensitive.
A n-char-sequence is a non-empty string containing only digits, Latin letters and the un-
erscore (0, 1, 2, . . . , 9, a, b, . . . , z, A, B, . . . , Z, _). Note: one has an optional sign for all
data, even NaN.

void mpfr_set_inf (mpfr_t x, int sign) [Function]
void mpfr_set_nan (mpfr_t x) [Function]
Set the variable x to infinity or NaN (Not-a-Number) respectively. In mpfr_set_inf, x is
set to plus infinity if sign is nonnegative.

void mpfr_swap (mpfr_t x, mpfr_t y) [Function]
Swap the values x and y efficiently. Warning: the precisions are exchanged too; in case the
precisions are different, mpfr_swap is thus not equivalent to three mpfr_set calls using a
third auxiliary variable.

5.3 Combined Initialization and Assignment Functions

int mpfr_init_set (mpfr_t rop, mpfr_t op, mpn rnd_t rnd) [Macro]
int mpfr_init_set_ui (mpfr_t rop, unsigned long int op, mpn rnd_t rnd) [Macro]
int mpfr_init_set_si (mpfr_t rop, signed long int op, mpn rnd_t rnd) [Macro]
int mpfr_init_set_d (mpfr_t rop, double op, mpn rnd_t rnd) [Macro]
int mpfr_init_set_ld (mpfr_t rop, long double op, mpn rnd_t rnd) [Macro]
int mpfr_init_set_z (mpfr_t rop, mpz_t op, mpn rnd_t rnd) [Macro]
int mpfr_init_set_q (mpfr_t rop, mpq_t op, mpn rnd_t rnd) [Macro]
int mpfr_init_set_f (mpfr_t rop, mpf_t op, mpn rnd_t rnd) [Macro]
Initialize rop and set its value from op, rounded in the direction rnd. The precision of rop
will be taken from the active default precision, as set by mpfr_set_default_prec.

int mpfr_init_set_str (mpfr_t x, const char *s, int base, mpn rnd_t rnd) [Function]
Initialize x and set its value from the string s in base base, rounded in the direction rnd. See
mpfr_set_str.

5.4 Conversion Functions

double mpfr_get_d (mpfr_t op, mpn rnd_t rnd) [Function]
long double mpfr_get_ld (mpfr_t op, mp_rnd_t rnd)

_DOUBLE64 mpfr_get_decimal64 (mpfr_t op, mp_rnd_t rnd)

Convert op to a double (respectively _Decimal64 or long double), using the rounding mode rnd. If op is NaN, some fixed NaN (either quiet or signaling) or the result of 0.0/0.0 is returned. If op is ±Inf, an infinity of the same sign or the result of ±1.0/0.0 is returned. If op is zero, these functions return a zero, trying to preserve its sign, if possible. The mpfr_get_decimal64 function is built only under some conditions: see the documentation of mpfr_set_decimal64.

double mpfr_get_d_2exp (long *exp, mpfr_t op, mp_rnd_t rnd)
long double mpfr_get_ld_2exp (long *exp, mpfr_t op, mp_rnd_t rnd)

Return d and set exp such that 0.5 ≤ |d| < 1 and d × 2exp equals op rounded to double (resp. long double) precision, using the given rounding mode. If op is zero, then a zero of the same sign (or an unsigned zero, if the implementation does not have signed zeros) is returned, and exp is set to 0. If op is NaN or an infinity, then the corresponding double precision (resp. long-double precision) value is returned, and exp is undefined.

long mpfr_get_si (mpfr_t op, mp_rnd_t rnd)
unsigned long mpfr_get_ui (mpfr_t op, mp_rnd_t rnd)
intmax_t mpfr_get_sj (mpfr_t op, mp_rnd_t rnd)
uintmax_t mpfr_get_uj (mpfr_t op, mp_rnd_t rnd)

Convert op to a long, an unsigned long, an intmax_t or an uintmax_t (respectively) after rounding it with respect to rnd. If op is NaN, the result is undefined. If op is too big for the return type, it returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow. The flag erange is set too. See also mpfr.fits_ulong_p, mpfr.fits_intmax_p and mpfr.fits_uintmax_p.

mp_exp_t mpfr_get_z_exp (mpz_t rop, mpfr_t op)

Put the scaled significand of op (regarded as an integer, with the precision of op) into rop, and return the exponent exp (which may be outside the current exponent range) such that op exactly equals rop × 2exp. If the exponent is not representable in the mp_exp_t type, the behavior is undefined.

void mpfr_get_z (mpz_t rop, mpfr_t op, mp_rnd_t rnd)

Convert op to a mpz_t, after rounding it with respect to rnd. If op is NaN or Inf, the result is undefined.

int mpfr_get_f (mpf_t rop, mpfr_t op, mp_rnd_t rnd)

Convert op to a mpf_t, after rounding it with respect to rnd. Return zero iff no error occurred, in particular a non-zero value is returned if op is NaN or Inf, which do not exist in mpf.

char * mpfr_get_str (char *str, mp_exp_t *expptr, int b, size_t n, mpfr_t op, mp_rnd_t rnd)

Convert op to a string of digits in base b, with rounding in the direction rnd, where n is either zero (see below) or the number of significant digits; in the latter case, n must be greater or equal to 2. The base may vary from 2 to 36.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number −3.1416 would be returned as "−31416" in the string and 1 written at expptr. If rnd is to nearest, and op is exactly in the middle of two possible outputs, the one with an even last digit is chosen (for an odd base, this may not correspond to an even significand).
If \( n \) is zero, the number of digits of the significand is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding to nearest, will recover the original value of \( op \). More precisely, in most cases, the chosen precision of \( str \) is the minimal precision depending on \( n \) and \( b \) only that satisfies the above property, i.e., \( m = 1 + \lceil n \log_2 b \rceil \), but in some very rare cases, it might be \( m + 1 \).

If \( str \) is a null pointer, space for the significand is allocated using the current allocation function, and a pointer to the string is returned. To free the returned string, you must use \texttt{mpfr_free_str}.

If \( str \) is not a null pointer, it should point to a block of storage large enough for the significand, i.e., at least \( \max(n + 2, 7) \). The extra two bytes are for a possible minus sign, and for the terminating null character.

If the input number is an ordinary number, the exponent is written through the pointer \( \text{expptr} \) (the current minimal exponent for 0).

