GNU MPFR

The Multiple Precision Floating-Point Reliable Library
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This manual documents how to install and use the Multiple Precision Floating-Point Reliable Library, version 3.1.6.

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1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to provide a class of floating-point numbers with precise semantics. The main characteristics of MPFR, which make it differ from most arbitrary precision floating-point software tools, are:

- the MPFR code is portable, i.e., the result of any operation does not depend on the machine word size `mp_bits_per_limb` (64 on most current processors);
- the precision in bits can be set *exactly* to any valid value for each variable (including very small precision);
- MPFR provides the four rounding modes from the IEEE 754-1985 standard, plus away-from-zero, as well as for basic operations as for other mathematical functions.

In particular, with a precision of 53 bits, MPFR is able to exactly reproduce all computations with double-precision machine floating-point numbers (e.g., `double` type in C, with a C implementation that rigorously follows Annex F of the ISO C99 standard and `FP_CONTRACT` pragma set to `OFF`) on the four arithmetic operations and the square root, except the default exponent range is much wider and subnormal numbers are not implemented (but can be emulated).

This version of MPFR is released under the GNU Lesser General Public License, version 3 or any later version. It is permitted to link MPFR to most non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

1.1 How to Use This Manual

Everyone should read Chapter 4 [MPFR Basics], page 6. If you need to install the library yourself, you need to read Chapter 2 [Installing MPFR], page 3, too. To use the library you will need to refer to Chapter 5 [MPFR Interface], page 11.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.

2 Installing MPFR

The MPFR library is already installed on some GNU/Linux distributions, but the development files necessary to the compilation such as `mpfr.h` are not always present. To check that MPFR is fully installed on your computer, you can check the presence of the file `mpfr.h` in `/usr/include`, or try to compile a small program having `#include <mpfr.h>` (since `mpfr.h` may be installed somewhere else). For instance, you can try to compile:

```
#include <stdio.h>
#include <mpfr.h>
int main (void)
{
  printf ("MPFR library: %-12s\nMPFR header:  %s (based on %d.%d.%d)\n",
          mpfr_get_version (), MPFR_VERSION_STRING, MPFR_VERSION_MAJOR,
          MPFR_VERSION_MINOR, MPFR_VERSION_PATCHLEVEL);
  return 0;
}
```

with

```
cc -o version version.c -lmpfr -lgmp
```

and if you get errors whose first line looks like

```
version.c:2:19: error: mpfr.h: No such file or directory
```

then MPFR is probably not installed. Running this program will give you the MPFR version.

If MPFR is not installed on your computer, or if you want to install a different version, please follow the steps below.

2.1 How to Install

Here are the steps needed to install the library on Unix systems (more details are provided in the `INSTALL` file):

1. To build MPFR, you first have to install GNU MP (version 4.1 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need the standard Unix '`make`' command, plus some other standard Unix utility commands.

   Then, in the MPFR build directory, type the following commands.
2. '`./configure`'

   This will prepare the build and setup the options according to your system. You can give options to specify the install directories (instead of the default `/usr/local`), threading support, and so on. See the `INSTALL` file and/or the output of '`./configure --help`' for more information, in particular if you get error messages.
3. '`make`'

   This will compile MPFR, and create a library archive file `libmpfr.a`. On most platforms, a dynamic library will be produced too.
4. '`make check`'

   This will make sure that MPFR was built correctly. If any test fails, information about this failure can be found in the `tests/test-suite.log` file. If you want the contents of this file to be automatically output in case of failure, you can set the '`VERBOSE`' environment variable to 1 before running '`make check`', for instance by typing:

   '`VERBOSE=1 make check`'

In case of failure, you may want to check whether the problem is already known. If not, please report this failure to the MPFR mailing-list '`mpfr@inria.fr`'. For details, See Chapter 3 [Reporting Bugs], page 5.

5. '`make install`'

   This will copy the files `mpfr.h` and `mpf2mpfr.h` to the directory `/usr/local/include`, the library files (`libmpfr.a` and possibly others) to the directory `/usr/local/lib`, the file `mpfr.info` to the directory `/usr/local/share/info`, and some other documentation files to the directory `/usr/local/share/doc/mpfr` (or if you passed the '`--prefix`' option to `configure`, using the prefix directory given as argument to '`--prefix`' instead of `/usr/local`).

2.2 Other 'make' Targets

There are some other useful make targets:

- '`mpfr.info`' or '`info`'

  Create or update an info version of the manual, in `mpfr.info`.

  This file is already provided in the MPFR archives.
- '`mpfr.pdf`' or '`pdf`'

  Create a PDF version of the manual, in `mpfr.pdf`.
- '`mpfr.dvi`' or '`dvi`'

  Create a DVI version of the manual, in `mpfr.dvi`.
- '`mpfr.ps`' or '`ps`'

  Create a Postscript version of the manual, in `mpfr.ps`.
- '`mpfr.html`' or '`html`'

  Create a HTML version of the manual, in several pages in the directory `doc/mpfr.html`; if you want only one output HTML file, then type '`makeinfo --html --no-split mpfr.texi`' from the '`doc`' directory instead.
- '`clean`'

  Delete all object files and archive files, but not the configuration files.
- '`distclean`'

  Delete all generated files not included in the distribution.
- '`uninstall`'

  Delete all files copied by '`make install`'.

2.3 Build Problems

In case of problem, please read the `INSTALL` file carefully before reporting a bug, in particular section "In case of problem". Some problems are due to bad configuration on the user side (not specific to MPFR). Problems are also mentioned in the FAQ `http://www.mpfr.org/faq.html`.

Please report problems to the MPFR mailing-list '`mpfr@inria.fr`'. See Chapter 3 [Reporting Bugs], page 5. Some bug fixes are available on the MPFR 3.1.6 web page `http://www.mpfr.org/mpfr-3.1.6/`.

2.4 Getting the Latest Version of MPFR

The latest version of MPFR is available from `https://ftp.gnu.org/gnu/mpfr/` or `http://www.mpfr.org/`.

3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR 3.1.6 web page `http://www.mpfr.org/mpfr-3.1.6/` and the FAQ `http://www.mpfr.org/faq.html`: perhaps this bug is already known, in which case you may find there a workaround for it. You might also look in the archives of the MPFR mailing-list: `https://sympa.inria.fr/sympa/arc/mpfr`. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you, to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug, i.e., a small self-content program, using no other library than MPFR. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results you get are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using '`cc -V`' on some machines, or, if you're using GCC, '`gcc -v`'. Also, include the output from '`uname -a`' and the MPFR version (the GMP version may be useful too). If you get a failure while running '`make`' or '`make check`', please include the `config.log` file in your bug report, and in case of test failure, the `tests/test-suite.log` file too.

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we will not do anything about it (aside of chiding you to send better bug reports).

Send your bug report to the MPFR mailing-list '`mpfr@inria.fr`'.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.

4 MPFR Basics

4.1 Headers and Libraries

All declarations needed to use MPFR are collected in the include file `mpfr.h`. It is designed to work with both C and C++ compilers. You should include that file in any program using the MPFR library:

```
#include <mpfr.h>
```

Note however that prototypes for MPFR functions with `FILE *` parameters are provided only if `<stdio.h>` is included too (before `mpfr.h`):

```
#include <stdio.h>
#include <mpfr.h>
```

Likewise `<stdarg.h>` (or `<varargs.h>`) is required for prototypes with `va_list` parameters, such as `mpfr_vprintf`.