A pointer to the string is returned, unless there is an error, in which case a null pointer is returned.

\begin{verbatim}
void mpfr_free_str (char *str) [Function]
Free a string allocated by mpfr_get_str using the current unallocation function (preliminary interface). The block is assumed to be strlen\((str)\)+1 bytes. For more information about how it is done: see Section “Custom Allocation” in GNU MP.
\end{verbatim}

\begin{verbatim}
int mpfr_fits_ulong_p (mpfr_t op, mpfr_t rnd) [Function]
Return non-zero if \( op \) would fit in the respective C data type, when rounded to an integer in the direction \( \text{rnd} \).
\end{verbatim}

5.5 Basic Arithmetic Functions

\begin{verbatim}
int mpfr_add (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t rnd) [Function]
int mpfr_add_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_t rnd) [Function]
int mpfr_sub (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t rnd) [Function]
int mpfr_add_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_t rnd) [Function]
int mpfr_sub_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_t rnd) [Function]
int mpfr_add_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_t rnd) [Function]
int mpfr_add_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_t rnd) [Function]
int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_t rnd) [Function]
int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_t rnd) [Function]
\end{verbatim}

Set \( rop \) to \( op1 + op2 \) rounded in the direction \( \text{rnd} \). For types having no signed zero, it is considered unsigned (i.e. \((+0) + 0 = (+0)\) and \((-0) + 0 = (-0))\).

\begin{verbatim}
int mpfr_add_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_t rnd) [Function]
int mpfr_sub_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_t rnd) [Function]
int mpfr_add_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_t rnd) [Function]
int mpfr_add_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_t rnd) [Function]
int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_t rnd) [Function]
int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_t rnd) [Function]
\end{verbatim}
int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)  [Function]
int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)  [Function]

Set rop to \(op1 - op2\) rounded in the direction rnd. For types having no signed zero, it is considered unsigned (i.e. \(+0 - 0 = (+0), (-0) - 0 = (-0), 0 - (+0) = (-0)\) and \(0 - (-0) = (+0)\)).

int mpfr_mul (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)  [Function]
int mpfr_mul_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t rnd)  [Function]
int mpfr_mul_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)  [Function]
int mpfr_mul_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)  [Function]

Set rop to \(op1 \times op2\) rounded in the direction rnd. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zero, it is considered positive).

int mpfr_sqr (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)  [Function]

Set rop to \(op^2\) rounded in the direction rnd.

int mpfr_div (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)  [Function]
int mpfr_ui_div (mpfr_t rop, unsigned long int op1, mpfr_t op2, mp_rnd_t rnd)  [Function]
int mpfr_div_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t rnd)  [Function]
int mpfr_div_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)  [Function]
int mpfr_div_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mp_rnd_t rnd)  [Function]

Set rop to \(op1/op2\) rounded in the direction rnd. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zero, it is considered positive).

int mpfr_sqrt (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)  [Function]
int mpfr_sqrt_ui (mpfr_t rop, unsigned long int op, mp_rnd_t rnd)  [Function]

Set rop to \(\sqrt{op}\) rounded in the direction rnd. Return \(-0\) if op is \(-0\) (to be consistent with the IEEE 754-1985 standard). Set rop to NaN if op is negative.

int mpfr_cbrt (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)  [Function]
int mpfr_root (mpfr_t rop, mpfr_t op, unsigned long int k, mp_rnd_t rnd)  [Function]

Set rop to the cubic root (resp. the kth root) of op rounded in the direction rnd. An odd (resp. even) root of a negative number (including \(-\text{Inf}\)) returns a negative number (resp. NaN). The kth root of \(-0\) is defined to be \(-0\), whatever the parity of k.

int mpfr_pow (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)  [Function]
int mpfr_pow_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t rnd)  [Function]
int mpfr_pow_si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd)  [Function]
int mpfr_pow_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mp_rnd_t rnd)  [Function]
int mpfr_ui_pow_ui (mpfr_t rop, unsigned long int op1, unsigned long int op2, mp_rnd_t rnd)  [Function]
int mpfr_ui_pow (mpfr_t rop, unsigned long int op1, mpfr_t op2, mp_rnd_t rnd)  [Function]

Set rop to \(op1^{op2}\), rounded in the direction rnd. Special values are currently handled as described in the ISO C99 standard for the pow function (note this may change in future versions):
- `pow(±0, y)` returns plus or minus infinity for `y` a negative odd integer.
- `pow(±0, y)` returns plus infinity for `y` negative and not an odd integer.
- `pow(±0, y)` returns plus or minus zero for `y` a positive odd integer.
- `pow(±0, y)` returns plus zero for `y` positive and not an odd integer.
- `pow(-1, ±Inf)` returns 1.
- `pow(+1, y)` returns 1 for any `y`, even a NaN.
- `pow(x, ±0)` returns 1 for any `x`, even a NaN.
- `pow(x, y)` returns NaN for finite negative `x` and finite non-integer `y`.
- `pow(x, -Inf)` returns plus infinity for 0 < |x| < 1, and plus zero for |x| > 1.
- `pow(x, +Inf)` returns plus zero for 0 < |x| < 1, and plus infinity for |x| > 1.
- `pow(-Inf, y)` returns minus zero for `y` a negative odd integer.
- `pow(-Inf, y)` returns plus zero for `y` negative and not an odd integer.
- `pow(-Inf, y)` returns minus infinity for `y` a positive odd integer.
- `pow(-Inf, y)` returns plus infinity for `y` positive and not an odd integer.
- `pow(+Inf, y)` returns plus zero for `y` negative, and plus infinity for `y` positive.

```c
int mpfr_neg (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to −op rounded in the direction rnd. Just changes the sign if rop and op are
the same variable.
```

```c
int mpfr_abs (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the absolute value of op, rounded in the direction rnd. Just changes the sign if
rop and op are the same variable.
```

```c
int mpfr_dim (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd) [Function]
Set rop to the positive difference of op1 and op2, i.e., op1 − op2 rounded in the direction rnd
if op1 > op2, and +0 otherwise. Returns NaN when op1 or op2 is NaN.
```

```c
int mpfr_mul_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t rnd) [Function]
int mpfr_mul_2si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd) [Function]
Set rop to op1 × 2^op2 rounded in the direction rnd. Just increases the exponent by op2 when
rop and op1 are identical.
```

```c
int mpfr_div_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mp_rnd_t rnd) [Function]
int mpfr_div_2si (mpfr_t rop, mpfr_t op1, long int op2, mp_rnd_t rnd) [Function]
Set rop to op1/2^op2 rounded in the direction rnd. Just decreases the exponent by op2 when
rop and op1 are identical.
```

### 5.6 Comparison Functions

```c
int mpfr_cmp (mpfr_t op1, mpfr_t op2) [Function]
int mpfr_cmp_ui (mpfr_t op1, unsigned long int op2) [Function]
int mpfr_cmp_si (mpfr_t op1, signed long int op2) [Function]
int mpfr_cmp_d (mpfr_t op1, double op2) [Function]
int mpfr_cmp_ld (mpfr_t op1, long double op2) [Function]
int mpfr_cmp_z (mpfr_t op1, mpz_t op2) [Function]
int mpfr_cmp_q (mpfr_t op1, mpq_t op2) [Function]
```
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**int mpfr_cmp_f (mpfr_t op1, mpf_t op2)**

Compare op1 and op2. Return a positive value if op1 > op2, zero if op1 = op2, and a negative value if op1 < op2. Both op1 and op2 are considered to their full own precision, which may differ. If one of the operands is NaN, set the erange flag and return zero.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., mpfr_equal_p for the equality) described below; they behave like the IEEE-754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).