And for any functions using `intmax_t`, you must include `<stdint.h>` or `<inttypes.h>` before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions. Moreover, users of C++ compilers under some platforms may need to define `MPFR_USE_INTMAX_T` (and should do it for portability) before `mpfr.h` has been included; of course, it is possible to do that on the command line, e.g., with `-DMPFR_USE_INTMAX_T`.

Note: If `mpfr.h` and/or `gmp.h` (used by `mpfr.h`) are included several times (possibly from another header file), `<stdio.h>` and/or `<stdarg.h>` (or `<varargs.h>`) should be included **before the first inclusion** of `mpfr.h` or `gmp.h`. Alternatively, you can define `MPFR_USE_FILE` (for MPFR I/O functions) and/or `MPFR_USE_VA_LIST` (for MPFR functions with `va_list` parameters) anywhere before the last inclusion of `mpfr.h`. As a consequence, if your file is a public header that includes `mpfr.h`, you need to use the latter method.

When calling a MPFR macro, it is not allowed to have previously defined a macro with the same name as some keywords (currently `do`, `while` and `sizeof`).

You can avoid the use of MPFR macros encapsulating functions by defining the `MPFR_USE_NO_MACRO` macro before `mpfr.h` is included. In general this should not be necessary, but this can be useful when debugging user code: with some macros, the compiler may emit spurious warnings with some warning options, and macros can prevent some prototype checking.

All programs using MPFR must link against both `libmpfr` and `libgmp` libraries. On a typical Unix-like system this can be done with '`-lmpfr -lgmp`' (in that order), for example:

```
gcc myprogram.c -lmpfr -lgmp
```

MPFR is built using Libtool and an application can use that to link if desired, see *GNU Libtool*.

If MPFR has been installed to a non-standard location, then it may be necessary to set up environment variables such as '`C_INCLUDE_PATH`' and '`LIBRARY_PATH`', or use '`-I`' and '`-L`' compiler options, in order to point to the right directories. For a shared library, it may also be necessary to set up some sort of run-time library path (e.g., '`LD_LIBRARY_PATH`') on some systems. Please read the `INSTALL` file for additional information.

4.2 Nomenclature and Types

A *floating-point number*, or *float* for short, is an arbitrary precision significand (also called mantissa) with a limited precision exponent. The C data type for such objects is `mpfr_t`

(internally defined as a one-element array of a structure, and `mpfr_ptr` is the C data type representing a pointer to this structure). A floating-point number can have three special values: Not-a-Number (NaN) or plus or minus Infinity. NaN represents an uninitialized object, the result of an invalid operation (like 0 divided by 0), or a value that cannot be determined (like +Infinity minus +Infinity). Moreover, like in the IEEE 754 standard, zero is signed, i.e., there are both +0 and −0; the behavior is the same as in the IEEE 754 standard and it is generalized to the other functions supported by MPFR. Unless documented otherwise, the sign bit of a NaN is unspecified.

The *precision* is the number of bits used to represent the significand of a floating-point number; the corresponding C data type is `mpfr_prec_t`. The precision can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. In the current implementation, `MPFR_PREC_MIN` is equal to 2.

Warning! MPFR needs to increase the precision internally, in order to provide accurate results (and in particular, correct rounding). Do not attempt to set the precision to any value near `MPFR_PREC_MAX`, otherwise MPFR will abort due to an assertion failure. Moreover, you may reach some memory limit on your platform, in which case the program may abort, crash or have undefined behavior (depending on your C implementation).

The *rounding mode* specifies the way to round the result of a floating-point operation, in case the exact result can not be represented exactly in the destination significand; the corresponding C data type is `mpfr_rnd_t`.

4.3 MPFR Variable Conventions

Before you can assign to an MPFR variable, you need to initialize it by calling one of the special initialization functions. When you're done with a variable, you need to clear it out, using one of the functions for that purpose. A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned to any number of times. For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited. You do not need to be concerned about allocating additional space for MPFR variables, since any variable has a significand of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator. MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication, `mpfr_mul`, can be used like this: `mpfr_mul (x, x, x, rnd)`. This computes the square of x with rounding mode `rnd` and puts the result back in x.

4.4 Rounding Modes

The following five rounding modes are supported:

- `MPFR_RNDN`: round to nearest (roundTiesToEven in IEEE 754-2008),
- `MPFR_RNDZ`: round toward zero (roundTowardZero in IEEE 754-2008),
- `MPFR_RNDU`: round toward plus infinity (roundTowardPositive in IEEE 754-2008),
- `MPFR_RNDD`: round toward minus infinity (roundTowardNegative in IEEE 754-2008),
- `MPFR_RNDA`: round away from zero.

The '`round to nearest`' mode works as in the IEEE 754 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number 2.5, which is represented by (10.1) in

binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the *drift* phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value of type `int`, called the *ternary value*. The value stored in the destination variable is correctly rounded, i.e., MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

As a consequence, in case of a non-zero real rounded result, the error on the result is less or equal to 1/2 ulp (unit in the last place) of that result in the rounding to nearest mode, and less than 1 ulp of that result in the directed rounding modes (a ulp is the weight of the least significant represented bit of the result after rounding).

Unless documented otherwise, functions returning an `int` return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the `MPFR_RNDU` rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an `int`.

Unless documented otherwise, functions returning as result the value `1` (or any other value specified in this manual) for special cases (like `acos(0)`) yield an overflow or an underflow if that value is not representable in the current exponent range.

4.5 Floating-Point Values on Special Numbers

This section specifies the floating-point values (of type `mpfr_t`) returned by MPFR functions (where by "returned" we mean here the modified value of the destination object, which should not be mixed with the ternary return value of type `int` of those functions). For functions returning several values (like `mpfr_sin_cos`), the rules apply to each result separately.

Functions can have one or several input arguments. An input point is a mapping from these input arguments to the set of the MPFR numbers. When none of its components are NaN, an input point can also be seen as a tuple in the extended real numbers (the set of the real numbers with both infinities).

When the input point is in the domain of the mathematical function, the result is rounded as described in Section "Rounding Modes" (but see below for the specification of the sign of an exact zero). Otherwise the general rules from this section apply unless stated otherwise in the description of the MPFR function (Chapter 5 [MPFR Interface], page 11).

When the input point is not in the domain of the mathematical function but is in its closure in the extended real numbers and the function can be extended by continuity, the result is the obtained limit. Examples: `mpfr_hypot` on (+Inf,0) gives +Inf. But `mpfr_pow` cannot be defined on (1,+Inf) using this rule, as one can find sequences (x_n,y_n) such that x_n goes to 1, y_n goes to +Inf and $(x_n)^{y_n}$ goes to any positive value when n goes to the infinity.

When the input point is in the closure of the domain of the mathematical function and an input argument is +0 (resp. -0), one considers the limit when the corresponding argument approaches 0 from above (resp. below), if possible. If the limit is not defined (e.g., `mpfr_sqrt` and `mpfr_`

`log` on -0), the behavior is specified in the description of the MPFR function, but must be consistent with the rule from the above paragraph (e.g., `mpfr_log` on ± 0 gives $-$Inf).

When the result is equal to 0, its sign is determined by considering the limit as if the input point were not in the domain: If one approaches 0 from above (resp. below), the result is +0 (resp. -0); for example, `mpfr_sin` on -0 gives -0 and `mpfr_acos` on 1 gives +0 (in all rounding modes). In the other cases, the sign is specified in the description of the MPFR function; for example `mpfr_max` on -0 and +0 gives +0.

When the input point is not in the closure of the domain of the function, the result is NaN. Example: `mpfr_sqrt` on -17 gives NaN.

When an input argument is NaN, the result is NaN, possibly except when a partial function is constant on the finite floating-point numbers; such a case is always explicitly specified in Chapter 5 [MPFR Interface], page 11. Example: `mpfr_hypot` on (NaN,0) gives NaN, but `mpfr_hypot` on (NaN,+Inf) gives +Inf (as specified in Section 5.7 [Special Functions], page 22), since for any finite or infinite input x, `mpfr_hypot` on (x,+Inf) gives +Inf.

4.6 Exceptions

MPFR supports 6 exception types:

- Underflow: An underflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent smaller than the minimum value of the current exponent range. (In the round-to-nearest mode, the halfway case is rounded toward zero.)

  Note: This is not the single possible definition of the underflow. MPFR chooses to consider the underflow *after* rounding. The underflow before rounding can also be defined. For instance, consider a function that has the exact result $7 \times 2^{e-4}$, where e is the smallest exponent (for a significand between 1/2 and 1), with a 2-bit target precision and rounding toward plus infinity. The exact result has the exponent $e-1$. With the underflow before rounding, such a function call would yield an underflow, as $e-1$ is outside the current exponent range. However, MPFR first considers the rounded result assuming an unbounded exponent range. The exact result cannot be represented exactly in precision 2, and here, it is rounded to 0.5×2^e, which is representable in the current exponent range. As a consequence, this will not yield an underflow in MPFR.
- Overflow: An overflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent larger than the maximum value of the current exponent range. In the round-to-nearest mode, the result is infinite. Note: unlike the underflow case, there is only one possible definition of overflow here.
- Divide-by-zero: An exact infinite result is obtained from finite inputs.
- NaN: A NaN exception occurs when the result of a function is NaN.
- Inexact: An inexact exception occurs when the result of a function cannot be represented exactly and must be rounded.
- Range error: A range exception occurs when a function that does not return a MPFR number (such as comparisons and conversions to an integer) has an invalid result (e.g., an argument is NaN in `mpfr_cmp`, or a conversion to an integer cannot be represented in the target type).

MPFR has a global flag for each exception, which can be cleared, set or tested by functions described in Section 5.13 [Exception Related Functions], page 35.

Differences with the ISO C99 standard:

- In C, only quiet NaNs are specified, and a NaN propagation does not raise an invalid exception. Unless explicitly stated otherwise, MPFR sets the NaN flag whenever a NaN is generated, even when a NaN is propagated (e.g., in NaN + NaN), as if all NaNs were signaling.
- An invalid exception in C corresponds to either a NaN exception or a range error in MPFR.

4.7 Memory Handling

MPFR functions may create caches, e.g., when computing constants such as π, either because the user has called a function like `mpfr_const_pi` directly or because such a function was called internally by the MPFR library itself to compute some other function.

At any time, the user can free the various caches with `mpfr_free_cache`. It is strongly advised to do that before terminating a thread, or before exiting when using tools like '`valgrind`' (to avoid memory leaks being reported).

MPFR internal data such as flags, the exponent range, the default precision and rounding mode, and caches (i.e., data that are not accessed via parameters) are either global (if MPFR has not been compiled as thread safe) or per-thread (thread local storage, TLS). The initial values of TLS data after a thread is created entirely depend on the compiler and thread implementation (MPFR simply does a conventional variable initialization, the variables being declared with an implementation-defined TLS specifier).

By default, MPFR uses the GMP memory allocators (see Section "Custom Allocation" in *GNU MP*) instead of `malloc` for all its allocations. But since caches have been introduced in MPFR and nothing had been documented about necessary clean-up if the GMP allocators had to be changed (by the application or another library), the current GMP allocators are memorized the first time they are used by MPFR, and MPFR will always use these allocators. This means that the allocators need to remain valid as long as MPFR is used. To avoid this requirement, future versions may handle memory allocation differently, and applications that change the allocators may have to be slightly modified.

5 MPFR Interface

The floating-point functions expect arguments of type `mpfr_t`.

The MPFR floating-point functions have an interface that is similar to the GNU MP functions. The function prefix for floating-point operations is `mpfr_`.

The user has to specify the precision of each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend on the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with "infinite accuracy"), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754 arithmetic. The results obtained on a given computer are identical to those obtained on a computer with a different word size, or with a different compiler or operating system.

MPFR *does not keep track* of the accuracy of a computation. This is left to the user or to a higher layer (for example the MPFI library for interval arithmetic). As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with large precision, then MPFR will still compute the result with full precision.

The value of the standard C macro `errno` may be set to non-zero by any MPFR function or macro, whether or not there is an error.

5.1 Initialization Functions

An `mpfr_t` object must be initialized before storing the first value in it. The functions `mpfr_init` and `mpfr_init2` are used for that purpose.

`void mpfr_init2` (*mpfr_t* `x`, *mpfr_prec_t* `prec`) [Function]

Initialize *x*, set its precision to be **exactly** *prec* bits and its value to NaN. (Warning: the corresponding MPF function initializes to zero instead.)

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. To change the precision of a variable which has already been initialized, use `mpfr_set_prec`. The precision *prec* must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined).

`void mpfr_inits2` (*mpfr_prec_t* `prec`, *mpfr_t* `x`, ...) [Function]

Initialize all the `mpfr_t` variables of the given variable argument `va_list`, set their precision to be **exactly** *prec* bits and their value to NaN. See `mpfr_init2` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from *x*, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).

`void mpfr_clear` (*mpfr_t* `x`) [Function]

Free the space occupied by the significand of *x*. Make sure to call this function for all `mpfr_t` variables when you are done with them.

`void mpfr_clears` (*mpfr_t* `x`, ...) [Function]

Free the space occupied by all the `mpfr_t` variables of the given `va_list`. See `mpfr_clear` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from *x*, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).

Here is an example of how to use multiple initialization functions (since `NULL` is not necessarily defined in this context, we use `(mpfr_ptr) 0` instead, but `(mpfr_ptr) NULL` is also correct).

```
{
  mpfr_t x, y, z, t;
  mpfr_inits2 (256, x, y, z, t, (mpfr_ptr) 0);
  ...
  mpfr_clears (x, y, z, t, (mpfr_ptr) 0);
}
```

`void mpfr_init` (*mpfr_t x*) [Function]

Initialize *x*, set its precision to the default precision, and set its value to NaN. The default precision can be changed by a call to `mpfr_set_default_prec`.

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use `mpfr_init2`.

`void mpfr_inits` (*mpfr_t x, ...*) [Function]

Initialize all the `mpfr_t` variables of the given `va_list`, set their precision to the default precision and their value to NaN. See `mpfr_init` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from *x*, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use `mpfr_inits2`.