**int mpfr_cmp_ui_2exp (mpfr_t op1, unsigned long int op2, mpfexp_t e)**

Compare op1 and op2 × 2^e. Similar as above.

**int mpfr_cmpabs (mpfr_t op1, mpfr_t op2)**

Compare |op1| and |op2|. Return a positive value if |op1| > |op2|, zero if |op1| = |op2|, and a negative value if |op1| < |op2|. If one of the operands is NaN, set the erange flag and return zero.

**int mpfr_nan_p (mpfr_t op)**

Return non-zero if op is respectively NaN, an infinity, an ordinary number (i.e. neither NaN nor an infinity) or zero. Return zero otherwise.

**int mpfr_inf_p (mpfr_t op)**

Return non-zero if op > 0, zero if op = 0, and a negative value if op < 0. If the operand is NaN, set the erange flag and return zero.

**int mpfr_number_p (mpfr_t op)**

Return non-zero if op1 > op2, zero otherwise.

**int mpfr_greater_equal_p (mpfr_t op1, mpfr_t op2)**

Return non-zero if op1 ≥ op2, zero otherwise.

**int mpfr_less_p (mpfr_t op1, mpfr_t op2)**

Return non-zero if op1 < op2, zero otherwise.

**int mpfr_less_equal_p (mpfr_t op1, mpfr_t op2)**

Return non-zero if op1 ≤ op2, zero otherwise.

**int mpfr_unordered_p (mpfr_t op1, mpfr_t op2)**

Return non-zero if op1 or op2 is a NaN (i.e. they cannot be compared), zero otherwise.
5.7 Special Functions

All those functions, except explicitly stated, return zero for an exact return value, a positive value for a return value larger than the exact result, and a negative value otherwise.

Important note: in some domains, computing special functions (either with correct or incorrect rounding) is expensive, even for small precision, for example the trigonometric and Bessel functions for large argument.

```c
int mpfr_log (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_log2 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_log10 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the natural logarithm of op, log2 op or log10 op, respectively, rounded in the direction rnd. Return -Inf if op is -0 (i.e. the sign of the zero has no influence on the result).

int mpfr_exp (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_exp2 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_exp10 (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the exponential of op, to 2^op or to 10^op, respectively, rounded in the direction rnd.

int mpfr_cos (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_sin (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_tan (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the cosine of op, sine of op, tangent of op, rounded in the direction rnd.

int mpfr_sec (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_csc (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_cot (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the secant of op, cosecant of op, cotangent of op, rounded in the direction rnd.

int mpfr_sin_cos (mpfr_t sop, mpfr_t cop, mpfr_t op, mp_rnd_t rnd) [Function]
Set simultaneously sop to the sine of op and cop to the cosine of op, rounded in the direction rnd with the corresponding precisions of sop and cop, which must be different variables. Return 0 iff both results are exact.

int mpfr_acos (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_asin (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_atan (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
Set rop to the arc-cosine, arc-sine or arc-tangent of op, rounded in the direction rnd. Note that since acos(-1) returns the floating-point number closest to π according to the given rounding mode, this number might not be in the output range 0 ≤ rop < π of the arc-cosine function: still, the result lies in the image of the output range by the rounding function. The same holds for asin(-1), asin(1), atan(-Inf), atan(+Inf).

int mpfr_atan2 (mpfr_t rop, mpfr_t y, mpfr_t x, mp_rnd_t rnd) [Function]
Set rop to the arc-tangent2 of y and x, rounded in the direction rnd: if x > 0, atan2(y, x) = atan (y/x); if x < 0, atan2(y, x) = sign(y)*(Pi - atan (|y/x|)). As for atan, in case the exact mathematical result is +π or −π, its rounded result might be outside the function output range.

atan2(y, 0) does not raise any floating-point exception. Special values are currently handled as described in the ISO C99 standard for the atan2 function (note this may change in future versions):
• \( \text{atan2}(+0, -0) \) returns \(+\pi\).
• \( \text{atan2}(-0, -0) \) returns \(-\pi\).
• \( \text{atan2}(+0, +0) \) returns \(+0\).
• \( \text{atan2}(-0, +0) \) returns \(-0\).
• \( \text{atan2}(+0, x) \) returns \(+\pi\) for \(x < 0\).
• \( \text{atan2}(-0, x) \) returns \(-\pi\) for \(x < 0\).
• \( \text{atan2}(+0, x) \) returns \(+0\) for \(x > 0\).
• \( \text{atan2}(-0, x) \) returns \(-0\) for \(x > 0\).
• \( \text{atan2}(y, 0) \) returns \(-\pi/2\) for \(y < 0\).
• \( \text{atan2}(y, 0) \) returns \(+\pi/2\) for \(y > 0\).
• \( \text{atan2}(+\text{Inf}, -\text{Inf}) \) returns \(+3\pi/4\).
• \( \text{atan2}(-\text{Inf}, -\text{Inf}) \) returns \(-3\pi/4\).
• \( \text{atan2}(+\text{Inf}, +\text{Inf}) \) returns \(+\pi/4\).
• \( \text{atan2}(-\text{Inf}, +\text{Inf}) \) returns \(-\pi/4\).
• \( \text{atan2}(y, -\text{Inf}) \) returns \(+\pi\) for finite \(y > 0\).
• \( \text{atan2}(y, -\text{Inf}) \) returns \(-\pi\) for finite \(y < 0\).
• \( \text{atan2}(y, +\text{Inf}) \) returns \(+0\) for finite \(y > 0\).
• \( \text{atan2}(y, +\text{Inf}) \) returns \(-0\) for finite \(y < 0\).

\[ \text{int mpfr_cosh} (\text{mpfr} \ t \ \text{rop}, \text{mpfr} \ t \ \text{op}, \text{mpfr} \ \text{rnd} \ t \ \text{rnd}) \]  
Set \( \text{rop} \) to the hyperbolic cosine, sine or tangent of \( \text{op} \), rounded in the direction \( \text{rnd} \).

\[ \text{int mpfr_sech} (\text{mpfr} \ t \ \text{rop}, \text{mpfr} \ t \ \text{op}, \text{mpfr} \ \text{rnd} \ t \ \text{rnd}) \]  
Set \( \text{rop} \) to the hyperbolic secant of \( \text{op} \), cosecant of \( \text{op} \), cotangent of \( \text{op} \), rounded in the direction \( \text{rnd} \).

\[ \text{int mpfr_fac_ui} (\text{mpfr} \ t \ \text{rop}, \text{mpfr} \ t \ \text{op}, \text{mpfr} \ \text{rnd} \ t \ \text{rnd}) \]  
Set \( \text{rop} \) to the factorial of the unsigned long int \( \text{op} \), rounded in the direction \( \text{rnd} \).

\[ \text{int mpfr_log1p} (\text{mpfr} \ t \ \text{rop}, \text{mpfr} \ t \ \text{op}, \text{mpfr} \ \text{rnd} \ t \ \text{rnd}) \]  
Set \( \text{rop} \) to the logarithm of one plus \( \text{op} \), rounded in the direction \( \text{rnd} \).

\[ \text{int mpfr_expm1} (\text{mpfr} \ t \ \text{rop}, \text{mpfr} \ t \ \text{op}, \text{mpfr} \ \text{rnd} \ t \ \text{rnd}) \]  
Set \( \text{rop} \) to the exponential of \( \text{op} \) minus one, rounded in the direction \( \text{rnd} \).

\[ \text{int mpfr_eint} (\text{mpfr} \ t \ \text{y}, \text{mpfr} \ t \ \text{x}, \text{mpfr} \ \text{rnd} \ t \ \text{rnd}) \]  
Set \( y \) to the exponential integral of \( x \), rounded in the direction \( \text{rnd} \). For positive \( x \), the exponential integral is the sum of Euler's constant, of the logarithm of \( x \), and of the sum for \( k \) from 1 to infinity of \( x^k/k/k! \). For negative \( x \), the returned value is NaN.
The arithmetic-geometric mean is the common limit of the sequences \( u[n] \) and \( v[n] \), where

\[
\text{AGM}(u[n], v[n]) = \lim_{n \to \infty} \frac{u[n] + v[n]}{2}^{1/2^{n-1}}
\]

\[
\text{AGM}(u[n], v[n]) = \lim_{n \to \infty} \frac{u[n] + v[n]}{2}^{1/2^{n-1}}
\]

int mpfr_gmma (mpfr_t rop, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the Gamma function on op, rounded in the direction rnd. When op is a negative integer, NaN is returned.

int mpfr_lgamma (mpfr_t rop, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the logarithm of the Gamma function on op, rounded in the direction rnd. When \(-2k-1 \leq x \leq -2k\), \(k\) being a non-negative integer, NaN is returned. See also mpfr_lgamma.

int mpfr_lngamma (mpfr_t rop, int *signp, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the logarithm of the absolute value of the Gamma function on op, rounded in the direction rnd. The sign (1 or \(-1\)) of Gamma(op) is returned in the object pointed to by signp. When \(op\) is an infinity or a non-positive integer, +Inf is returned. When op is NaN, -Inf or a negative integer, *signp is undefined, and when op is ±0, *signp is the sign of the zero.

int mpfr_zeta (mpfr_t rop, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the Riemann Zeta function on op, rounded in the direction rnd.

int mpfr_zeta_ui (mpfr_t rop, unsigned long op, mpfr_t rnd) [Function]
Set rop to the value of the second order Bessel function of order 0, 1 and \(n\) on \(op\), rounded in the direction rnd. When op is NaN, rop is always set to NaN. When op is plus or minus Infinity, rop is set to +0. When op is zero, and \(n\) is not zero, rop is +0 or -0 depending on the parity and sign of \(n\), and the sign of op.

int mpfr_j0 (mpfr_t rop, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the first order Bessel function of order 0, 1 and \(n\) on \(op\), rounded in the direction rnd. When op is NaN, rop is always set to NaN. When op is plus or minus Infinity, rop is set to +0. When op is zero, and \(n\) is not zero, rop is +0 or -0 depending on the parity and sign of \(n\).

int mpfr_y0 (mpfr_t rop, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the first order Bessel function of order 0, 1 and \(n\) on \(op\), rounded in the direction rnd. When op is NaN or negative, rop is always set to NaN. When op is +Inf, rop is +0. When op is zero, rop is +Inf or -Inf depending on the parity and sign of \(n\).

int mpfr_y1 (mpfr_t rop, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the second order Bessel function of order 0, 1 and \(n\) on \(op\), rounded in the direction rnd. When op is NaN or negative, rop is always set to NaN. When op is +Inf, rop is +0. When op is zero, rop is +Inf or -Inf depending on the parity and sign of \(n\).

int mpfr_yn (mpfr_t rop, long n, mpfr_t op, mpfr_t rnd) [Function]
Set rop to the value of the second order Bessel function of order 0, 1 and \(n\) on \(op\), rounded in the direction rnd. When op is NaN or negative, rop is always set to NaN. When op is +Inf, rop is +0. When op is zero, rop is +Inf or -Inf depending on the parity and sign of \(n\).

int mpfr_fma (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_t rnd) [Function]
Set rop to \((op1 \times op2) + op3\), rounded in the direction rnd.

int mpfr_fms (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_t rnd) [Function]
Set rop to \((op1 \times op2) - op3\), rounded in the direction rnd.

int mpfr_agm (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t rnd) [Function]
Set rop to the arithmetic-geometric mean of op1 and op2, rounded in the direction rnd. The arithmetic-geometric mean is the common limit of the sequences \(u[n]\) and \(v[n]\), where
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$u[0]=op1$, $v[0]=op2$, $u[n+1]$ is the arithmetic mean of $u[n]$ and $v[n]$, and $v[n+1]$ is the geometric mean of $u[n]$ and $v[n]$. If any operand is negative, the return value is NaN.

**Function**