`MPFR_DECL_INIT` (*name, prec*) [Macro]

This macro declares *name* as an automatic variable of type `mpfr_t`, initializes it and sets its precision to be **exactly** *prec* bits and its value to NaN. *name* must be a valid identifier. You must use this macro in the declaration section. This macro is much faster than using `mpfr_init2` but has some drawbacks:

- You **must not** call `mpfr_clear` with variables created with this macro (the storage is allocated at the point of declaration and deallocated when the brace-level is exited).
- You **cannot** change their precision.
- You **should not** create variables with huge precision with this macro.
- Your compiler must support '`Non-Constant Initializers`' (standard in C++ and ISO C99) and '`Token Pasting`' (standard in ISO C89). If *prec* is not a constant expression, your compiler must support '`variable-length automatic arrays`' (standard in ISO C99). GCC 2.95.3 and above supports all these features. If you compile your program with GCC in C89 mode and with '`-pedantic`', you may want to define the `MPFR_USE_EXTENSION` macro to avoid warnings due to the `MPFR_DECL_INIT` implementation.

`void mpfr_set_default_prec` (*mpfr_prec_t prec*) [Function]

Set the default precision to be **exactly** *prec* bits, where *prec* can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. The precision of a variable means the number of bits used to store its significand. All subsequent calls to `mpfr_init` or `mpfr_inits` will use this precision, but previously initialized variables are unaffected. The default precision is set to 53 bits initially.

Note: when MPFR is built with the `--enable-thread-safe` configure option, the default precision is local to each thread. See Section 4.7 [Memory Handling], page 10, for more information.

`mpfr_prec_t mpfr_get_default_prec` (*void*) [Function]

Return the current default MPFR precision in bits. See the documentation of `mpfr_set_default_prec`.

Here is an example on how to initialize floating-point variables:

```
{
  mpfr_t x, y;
  mpfr_init (x);              /* use default precision */
  mpfr_init2 (y, 256);        /* precision exactly 256 bits */
  ...
  /* When the program is about to exit, do ... */
  mpfr_clear (x);
  mpfr_clear (y);
  mpfr_free_cache ();         /* free the cache for constants like pi */
}
```

The following functions are useful for changing the precision during a calculation. A typical use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson, making the computation precision closely match the actual accurate part of the numbers.

`void mpfr_set_prec` (*mpfr_t* `x`, *mpfr_prec_t* `prec`) [Function]

Reset the precision of *x* to be **exactly** *prec* bits, and set its value to NaN. The previous value stored in *x* is lost. It is equivalent to a call to `mpfr_clear(x)` followed by a call to `mpfr_init2(x, prec)`, but more efficient as no allocation is done in case the current allocated space for the significand of *x* is enough. The precision *prec* can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. In case you want to keep the previous value stored in *x*, use `mpfr_prec_round` instead.

Warning! You must not use this function if *x* was initialized with `MPFR_DECL_INIT` or with `mpfr_custom_init_set` (see Section 5.15 [Custom Interface], page 38).

`mpfr_prec_t mpfr_get_prec` (*mpfr_t* `x`) [Function]

Return the precision of *x*, i.e., the number of bits used to store its significand.

5.2 Assignment Functions

These functions assign new values to already initialized floats (see Section 5.1 [Initialization Functions], page 11).

`int mpfr_set` (*mpfr_t* `rop`, *mpfr_t* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_ui` (*mpfr_t* `rop`, *unsigned long int* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_si` (*mpfr_t* `rop`, *long int* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_uj` (*mpfr_t* `rop`, *uintmax_t* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_sj` (*mpfr_t* `rop`, *intmax_t* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_flt` (*mpfr_t* `rop`, *float* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_d` (*mpfr_t* `rop`, *double* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_ld` (*mpfr_t* `rop`, *long double* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_decimal64` (*mpfr_t* `rop`, *_Decimal64* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_z` (*mpfr_t* `rop`, *mpz_t* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_q` (*mpfr_t* `rop`, *mpq_t* `op`, *mpfr_rnd_t* `rnd`) [Function]
`int mpfr_set_f` (*mpfr_t* `rop`, *mpf_t* `op`, *mpfr_rnd_t* `rnd`) [Function]

Set the value of *rop* from *op*, rounded toward the given direction *rnd*. Note that the input 0 is converted to +0 by `mpfr_set_ui`, `mpfr_set_si`, `mpfr_set_uj`, `mpfr_set_sj`,

mpfr_set_z, mpfr_set_q and mpfr_set_f, regardless of the rounding mode. If the system does not support the IEEE 754 standard, mpfr_set_flt, mpfr_set_d, mpfr_set_ld and mpfr_set_decimal64 might not preserve the signed zeros. The mpfr_set_decimal64 function is built only with the configure option '--enable-decimal-float', which also requires '--with-gmp-build', and when the compiler or system provides the '_Decimal64' data type (recent versions of GCC support this data type); to use mpfr_set_decimal64, one should define the macro MPFR_WANT_DECIMAL_FLOATS before including mpfr.h. mpfr_set_q might fail if the numerator (or the denominator) can not be represented as a mpfr_t.

Note: If you want to store a floating-point constant to a mpfr_t, you should use mpfr_set_str (or one of the MPFR constant functions, such as mpfr_const_pi for π) instead of mpfr_set_flt, mpfr_set_d, mpfr_set_ld or mpfr_set_decimal64. Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary (or decimal, for mpfr_set_decimal64) number before MPFR can work with it.

int mpfr_set_ui_2exp (*mpfr_t* **rop**, *unsigned long int* **op**, *mpfr_exp_t* **e**, *mpfr_rnd_t* **rnd**) [Function]
int mpfr_set_si_2exp (*mpfr_t* **rop**, *long int* **op**, *mpfr_exp_t* **e**, *mpfr_rnd_t* **rnd**) [Function]
int mpfr_set_uj_2exp (*mpfr_t* **rop**, *uintmax_t* **op**, *intmax_t* **e**, *mpfr_rnd_t* **rnd**) [Function]
int mpfr_set_sj_2exp (*mpfr_t* **rop**, *intmax_t* **op**, *intmax_t* **e**, *mpfr_rnd_t* **rnd**) [Function]
int mpfr_set_z_2exp (*mpfr_t* **rop**, *mpz_t* **op**, *mpfr_exp_t* **e**, *mpfr_rnd_t* **rnd**) [Function]

Set the value of *rop* from $op \times 2^e$, rounded toward the given direction *rnd*. Note that the input 0 is converted to +0.

int mpfr_set_str (*mpfr_t* **rop**, *const char* ***s**, *int* **base**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the value of the string *s* in base *base*, rounded in the direction *rnd*. See the documentation of mpfr_strtofr for a detailed description of the valid string formats. Contrary to mpfr_strtofr, mpfr_set_str requires the *whole* string to represent a valid floating-point number.

The meaning of the return value differs from other MPFR functions: it is 0 if the entire string up to the final null character is a valid number in base *base*; otherwise it is -1, and *rop* may have changed (users interested in the [ternary value], page 8 should use mpfr_strtofr instead).

Note: it is preferable to use mpfr_strtofr if one wants to distinguish between an infinite *rop* value coming from an infinite *s* or from an overflow.

int mpfr_strtofr (*mpfr_t* **rop**, *const char* ***nptr**, *char* ****endptr**, *int* **base**, *mpfr_rnd_t* **rnd**) [Function]

Read a floating-point number from a string *nptr* in base *base*, rounded in the direction *rnd*; *base* must be either 0 (to detect the base, as described below) or a number from 2 to 62 (otherwise the behavior is undefined). If *nptr* starts with valid data, the result is stored in *rop* and ***endptr** points to the character just after the valid data (if *endptr* is not a null pointer); otherwise *rop* is set to zero (for consistency with strtod) and the value of *nptr* is stored in the location referenced by *endptr* (if *endptr* is not a null pointer). The usual ternary value is returned.

Parsing follows the standard C strtod function with some extensions. After optional leading whitespace, one has a subject sequence consisting of an optional sign (+ or -), and either nu-

meric data or special data. The subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-whitespace character, that is of the expected form.

The form of numeric data is a non-empty sequence of significand digits with an optional decimal point, and an optional exponent consisting of an exponent prefix followed by an optional sign and a non-empty sequence of decimal digits. A significand digit is either a decimal digit or a Latin letter (62 possible characters), with `A` = 10, `B` = 11, ..., `Z` = 35; case is ignored in bases less or equal to 36, in bases larger than 36, `a` = 36, `b` = 37, ..., `z` = 61. The value of a significand digit must be strictly less than the base. The decimal point can be either the one defined by the current locale or the period (the first one is accepted for consistency with the C standard and the practice, the second one is accepted to allow the programmer to provide MPFR numbers from strings in a way that does not depend on the current locale). The exponent prefix can be `e` or `E` for bases up to 10, or `@` in any base; it indicates a multiplication by a power of the base. In bases 2 and 16, the exponent prefix can also be `p` or `P`, in which case the exponent, called *binary exponent*, indicates a multiplication by a power of 2 instead of the base (there is a difference only for base 16); in base 16 for example `1p2` represents 4 whereas `1@2` represents 256. The value of an exponent is always written in base 10.

If the argument *base* is 0, then the base is automatically detected as follows. If the significand starts with `0b` or `0B`, base 2 is assumed. If the significand starts with `0x` or `0X`, base 16 is assumed. Otherwise base 10 is assumed.

Note: The exponent (if present) must contain at least a digit. Otherwise the possible exponent prefix and sign are not part of the number (which ends with the significand). Similarly, if `0b`, `0B`, `0x` or `0X` is not followed by a binary/hexadecimal digit, then the subject sequence stops at the character `0`, thus 0 is read.

Special data (for infinities and NaN) can be `@inf@` or `@nan@(n-char-sequence-opt)`, and if *base* $\leq$ 16, it can also be `infinity`, `inf`, `nan` or `nan(n-char-sequence-opt)`, all case insensitive. A `n-char-sequence-opt` is a possibly empty string containing only digits, Latin letters and the underscore (0, 1, 2, ..., 9, a, b, ..., z, A, B, ..., Z, _). Note: one has an optional sign for all data, even NaN. For example, `-@nAn@(This_Is_Not_17)` is a valid representation for NaN in base 17.

`void mpfr_set_nan (mpfr_t x)` [Function]
`void mpfr_set_inf (mpfr_t x, int sign)` [Function]
`void mpfr_set_zero (mpfr_t x, int sign)` [Function]

Set the variable *x* to NaN (Not-a-Number), infinity or zero respectively. In `mpfr_set_inf` or `mpfr_set_zero`, *x* is set to plus infinity or plus zero iff *sign* is nonnegative; in `mpfr_set_nan`, the sign bit of the result is unspecified.

`void mpfr_swap (mpfr_t x, mpfr_t y)` [Function]

Swap the structures pointed to by *x* and *y*. In particular, the values are exchanged without rounding (this may be different from three `mpfr_set` calls using a third auxiliary variable).

Warning! Since the precisions are exchanged, this will affect future assignments. Moreover, since the significand pointers are also exchanged, you must not use this function if the allocation method used for *x* and/or *y* does not permit it. This is the case when *x* and/or *y* were declared and initialized with `MPFR_DECL_INIT`, and possibly with `mpfr_custom_init_set` (see Section 5.15 [Custom Interface], page 38).

5.3 Combined Initialization and Assignment Functions

`int mpfr_init_set` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Macro]
`int mpfr_init_set_ui` (*mpfr_t* **rop**, *unsigned long int* **op**, *mpfr_rnd_t* **rnd**) [Macro]
`int mpfr_init_set_si` (*mpfr_t* **rop**, *long int* **op**, *mpfr_rnd_t* **rnd**) [Macro]
`int mpfr_init_set_d` (*mpfr_t* **rop**, *double* **op**, *mpfr_rnd_t* **rnd**) [Macro]
`int mpfr_init_set_ld` (*mpfr_t* **rop**, *long double* **op**, *mpfr_rnd_t* **rnd**) [Macro]
`int mpfr_init_set_z` (*mpfr_t* **rop**, *mpz_t* **op**, *mpfr_rnd_t* **rnd**) [Macro]
`int mpfr_init_set_q` (*mpfr_t* **rop**, *mpq_t* **op**, *mpfr_rnd_t* **rnd**) [Macro]
`int mpfr_init_set_f` (*mpfr_t* **rop**, *mpf_t* **op**, *mpfr_rnd_t* **rnd**) [Macro]

Initialize *rop* and set its value from *op*, rounded in the direction *rnd*. The precision of *rop* will be taken from the active default precision, as set by `mpfr_set_default_prec`.

`int mpfr_init_set_str` (*mpfr_t* **x**, *const char* ***s**, *int* **base**, *mpfr_rnd_t* **rnd**) [Function]

Initialize *x* and set its value from the string *s* in base *base*, rounded in the direction *rnd*. See `mpfr_set_str`.

5.4 Conversion Functions

`float mpfr_get_flt` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`double mpfr_get_d` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`long double mpfr_get_ld` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`_Decimal64 mpfr_get_decimal64` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Convert *op* to a `float` (respectively `double`, `long double` or `_Decimal64`), using the rounding mode *rnd*. If *op* is NaN, some fixed NaN (either quiet or signaling) or the result of 0.0/0.0 is returned. If *op* is ±Inf, an infinity of the same sign or the result of ±1.0/0.0 is returned. If *op* is zero, these functions return a zero, trying to preserve its sign, if possible. The `mpfr_get_decimal64` function is built only under some conditions: see the documentation of `mpfr_set_decimal64`.

`long mpfr_get_si` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`unsigned long mpfr_get_ui` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`intmax_t mpfr_get_sj` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`uintmax_t mpfr_get_uj` (*mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Convert *op* to a `long`, an `unsigned long`, an `intmax_t` or an `uintmax_t` (respectively) after rounding it with respect to *rnd*. If *op* is NaN, 0 is returned and the *erange* flag is set. If *op* is too big for the return type, the function returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow; the *erange* flag is set too. See also `mpfr_fits_slong_p`, `mpfr_fits_ulong_p`, `mpfr_fits_intmax_p` and `mpfr_fits_uintmax_p`.

`double mpfr_get_d_2exp` (*long* ***exp**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`long double mpfr_get_ld_2exp` (*long* ***exp**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Return d and set *exp* (formally, the value pointed to by *exp*) such that $0.5 \leq |d| < 1$ and $d \times 2^{exp}$ equals *op* rounded to double (resp. long double) precision, using the given rounding mode. If *op* is zero, then a zero of the same sign (or an unsigned zero, if the implementation does not have signed zeros) is returned, and *exp* is set to 0. If *op* is NaN or an infinity, then the corresponding double precision (resp. long-double precision) value is returned, and *exp* is undefined.

`int mpfr_frexp` (*mpfr_exp_t* ***exp**, *mpfr_t* **y**, *mpfr_t* **x**, *mpfr_rnd_t* **rnd**) [Function]

Set *exp* (formally, the value pointed to by *exp*) and y such that $0.5 \leq |y| < 1$ and $y \times 2^{exp}$ equals x rounded to the precision of y, using the given rounding mode. If x is zero, then y

is set to a zero of the same sign and *exp* is set to 0. If x is NaN or an infinity, then y is set to the same value and *exp* is undefined.