```c
int mpfr_hypot (mpfr_t rop, mpfr_t x, mpfr_t y, mp_rnd_t rnd)
```

Set `rop` to the Euclidean norm of `x` and `y`, i.e. $\sqrt{x^2 + y^2}$, rounded in the direction `rnd`. Special values are currently handled as described in Section F.9.4.3 of the ISO C99 standard, for the `hypot` function (note this may change in future versions): If `x` or `y` is an infinity, then plus infinity is returned in `rop`, even if the other number is NaN.

**Function**

```c
int mpfr_const_log2 (mpfr_t rop, mp_rnd_t rnd)
```

Set `rop` to the logarithm of 2, the value of $\pi$, of Euler’s constant $0.577\ldots$, of Catalan’s constant $0.915\ldots$, respectively, rounded in the direction `rnd`. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use `mpfr_free_cache`.

**Function**

```c
void mpfr_free_cache (void)
```

Free various caches used by MPFR internally, in particular the caches used by the functions computing constants (currently `mpfr_const_log2`, `mpfr_const_pi`, `mpfr_const_euler` and `mpfr_const_catalan`). You should call this function before terminating a thread, even if you did not call these functions directly (they could have been called internally).

**Function**

```c
int mpfr_sum (mpfr_t rop, mpfr_ptr const tab[], unsigned long n, mp_rnd_t rnd)
```

Set `ret` to the sum of all elements of `tab` whose size is `n`, rounded in the direction `rnd`. Warning, `tab` is a table of pointers to mpfr_t, not a table of mpfr_t (preliminary interface). The returned `int` value is zero when the computed value is the exact value, and non-zero when this cannot be guaranteed, without giving the direction of the error as the other functions do.

### 5.8 Input and Output Functions

This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a `stream` argument to any of these functions will make them read from `stdin` and write to `stdout`, respectively.

When using any of these functions, you must include the `<stdio.h>` standard header before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions.

**Function**