`mpfr_exp_t mpfr_get_z_2exp` (*mpz_t* **rop**, *mpfr_t* **op**) [Function]

Put the scaled significand of *op* (regarded as an integer, with the precision of *op*) into *rop*, and return the exponent *exp* (which may be outside the current exponent range) such that *op* exactly equals $rop \times 2^{\exp}$. If *op* is zero, the minimal exponent `emin` is returned. If *op* is NaN or an infinity, the *erange* flag is set, *rop* is set to 0, and the the minimal exponent `emin` is returned. The returned exponent may be less than the minimal exponent `emin` of MPFR numbers in the current exponent range; in case the exponent is not representable in the `mpfr_exp_t` type, the *erange* flag is set and the minimal value of the `mpfr_exp_t` type is returned.

`int mpfr_get_z` (*mpz_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Convert *op* to a `mpz_t`, after rounding it with respect to *rnd*. If *op* is NaN or an infinity, the *erange* flag is set, *rop* is set to 0, and 0 is returned.

`int mpfr_get_f` (*mpf_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Convert *op* to a `mpf_t`, after rounding it with respect to *rnd*. The *erange* flag is set if *op* is NaN or an infinity, which do not exist in MPF. If *op* is NaN, then *rop* is undefined. If *op* is +Inf (resp. −Inf), then *rop* is set to the maximum (resp. minimum) value in the precision of the MPF number; if a future MPF version supports infinities, this behavior will be considered incorrect and will change (portable programs should assume that *rop* is set either to this finite number or to an infinite number). Note that since MPFR currently has the same exponent type as MPF (but not with the same radix), the range of values is much larger in MPF than in MPFR, so that an overflow or underflow is not possible.

`char * mpfr_get_str` (*char* ***str**, *mpfr_exp_t* ***expptr**, *int* **b**, *size_t* **n**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Convert *op* to a string of digits in base b, with rounding in the direction *rnd*, where n is either zero (see below) or the number of significant digits output in the string; in the latter case, n must be greater or equal to 2. The base may vary from 2 to 62; otherwise the function does nothing and immediately returns a null pointer. If the input number is an ordinary number, the exponent is written through the pointer *expptr* (for input 0, the current minimal exponent is written); the type `mpfr_exp_t` is large enough to hold the exponent in all cases.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number −3.1416 would be returned as `"−31416"` in the string and 1 written at *expptr*. If *rnd* is to nearest, and *op* is exactly in the middle of two consecutive possible outputs, the one with an even significand is chosen, where both significands are considered with the exponent of *op*. Note that for an odd base, this may not correspond to an even last digit: for example with 2 digits in base 7, (14) and a half is rounded to (15) which is 12 in decimal, (16) and a half is rounded to (20) which is 14 in decimal, and (26) and a half is rounded to (26) which is 20 in decimal.

If n is zero, the number of digits of the significand is chosen large enough so that re-reading the printed value with the same precision, assuming both output and input use rounding to nearest, will recover the original value of *op*. More precisely, in most cases, the chosen precision of *str* is the minimal precision m depending only on $p = \mathrm{PREC}(op)$ and b that satisfies the above property, i.e., $m = 1 + \lceil p\frac{\log 2}{\log b}\rceil$, with p replaced by $p-1$ if b is a power of 2, but in some very rare cases, it might be $m + 1$ (the smallest case for bases up to 62 is when p equals 186564318007 for bases 7 and 49).

If *str* is a null pointer, space for the significand is allocated using the allocation function (see Section 4.7 [Memory Handling], page 10) and a pointer to the string is returned (unless the base is invalid). To free the returned string, you must use `mpfr_free_str`.

If *str* is not a null pointer, it should point to a block of storage large enough for the significand, i.e., at least `max(n + 2, 7)`. The extra two bytes are for a possible minus sign, and for the terminating null character, and the value 7 accounts for `-@Inf@` plus the terminating null character. The pointer to the string *str* is returned (unless the base is invalid).

Note: The NaN and inexact flags are currently not set when need be; this will be fixed in future versions. Programmers should currently assume that whether the flags are set by this function is unspecified.

`void mpfr_free_str` (*char *str*) [Function]

Free a string allocated by `mpfr_get_str` using the unallocation function (see Section 4.7 [Memory Handling], page 10). The block is assumed to be `strlen(str)+1` bytes.

`int mpfr_fits_ulong_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_fits_slong_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_fits_uint_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_fits_sint_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_fits_ushort_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_fits_sshort_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_fits_uintmax_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_fits_intmax_p` (*mpfr_t op, mpfr_rnd_t rnd*) [Function]

Return non-zero if *op* would fit in the respective C data type, respectively `unsigned long`, `long`, `unsigned int`, `int`, `unsigned short`, `short`, `uintmax_t`, `intmax_t`, when rounded to an integer in the direction *rnd*.

5.5 Basic Arithmetic Functions

`int mpfr_add` (*mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_add_ui` (*mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_add_si` (*mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_add_d` (*mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_add_z` (*mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_add_q` (*mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd*) [Function]

Set *rop* to *op1* + *op2* rounded in the direction *rnd*. The IEEE-754 rules are used, in particular for signed zeros. But for types having no signed zeros, 0 is considered unsigned (i.e., (+0) + 0 = (+0) and (−0) + 0 = (−0)). The `mpfr_add_d` function assumes that the radix of the `double` type is a power of 2, with a precision at most that declared by the C implementation (macro `IEEE_DBL_MANT_DIG`, and if not defined 53 bits).

`int mpfr_sub` (*mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_ui_sub` (*mpfr_t rop, unsigned long int op1, mpfr_t op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_sub_ui` (*mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_si_sub` (*mpfr_t rop, long int op1, mpfr_t op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_sub_si` (*mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_d_sub` (*mpfr_t rop, double op1, mpfr_t op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_sub_d` (*mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd*) [Function]
`int mpfr_z_sub` (*mpfr_t rop, mpz_t op1, mpfr_t op2, mpfr_rnd_t rnd*) [Function]

`int mpfr_sub_z` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpz_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_sub_q` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpq_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to *op1* − *op2* rounded in the direction *rnd*. The IEEE-754 rules are used, in particular for signed zeros. But for types having no signed zeros, 0 is considered unsigned (i.e., $(+0) - 0 = (+0)$, $(-0) - 0 = (-0)$, $0 - (+0) = (-0)$ and $0 - (-0) = (+0)$). The same restrictions than for `mpfr_add_d` apply to `mpfr_d_sub` and `mpfr_sub_d`.

`int mpfr_mul` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_mul_ui` (*mpfr_t* **rop**, *mpfr_t* **op1**, *unsigned long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_mul_si` (*mpfr_t* **rop**, *mpfr_t* **op1**, *long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_mul_d` (*mpfr_t* **rop**, *mpfr_t* **op1**, *double* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_mul_z` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpz_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_mul_q` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpq_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to *op1* × *op2* rounded in the direction *rnd*. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zeros, 0 is considered positive). The same restrictions than for `mpfr_add_d` apply to `mpfr_mul_d`.

`int mpfr_sqr` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to op^2 rounded in the direction *rnd*.

`int mpfr_div` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_ui_div` (*mpfr_t* **rop**, *unsigned long int* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_div_ui` (*mpfr_t* **rop**, *mpfr_t* **op1**, *unsigned long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_si_div` (*mpfr_t* **rop**, *long int* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_div_si` (*mpfr_t* **rop**, *mpfr_t* **op1**, *long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_d_div` (*mpfr_t* **rop**, *double* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_div_d` (*mpfr_t* **rop**, *mpfr_t* **op1**, *double* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_div_z` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpz_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_div_q` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpq_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to *op1*/*op2* rounded in the direction *rnd*. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zeros, 0 is considered positive). The same restrictions than for `mpfr_add_d` apply to `mpfr_d_div` and `mpfr_div_d`.

`int mpfr_sqrt` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_sqrt_ui` (*mpfr_t* **rop**, *unsigned long int* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to $\sqrt{op}$ rounded in the direction *rnd*. Set *rop* to −0 if *op* is −0, to be consistent with the IEEE 754 standard. Set *rop* to NaN if *op* is negative.

`int mpfr_rec_sqrt` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to $1/\sqrt{op}$ rounded in the direction *rnd*. Set *rop* to +Inf if *op* is ±0, +0 if *op* is +Inf, and NaN if *op* is negative. Warning! Therefore the result on −0 is different from the one of the rSqrt function recommended by the IEEE 754-2008 standard (Section 9.2.1), which is −Inf instead of +Inf.

`int mpfr_cbrt` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_root` (*mpfr_t* **rop**, *mpfr_t* **op**, *unsigned long int* **k**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the cubic root (resp. the kth root) of *op* rounded in the direction *rnd*. For $k = 0$, set *rop* to NaN. For k odd (resp. even) and *op* negative (including −Inf), set *rop* to a negative number (resp. NaN). The kth root of −0 is defined to be −0, whatever the parity of k (different from zero).

`int mpfr_pow` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_pow_ui` (*mpfr_t* **rop**, *mpfr_t* **op1**, *unsigned long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_pow_si` (*mpfr_t* **rop**, *mpfr_t* **op1**, *long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_pow_z` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpz_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_ui_pow_ui` (*mpfr_t* **rop**, *unsigned long int* **op1**, *unsigned long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_ui_pow` (*mpfr_t* **rop**, *unsigned long int* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to $op1^{op2}$, rounded in the direction *rnd*. Special values are handled as described in the ISO C99 and IEEE 754-2008 standards for the `pow` function:

- `pow(±0, y)` returns plus or minus infinity for y a negative odd integer.
- `pow(±0, y)` returns plus infinity for y negative and not an odd integer.
- `pow(±0, y)` returns plus or minus zero for y a positive odd integer.
- `pow(±0, y)` returns plus zero for y positive and not an odd integer.
- `pow(-1, ±Inf)` returns 1.
- `pow(+1, y)` returns 1 for any y, even a NaN.
- `pow(x, ±0)` returns 1 for any x, even a NaN.
- `pow(x, y)` returns NaN for finite negative x and finite non-integer y.
- `pow(x, -Inf)` returns plus infinity for $0 < |x| < 1$, and plus zero for $|x| > 1$.
- `pow(x, +Inf)` returns plus zero for $0 < |x| < 1$, and plus infinity for $|x| > 1$.
- `pow(-Inf, y)` returns minus zero for y a negative odd integer.
- `pow(-Inf, y)` returns plus zero for y negative and not an odd integer.
- `pow(-Inf, y)` returns minus infinity for y a positive odd integer.
- `pow(-Inf, y)` returns plus infinity for y positive and not an odd integer.
- `pow(+Inf, y)` returns plus zero for y negative, and plus infinity for y positive.

`int mpfr_neg` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_abs` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to $-op$ and the absolute value of *op* respectively, rounded in the direction *rnd*. Just changes or adjusts the sign if *rop* and *op* are the same variable, otherwise a rounding might occur if the precision of *rop* is less than that of *op*.

`int mpfr_dim` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the positive difference of *op1* and *op2*, i.e., $op1 - op2$ rounded in the direction *rnd* if $op1 > op2$, +0 if $op1 \leq op2$, and NaN if *op1* or *op2* is NaN.

`int mpfr_mul_2ui` (*mpfr_t* **rop**, *mpfr_t* **op1**, *unsigned long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_mul_2si` (*mpfr_t* **rop**, *mpfr_t* **op1**, *long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to $op1 \times 2^{op2}$ rounded in the direction *rnd*. Just increases the exponent by *op2* when *rop* and *op1* are identical.

`int mpfr_div_2ui` (*mpfr_t* **rop**, *mpfr_t* **op1**, *unsigned long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_div_2si` (*mpfr_t* **rop**, *mpfr_t* **op1**, *long int* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to $op1/2^{op2}$ rounded in the direction *rnd*. Just decreases the exponent by *op2* when *rop* and *op1* are identical.

5.6 Comparison Functions

`int mpfr_cmp` (*mpfr_t op1, mpfr_t op2*) [Function]
`int mpfr_cmp_ui` (*mpfr_t op1, unsigned long int op2*) [Function]
`int mpfr_cmp_si` (*mpfr_t op1, long int op2*) [Function]
`int mpfr_cmp_d` (*mpfr_t op1, double op2*) [Function]
`int mpfr_cmp_ld` (*mpfr_t op1, long double op2*) [Function]
`int mpfr_cmp_z` (*mpfr_t op1, mpz_t op2*) [Function]
`int mpfr_cmp_q` (*mpfr_t op1, mpq_t op2*) [Function]
`int mpfr_cmp_f` (*mpfr_t op1, mpf_t op2*) [Function]

Compare *op1* and *op2*. Return a positive value if $op1 > op2$, zero if $op1 = op2$, and a negative value if $op1 < op2$. Both *op1* and *op2* are considered to their full own precision, which may differ. If one of the operands is NaN, set the *erange* flag and return zero.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., `mpfr_equal_p` for the equality) described below; they behave like the IEEE 754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).

`int mpfr_cmp_ui_2exp` (*mpfr_t op1, unsigned long int op2, mpfr_exp_t e*) [Function]
`int mpfr_cmp_si_2exp` (*mpfr_t op1, long int op2, mpfr_exp_t e*) [Function]

Compare *op1* and $op2 \times 2^e$. Similar as above.

`int mpfr_cmpabs` (*mpfr_t op1, mpfr_t op2*) [Function]

Compare $|op1|$ and $|op2|$. Return a positive value if $|op1| > |op2|$, zero if $|op1| = |op2|$, and a negative value if $|op1| < |op2|$. If one of the operands is NaN, set the *erange* flag and return zero.

`int mpfr_nan_p` (*mpfr_t op*) [Function]
`int mpfr_inf_p` (*mpfr_t op*) [Function]
`int mpfr_number_p` (*mpfr_t op*) [Function]
`int mpfr_zero_p` (*mpfr_t op*) [Function]
`int mpfr_regular_p` (*mpfr_t op*) [Function]

Return non-zero if *op* is respectively NaN, an infinity, an ordinary number (i.e., neither NaN nor an infinity), zero, or a regular number (i.e., neither NaN, nor an infinity nor zero). Return zero otherwise.

`int mpfr_sgn` (*mpfr_t op*) [Macro]

Return a positive value if $op > 0$, zero if $op = 0$, and a negative value if $op < 0$. If the operand is NaN, set the *erange* flag and return zero. This is equivalent to `mpfr_cmp_ui (op, 0)`, but more efficient.