```c
size_t mpfr_out_str (FILE *stream, int base, size_t n, mpfr_t op, mp_rnd_t rnd)
```

Output `op` on stream `stream`, as a string of digits in base `base`, rounded in the direction `rnd`. The base may vary from 2 to 36. Print `n` significant digits exactly, or if `n` is 0, enough digits so that `op` can be read back exactly (see `mpfr_get_str`).

In addition to the significant digits, a decimal point (defined by the current locale) at the right of the first digit and a trailing exponent in base 10, in the form ‘eNNN’, are printed. If `base` is greater than 10, ‘@’ will be used instead of ‘e’ as exponent delimiter.

Return the number of bytes written, or if an error occurred, return 0.
size_t mpfr_inp_str (mpfr_t rop, FILE *stream, int base, mp_rnd_t rnd)

Input a string in base base from stream stream, rounded in the direction rnd, and put the read float in rop.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using mpfr_set_str (it may change). See the documentation of mpfr_strtofr for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

5.9 Integer Related Functions

int mpfr_rint (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)      [Function]
int mpfr ceil (mpfr_t rop, mpfr_t op)                    [Function]
int mpfr_floor (mpfr_t rop, mpfr_t op)                   [Function]
int mpfr_round (mpfr_t rop, mpfr_t op)                   [Function]
int mpfr trunc (mpfr_t rop, mpfr_t op)                   [Function]

Set rop to op rounded to an integer. mpfr_rint rounds to the nearest representable integer in the given rounding mode, mpfr ceil rounds to the next higher or equal representable integer, mpfr_floor to the next lower or equal representable integer, mpfr_round to the nearest representable integer, rounding halfway cases away from zero, and mpfr trunc to the next representable integer toward zero.

The returned value is zero when the result is exact, positive when it is greater than the original value of op, and negative when it is smaller. More precisely, the returned value is 0 when op is an integer representable in rop, 1 or −1 when op is an integer that is not representable in rop, 2 or −2 when op is not an integer.

Note that mpfr_round is different from mpfr_rint called with the rounding to nearest mode (where halfway cases are rounded to an even integer or significand). Note also that no double rounding is performed; for instance, 4.5 (100.1 in binary) is rounded by mpfr_round to 4 (100 in binary) in 2-bit precision, though round(4.5) is equal to 5 and 5 (101 in binary) is rounded to 6 (110 in binary) in 2-bit precision.

int mpfr_rint ceil (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_rint floor (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_rint round (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]
int mpfr_rint trunc (mpfr_t rop, mpfr_t op, mp_rnd_t rnd) [Function]

Set rop to op rounded to an integer. mpfr_rint ceil rounds to the next higher or equal integer, mpfr_rint floor to the next lower or equal integer, mpfr_rint round to the nearest integer, rounding halfway cases away from zero, and mpfr_rint trunc to the next integer toward zero. If the result is not representable, it is rounded in the direction rnd. The returned value is the ternary value associated with the considered round-to-integer function (regarded in the same way as any other mathematical function).

int mpfr frac (mpfr_t rop, mpfr_t op, mp_rnd_t rnd)     [Function]

Set rop to the fractional part of op, having the same sign as op, rounded in the direction rnd (unlike in mpfr_rint, rnd affects only how the exact fractional part is rounded, not how the fractional part is generated).
int mpfr_remainder (mpfr_t r, mpfr_t x, mpfr_t y, mpfr_t rnd)

int mpfr_remquo (mpfr_t r, long*q, mpfr_t x, mpfr_t y, mpfr_t rnd)
Set r to the remainder of the division of x by y, with quotient rounded to the nearest integer (ties rounded to even), and r rounded according to the direction rnd. If r is zero, it has the sign of x. The return value is the ternary value corresponding to r. Additionally, mpfr_remquo stores the low significant bits from the quotient in *q (more precisely the number of bits in a long minus one), with the sign of x divided by y (except if those low bits are all zero, in which case zero is returned). Note that x may be so large in magnitude relative to y that an exact representation of the quotient is not practical. These functions are useful for additive argument reduction.

int mpfr_integer_p (mpfr_t op)
Return non-zero iff op is an integer.

5.10 Miscellaneous Functions

void mpfr_nexttoward (mpfr_t x, mpfr_t y)
If x or y is NaN, set x to NaN. Otherwise, if x is different from y, replace x by the next floating-point number (with the precision of x and the current exponent range) in the direction of y, if there is one (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow or overflow is generated.

void mpfr_nextabove (mpfr_t x)
Equivalent to mpfr_nexttoward where y is plus infinity.

void mpfr_nextbelow (mpfr_t x)
Equivalent to mpfr_nexttoward where y is minus infinity.

int mpfr_min (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t rnd)
Set rop to the minimum of op1 and op2. If op1 and op2 are both NaN, then rop is set to NaN. If op1 or op2 is NaN, then rop is set to the numeric value. If op1 and op2 are zeros of different signs, then rop is set to −0.

int mpfr_max (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t rnd)
Set rop to the maximum of op1 and op2. If op1 and op2 are both NaN, then rop is set to NaN. If op1 or op2 is NaN, then rop is set to the numeric value. If op1 and op2 are zeros of different signs, then rop is set to +0.

int mpfr_urandomb (mpfr_t rop, gmp_randstate_t state)
Generate a uniformly distributed random float in the interval 0 ≤ rop < 1. Return 0, unless the exponent is not in the current exponent range, in which case rop is set to NaN and a non-zero value is returned. The second argument is a gmp_randstate_t structure which should be created using the GMP gmp_randinit function, see the GMP manual.

void mpfr_random (mpfr_t rop)
Generate a uniformly distributed random float in the interval 0 ≤ rop < 1. This function is deprecated; mpfr_urandomb should be used instead.

void mpfr_random2 (mpfr_t rop, mp_size_t size, mp_exp_t exp)
Generate a random float of at most size limbs, with long strings of zeros and ones in the binary representation. The exponent of the number is in the interval −exp to exp. This function is useful for testing functions and algorithms, since this kind of random numbers
have proven to be more likely to trigger corner-case bugs. Negative random numbers are generated when size is negative. Put +0 in rop when size if zero. The internal state of the default pseudorandom number generator is modified by a call to this function (the same one as GMP if MPFR was built using ‘--with-gmp-build’).

**mpf_exp_t mpfr_get_exp (mpfr_t x)**

Get the exponent of x, assuming that x is a non-zero ordinary number and the significand is chosen in [1/2,1). The behavior for NaN, infinity or zero is undefined.

**int mpfr_set_exp (mpfr_t x, mp_exp_t e)**

Set the exponent of x if e is in the current exponent range, and return 0 (even if x is not a non-zero ordinary number); otherwise, return a non-zero value. The significand is assumed to be in [1/2,1).

**int mpfr_signbit (mpfr_t op)**

Return a non-zero value iff op has its sign bit set (i.e. if it is negative, −0, or a NaN whose representation has its sign bit set).

**int mpfr_setsign (mpfr_t rop, mpfr_t op, int s, mp_rnd_t rnd)**

Set the value of rop from op, rounded toward the given direction rnd, then set (resp. clear) its sign bit if s is non-zero (resp. zero), even when op is a NaN.

**int mpfr_copysign (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd)**

Set the value of rop from op1, rounded toward the given direction rnd, then set its sign bit to that of op2 (even when op1 or op2 is a NaN). This function is equivalent to mpfr_setsign (rop, op1, mpfr_signbit (op2), rnd).

**const char * mpfr_get_version (void)**

Return the MPFR version, as a null-terminated string.

**MPFR_VERSION**
**MPFR_VERSION_MAJOR**
**MPFR_VERSION_MINOR**
**MPFR_VERSION_PATCHLEVEL**
**MPFR_VERSION_STRING**

**MPFR_VERSION** is the version of MPFR as a preprocessing constant. **MPFR_VERSION_MAJOR**, **MPFR_VERSION_MINOR** and **MPFR_VERSION_PATCHLEVEL** are respectively the major, minor and patch level of MPFR version, as preprocessing constants. **MPFR_VERSION_STRING** is the version (with an optional suffix, used in development and pre-release versions) as a string constant, which can be compared to the result of mpfr_get_version to check at run time the header file and library used match:

```c
if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))
    fprintf (stderr, "Warning: header and library do not match\n");
```

Note: Obtaining different strings is not necessarily an error, as in general, a program compiled with some old MPFR version can be dynamically linked with a newer MPFR library version (if allowed by the library versioning system).

**long MPFR_VERSION_NUM (major, minor, patchlevel)**

Create an integer in the same format as used by **MPFR_VERSION** from the given major, minor and patchlevel. Here is an example of how to check the MPFR version at compile time:
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```c
#ifndef (defined(MPFR_VERSION) || (MPFR_VERSION<MPFR_VERSION_NUM(2,1,0)))
#error "Wrong MPFR version."
#endif

const char * mpfr_get_patches (void) [Function]
Return a null-terminated string containing the ids of the patches applied to the MPFR library
(contents of the 'PATCHES' file), separated by spaces. Note: If the program has been compiled
with an older MPFR version and is dynamically linked with a new MPFR library version, the
ids of the patches applied to the old (compile-time) MPFR version are not available (however
this information should not have much interest in general).

5.11 Rounding Mode Related Functions

void mpfr_set_default_rounding_mode (mp_rnd_t rnd) [Function]
Set the default rounding mode to rnd. The default rounding mode is to nearest initially.

mp_rnd_t mpfr_get_default_rounding_mode (void) [Function]
Get the default rounding mode.

int mpfr_prec_round (mpfr_t x, mpfr_t prec, mp_rnd_t rnd) [Function]
Round x according to rnd with precision prec, which must be an integer between MPFR_PREC_MIN
and MPFR_PREC_MAX (otherwise the behavior is undefined). If prec is greater or equal to
the precision of x, then new space is allocated for the significand, and it is filled with zeros.
Otherwise, the significand is rounded to precision prec with the given direction. In both
cases, the precision of x is changed to prec.

int mpfr_round_prec (mpfr_t x, mp_rnd_t rnd, mpfr_t prec) [Function]
[This function is obsolete. Please use mpfr_prec_round instead.]

const char * mpfr_print_rnd_mode (mp_rnd_t rnd) [Function]
Return the input string (GMP_RNDN, GMP_RNDU, GMP_RNDD, GMP_RNDZ) corre-
sponding to the rounding mode rnd or a null pointer if rnd is an invalid rounding mode.

5.12 Exception Related Functions

mp_exp_t mpfr_get_emin (void) [Function]
mp_exp_t mpfr_get_emax (void) [Function]
Return the (current) smallest and largest exponents allowed for a floating-point variable. The
smallest positive value of a floating-point variable is $1/2 \times 2^{emin}$ and the largest value has the
form $(1-\varepsilon) \times 2^{emax}$.

int mpfr_set_emin (mp_exp_t exp) [Function]
int mpfr_set_emax (mp_exp_t exp) [Function]
Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero
value when exp is not in the range accepted by the implementation (in that case the smallest
or largest exponent is not changed), and zero otherwise. If the user changes the exponent
range, it is her/his responsibility to check that all current floating-point variables are in the
new allowed range (for example using mpfr_check_range), otherwise the subsequent behavior
will be undefined, in the sense of the ISO C standard.

mp_exp_t mpfr_get_emin_min (void) [Function]
mp_exp_t mpfr_get_emin_max (void) [Function]
mp_exp_t mpfr_get_emax_min (void)  
mp_exp_t mpfr_get_emax_max (void)  

Return the minimum and maximum of the smallest and largest exponents allowed for mpfr_set_emin and mpfr_set_emax. These values are implementation dependent; it is possible to create a non portable program by writing mpfr_set_emax(mpfr_get_emax_max()) and mpfr_set_emin(mpfr_get_emin_min()) since the values of the smallest and largest exponents become implementation dependent.

int mpfr_check_range (mpfr_t x, int t, mpfr_t rnd)  

This function forces x to be in the current range of acceptable values, t being the current ternary value: negative if x is smaller than the exact value, positive if x is larger than the exact value and zero if x is exact (before the call). It generates an underflow or an overflow if the exponent of x is outside the current allowed range; the value of t may be used to avoid a double rounding. This function returns zero if the rounded result is equal to the exact one, a positive value if the rounded result is larger than the exact one, a negative value if the rounded result is smaller than the exact one. Note that unlike most functions, the result is compared to the exact one, not the input value x, i.e. the ternary value is propagated.

Note: If x is an infinity and t is different from zero (i.e., if the rounded result is an inexact infinity), then the overflow flag is set. This is useful because mpfr_check_range is typically called (at least in MPFR functions) after restoring the flags that could have been set due to internal computations.

int mpfr_subnormalize (mpfr_t x, int t, mpfr_t rnd)  

This function rounds x emulating subnormal number arithmetic: if x is outside the subnormal exponent range, it just propagates the ternary value t; otherwise, it rounds x to precision EXP(x)-emin+1 according to rounding mode rnd and previous ternary value t, avoiding double rounding problems. More precisely in the subnormal domain, denoting by e the value ofemin, x is rounded in fixed-point arithmetic to an integer multiple of 2e−1; as a consequence, 1.5 × 2e−1 when t is zero is rounded to 2e with rounding to nearest.

PREC(x) is not modified by this function. rnd and t must be the used rounding mode for computing x and the returned ternary value when computing x. The subnormal exponent range is from emin to emin+PREC(x)-1. If the result cannot be represented in the current exponent range (due to a too small emax), the behavior is undefined. Note that unlike most functions, the result is compared to the exact one, not the input value x, i.e. the ternary value is propagated. This is a preliminary interface.

This is an example of how to emulate double IEEE-754 arithmetic using MPFR:

```c
{
    mpfr_t xa, xb;
    int i;
    volatile double a, b;

    mpfr_set_default_prec (53);
    mpfr_set_emin (-1073);
    mpfr_set_emax (1024);

    mpfr_init (xa); mpfr_init (xb);

    b = 34.3; mpfr_set_d (xb, b, GMP_RNDN);
    a = 0x1.1235P-1021; mpfr_set_d (xa, a, GMP_RNDN);
}```
a /= b;
i = mpfr_div (xa, xa, xb, GMP_RNDN);
i = mpfr_subnormalize (xa, i, GMP_RNDN);

mpfr_clear (xa); mpfr_clear (xb);
}

Warning: this emulates a double IEEE-754 arithmetic with correct rounding in the subnormal range, which may not be the case for your hardware.

void mpfr_clear_underflow (void) [Function]
void mpfr_clear_overflow (void) [Function]
void mpfr_clear_nanflag (void) [Function]
void mpfr_clear_inexflag (void) [Function]
void mpfr_clear_erangeflag (void) [Function]

Clear the underflow, overflow, invalid, inexact and erange flags.

void mpfr_set_underflow (void) [Function]
void mpfr_set_overflow (void) [Function]
void mpfr_set_nanflag (void) [Function]
void mpfr_set_inexflag (void) [Function]
void mpfr_set_erangeflag (void) [Function]

Set the underflow, overflow, invalid, inexact and erange flags.

void mpfr_clear_flags (void) [Function]

Clear all global flags (underflow, overflow, inexact, invalid, erange).

int mpfr_underflow_p (void) [Function]
int mpfr_overflow_p (void) [Function]
int mpfr_nanflag_p (void) [Function]
int mpfr_inexflag_p (void) [Function]
int mpfr_erangeflag_p (void) [Function]

Return the corresponding (underflow, overflow, invalid, inexact, erange) flag, which is non-zero iff the flag is set.

5.13 Advanced Functions

All the given interfaces are preliminary. They might change incompatibly in future revisions.

MPFR_DECL_INIT (name, prec) [Macro]

This macro declares name as an automatic variable of type mpfr_t, initializes it and sets its precision to be exactly prec bits and its value to NaN. name must be a valid identifier. You must use this macro in the declaration section. This macro is much faster than using mpfr_init2 but has some drawbacks:

- You must not call mpfr_clear with variables created with this macro (The storage is allocated at the point of declaration and deallocated when the brace-level is exited.).
- You can not change their precision.
- You should not create variables with huge precision with this macro.
- Your compiler must support ‘Non-Constant Initializers’ (standard in C++ and ISO C99) and ‘Token Pasting’ (standard in ISO C89). If prec is not a compiler constant, your compiler must support ‘Variable-length automatic arrays’ (standard in ISO C99). ‘GCC 2.95.3’ supports all these features. If you compile your program with gcc
in c89 mode and with ‘-pedantic’, you may want to define the MPFR_USE_EXTENSION macro to avoid warnings due to the MPFRDECL_INIT implementation.

```c
void mpfr_inits (mpfr_t x, ...)  
[Function]  
Initialize all the mpfr_t variables of the given va_list, set their precision to be the default precision and their value to NaN. See mpfr_init for more details. The va_list is assumed to be composed only of type mpfr_t (or equivalently mpfr_ptr). It begins from x. It ends when it encounters a null pointer (whose type must also be mpfr_ptr).

void mpfr_inits2 (mp_prec_t prec, mpfr_t x, ...)  
[Function]  
Initialize all the mpfr_t variables of the given va_list, set their precision to be exactly prec bits and their value to NaN. See mpfr_init2 for more details. The va_list is assumed to be composed only of type mpfr_t (or equivalently mpfr_ptr). It begins from x. It ends when it encounters a null pointer (whose type must also be mpfr_ptr).

void mpfr_clears (mpfr_t x, ...)  
[Function]  
Free the space occupied by all the mpfr_t variables of the given va_list. See mpfr_clear for more details. The va_list is assumed to be composed only of type mpfr_t (or equivalently mpfr_ptr). It begins from x. It ends when it encounters a null pointer (whose type must also be mpfr_ptr).
```

Here is an example of how to use multiple initialization functions:

```c
{  
    mpfr_t x, y, z, t;
    mpfr_inits2 (256, x, y, z, t, (mpfr_ptr) 0);
    ...
    mpfr_clears (x, y, z, t, (mpfr_ptr) 0);
}
```

### 5.14 Compatibility With MPF

A header file `mpf2mpfr.h` is included in the distribution of MPFR for compatibility with the GNU MP class MPF. After inserting the following two lines after the `#include <gmp.h>` line,

```c
#include <mpfr.h>
#include <mpf2mpfr.h>
```

any program written for MPF can be compiled directly with MPFR without any changes. All operations are then performed with the default MPFR rounding mode, which can be reset with `mpfr_set_default_rounding_mode`.

Warning: the `mpf_init` and `mpf_init2` functions initialize to zero, whereas the corresponding MPFR functions initialize to NaN: this is useful to detect uninitialized values, but is slightly incompatible with mpf.

```c
void mpfr_set_prec_raw (mpfr_t x, mp_prec_t prec)  
[Function]  
Reset the precision of x to be exactly prec bits. The only difference with mpfr_set_prec is that prec is assumed to be small enough so that the significand fits into the current allocated memory space for x. Otherwise the behavior is undefined.

int mpfr_eq (mpfr_t op1, mpfr_t op2, unsigned long int op3)  
[Function]  
Return non-zero if op1 and op2 are both non-zero ordinary numbers with the same exponent and the same first op3 bits, both zero, or both infinities of the same sign. Return zero
otherwise. This function is defined for compatibility with mpfr. Do not use it if you want to
know whether two numbers are close to each other; for instance, 1.011111 and 1.100000 are
regarded as different for any value of op3 larger than 1.

```c
void mpfr_reldiff (mpfr_t rop, mpfr_t op1, mpfr_t op2, mp_rnd_t rnd) [Function]
Compute the relative difference between op1 and op2 and store the result in rop. This
function does not guarantee the correct rounding on the relative difference; it just computes
|op1 − op2|/op1, using the rounding mode rnd for all operations and the precision of rop.
```

```c
int mpfr_mul_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2,
                    mp_rnd_t rnd) [Function]
int mpfr_div_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2,
                   mp_rnd_t rnd) [Function]
See mpfr_mul_2ui and mpfr_div_2ui. These functions are only kept for compatibility with
MPF.
```

### 5.15 Custom Interface

Some applications use a stack to handle the memory and their objects. However, the MPFR
memory design is not well suited for such a thing. So that such applications are able to use
MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows them to use MPFR in two ways:

- Either they directly store the MPFR FP number as a mpfr_t on the stack.
- Either they store their own representation of a FP number on the stack and construct a
  new temporary mpfr_t each time it is needed.

Nothing has to be done to destroy the FP numbers except garbageing the used memory: all the
memory stuff (allocating, destroying, garbageing) is kept to the application.

Each function in this interface is also implemented as a macro for efficiency reasons: for ex-
ample mpfr_custom_init (s, p) uses the macro, while (mpfr_custom_init) (s, p) uses the
function.

Note 1: MPFR functions may still initialize temporary FP numbers using standard mpfr_init.
See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (mpfr_const_pi for example), even if they
are not explicitly called. You have to call mpfr_free_cache each time you garbage the memory
iff mpfr_init, through GMP Custom Allocation, allocates its memory on the application stack.

Note 3: This interface is preliminary.

```c
size_t mpfr_custom_get_size (mp_prec_t prec) [Function]
Return the needed size in bytes to store the significand of a FP number of precision prec.
```

```c
void mpfr_custom_init (void *significand, mp_prec_t prec) [Function]
Initialize a significand of precision prec. significand must be an area of mpfr_custom_get_-
size (prec) bytes at least and be suitably aligned for an array of mp_limb_t.
```

```c
void mpfr_custom_init_set (mpfr_t x, int kind, mp_exp_t exp, mp_prec_t
                           prec, void *significand) [Function]
Perform a dummy initialization of a mpfr_t and set it to:
- if ABS(kind) == MPFR_NAN_KIND, x is set to NaN;
```
• if \( \text{ABS}(\text{kind}) == \text{MPFR\_INF\_KIND} \), \( x \) is set to the infinity of sign \( \text{sign}(\text{kind}) \);
• if \( \text{ABS}(\text{kind}) == \text{MPFR\_ZERO\_KIND} \), \( x \) is set to the zero of sign \( \text{sign}(\text{kind}) \);
• if \( \text{ABS}(\text{kind}) == \text{MPFR\_REGULAR\_KIND} \), \( x \) is set to a regular number: \( x = \text{sign}(\text{kind})\text{significand} \times 2^\text{exp} \)

In all cases, it uses \textit{significand} directly for further computing involving \( x \). It will not allocate anything. A FP number initialized with this function cannot be resized using \texttt{mpfr\_set\_prec}, or cleared using \texttt{mpfr\_clear}! \textit{Significand} must have been initialized with \texttt{mpfr\_custom\_init} using the same precision \( \text{prec} \).

**Function**

```c
int mpfr\_custom\_get\_kind (mpfr\_t \( x \))
```

Return the current kind of a \( \text{mpfr\_t} \) as used by \texttt{mpfr\_custom\_init\_set}. The behavior of this function for any \( \text{mpfr\_t} \) not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

**Function**

```c
void * mpfr\_custom\_get\_mantissa (mpfr\_t \( x \))
```

Return a pointer to the significand used by a \( \text{mpfr\_t} \) initialized with \texttt{mpfr\_custom\_init\_set}. The behavior of this function for any \( \text{mpfr\_t} \) not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

**Function**

```c
mp\_exp\_t mpfr\_custom\_get\_exp (mpfr\_t \( x \))
```

Return the exponent of \( x \), assuming that \( x \) is a non-zero ordinary number. The return value for NaN, Infinity or Zero is unspecified but doesn’t produce any trap. The behavior of this function for any \( \text{mpfr\_t} \) not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

**Function**

```c
void mpfr\_custom\_move (mpfr\_t \( x \), void *\texttt{new\_position})
```

Inform MPFR that the significand has moved due to a garbage collect and update its new position to \texttt{new\_position}. However the application has to move the significand and the \( \text{mpfr\_t} \) itself. The behavior of this function for any \( \text{mpfr\_t} \) not initialized with \texttt{mpfr\_custom\_init\_set} is undefined.

See the test suite for examples.

### 5.16 Internals

The following types and functions were mainly designed for the implementation of MPFR, but may be useful for users too. However no upward compatibility is guaranteed. You may need to include ‘\texttt{mpfr-impl.h}’ to use them.

The \texttt{mpfr\_t} type consists of four fields.

- The \texttt{_mpfr\_prec} field is used to store the precision of the variable (in bits); this is not less than \texttt{MPFR\_PREC\_MIN}.
- The \texttt{_mpfr\_sign} field is used to store the sign of the variable.
- The \texttt{_mpfr\_exp} field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values \( n \) are a multiplier \( 2^n \) relative to that point. A NaN, an infinity and a zero are indicated by a special value of the exponent.
- Finally, the \texttt{_mpfr\_d} is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by \texttt{_mpfr\_prec}, namely \( \text{ceil}(\text{mpfr\_prec}/\text{mp_bits\_per\_limb}) \). Non-singular values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zero.
int mpfr_can_round (mpfr_t b, mpfr_t err, mpfr_t rnd1, mpfr_t rnd2, mpfr_t prec) [Function]

Assuming $b$ is an approximation of an unknown number $x$ in the direction $\text{rnd1}$ with error at most two to the power $E(b)-\text{err}$ where $E(b)$ is the exponent of $b$, return a non-zero value if one is able to round correctly $x$ to precision $\text{prec}$ with the direction $\text{rnd2}$, and 0 otherwise (including for NaN and Inf). This function does not modify its arguments.

Note: if one wants to also determine the correct ternary value when rounding $b$ to precision $\text{prec}$, a useful trick is the following:

$$\text{if (mpfr\_can\_round (b, err, \text{rnd1}, \text{GMP\_RNDZ}, \text{prec} + (\text{rnd2} == \text{GMP\_RNDN})))}$$

... Indeed, if $\text{rnd2}$ is $\text{GMP\_RNDN}$, this will check if one can round to $\text{prec}+1$ bits with a directed rounding: if so, one can surely round to nearest to $\text{prec}$ bits, and in addition one can determine the correct ternary value, which would not be the case when $b$ is near from a value exactly representable on $\text{prec}$ bits.

double mpfr_get_d1 (mpfr_t op) [Function]

Convert $op$ to a double, using the default MPFR rounding mode (see function mpfr_set_default_rounding_mode). This function is obsolete.
Contributors

The main developers of MPFR are Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier, Philippe Théveny and Paul Zimmermann.

Sylvie Boldo from ENS-Lyon, France, contributed the functions \texttt{mpfr\_agm} and \texttt{mpfr\_log}. Emmanuel Jeandel, from ENS-Lyon too, contributed the generic hypergeometric code in \texttt{generic.c}, as well as the \texttt{mpfr\_exp3}, a first implementation of the sine and cosine, and improved versions of \texttt{mpfr\_const\_log2} and \texttt{mpfr\_const\_pi}. Mathieu Dutour contributed the functions \texttt{mpfr\_atan} and \texttt{mpfr\_asin}, and a previous version of \texttt{mpfr\_gamma}; David Daney contributed the hyperbolic and inverse hyperbolic functions, the base-2 exponential, and the factorial function. Fabrice Rouillier contributed the original version of \texttt{mul\_ui.c}, the \texttt{gmp\_op.c} file, and helped to the Microsoft Windows porting. Jean-Luc Rémy contributed the \texttt{mpfr\_zeta} code. Ludovic Meunier helped in the design of the \texttt{mpfr\_erf} code. Damien Stehlé contributed the \texttt{mpfr\_get\_ld\_2exp} function.

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References

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