`int mpfr_greater_p` (*mpfr_t op1, mpfr_t op2*) [Function]
`int mpfr_greaterequal_p` (*mpfr_t op1, mpfr_t op2*) [Function]
`int mpfr_less_p` (*mpfr_t op1, mpfr_t op2*) [Function]
`int mpfr_lessequal_p` (*mpfr_t op1, mpfr_t op2*) [Function]
`int mpfr_equal_p` (*mpfr_t op1, mpfr_t op2*) [Function]

Return non-zero if $op1 > op2$, $op1 \geq op2$, $op1 < op2$, $op1 \leq op2$, $op1 = op2$ respectively, and zero otherwise. Those functions return zero whenever *op1* and/or *op2* is NaN.

`int mpfr_lessgreater_p` (*mpfr_t op1, mpfr_t op2*) [Function]
Return non-zero if *op1* < *op2* or *op1* > *op2* (i.e., neither *op1*, nor *op2* is NaN, and *op1* ≠ *op2*), zero otherwise (i.e., *op1* and/or *op2* is NaN, or *op1* = *op2*).

`int mpfr_unordered_p` (*mpfr_t op1, mpfr_t op2*) [Function]
Return non-zero if *op1* or *op2* is a NaN (i.e., they cannot be compared), zero otherwise.

5.7 Special Functions

All those functions, except explicitly stated (for example `mpfr_sin_cos`), return a [ternary value], page 8, i.e., zero for an exact return value, a positive value for a return value larger than the exact result, and a negative value otherwise.

Important note: in some domains, computing special functions (either with correct or incorrect rounding) is expensive, even for small precision, for example the trigonometric and Bessel functions for large argument.

`int mpfr_log` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_log2` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_log10` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
Set *rop* to the natural logarithm of *op*, $\log_2 op$ or $\log_{10} op$, respectively, rounded in the direction *rnd*. Set *rop* to +0 if *op* is 1 (in all rounding modes), for consistency with the ISO C99 and IEEE 754-2008 standards. Set *rop* to −Inf if *op* is ±0 (i.e., the sign of the zero has no influence on the result).

`int mpfr_exp` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_exp2` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_exp10` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
Set *rop* to the exponential of *op*, to 2^{op} or to 10^{op}, respectively, rounded in the direction *rnd*.

`int mpfr_cos` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_sin` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_tan` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
Set *rop* to the cosine of *op*, sine of *op*, tangent of *op*, rounded in the direction *rnd*.

`int mpfr_sin_cos` (*mpfr_t sop, mpfr_t cop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
Set simultaneously *sop* to the sine of *op* and *cop* to the cosine of *op*, rounded in the direction *rnd* with the corresponding precisions of *sop* and *cop*, which must be different variables. Return 0 iff both results are exact, more precisely it returns $s + 4c$ where $s = 0$ if *sop* is exact, $s = 1$ if *sop* is larger than the sine of *op*, $s = 2$ if *sop* is smaller than the sine of *op*, and similarly for c and the cosine of *op*.

`int mpfr_sec` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_csc` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_cot` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
Set *rop* to the secant of *op*, cosecant of *op*, cotangent of *op*, rounded in the direction *rnd*.

`int mpfr_acos` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_asin` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
`int mpfr_atan` (*mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd*) [Function]
Set *rop* to the arc-cosine, arc-sine or arc-tangent of *op*, rounded in the direction *rnd*. Note that since `acos(-1)` returns the floating-point number closest to π according to the given rounding mode, this number might not be in the output range $0 \le rop < \pi$ of the arc-cosine

function; still, the result lies in the image of the output range by the rounding function. The same holds for `asin(-1)`, `asin(1)`, `atan(-Inf)`, `atan(+Inf)` or for `atan(op)` with large *op* and small precision of *rop*.

`int mpfr_atan2` (*mpfr_t* **rop**, *mpfr_t* **y**, *mpfr_t* **x**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the arc-tangent2 of *y* and *x*, rounded in the direction *rnd*: if `x > 0`, `atan2(y, x) = atan (y/x)`; if `x < 0`, `atan2(y, x) = sign(y)*(Pi - atan (|y/x|))`, thus a number from $-\pi$ to π. As for `atan`, in case the exact mathematical result is $+\pi$ or $-\pi$, its rounded result might be outside the function output range.

`atan2(y, 0)` does not raise any floating-point exception. Special values are handled as described in the ISO C99 and IEEE 754-2008 standards for the `atan2` function:

- `atan2(+0, -0)` returns $+\pi$.
- `atan2(-0, -0)` returns $-\pi$.
- `atan2(+0, +0)` returns +0.
- `atan2(-0, +0)` returns -0.
- `atan2(+0, x)` returns $+\pi$ for $x < 0$.
- `atan2(-0, x)` returns $-\pi$ for $x < 0$.
- `atan2(+0, x)` returns +0 for $x > 0$.
- `atan2(-0, x)` returns -0 for $x > 0$.
- `atan2(y, 0)` returns $-\pi/2$ for $y < 0$.
- `atan2(y, 0)` returns $+\pi/2$ for $y > 0$.
- `atan2(+Inf, -Inf)` returns $+3\pi/4$.
- `atan2(-Inf, -Inf)` returns $-3\pi/4$.
- `atan2(+Inf, +Inf)` returns $+\pi/4$.
- `atan2(-Inf, +Inf)` returns $-\pi/4$.
- `atan2(+Inf, x)` returns $+\pi/2$ for finite x.
- `atan2(-Inf, x)` returns $-\pi/2$ for finite x.
- `atan2(y, -Inf)` returns $+\pi$ for finite $y > 0$.
- `atan2(y, -Inf)` returns $-\pi$ for finite $y < 0$.
- `atan2(y, +Inf)` returns +0 for finite $y > 0$.
- `atan2(y, +Inf)` returns -0 for finite $y < 0$.

`int mpfr_cosh` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_sinh` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_tanh` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the hyperbolic cosine, sine or tangent of *op*, rounded in the direction *rnd*.

`int mpfr_sinh_cosh` (*mpfr_t* **sop**, *mpfr_t* **cop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set simultaneously *sop* to the hyperbolic sine of *op* and *cop* to the hyperbolic cosine of *op*, rounded in the direction *rnd* with the corresponding precision of *sop* and *cop*, which must be different variables. Return 0 iff both results are exact (see `mpfr_sin_cos` for a more detailed description of the return value).

`int mpfr_sech` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_csch` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_coth` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the hyperbolic secant of *op*, cosecant of *op*, cotangent of *op*, rounded in the direction *rnd*.

`int mpfr_acosh` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_asinh` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_atanh` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the inverse hyperbolic cosine, sine or tangent of *op*, rounded in the direction *rnd*.

`int mpfr_fac_ui` (*mpfr_t* **rop**, *unsigned long int* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the factorial of *op*, rounded in the direction *rnd*.

`int mpfr_log1p` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the logarithm of one plus *op*, rounded in the direction *rnd*.

`int mpfr_expm1` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to $e^{op} - 1$, rounded in the direction *rnd*.

`int mpfr_eint` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the exponential integral of *op*, rounded in the direction *rnd*. For positive *op*, the exponential integral is the sum of Euler's constant, of the logarithm of *op*, and of the sum for k from 1 to infinity of $op^k/(k \cdot k!)$. For negative *op*, *rop* is set to NaN (this definition for negative argument follows formula 5.1.2 from the Handbook of Mathematical Functions from Abramowitz and Stegun, a future version might use another definition).

`int mpfr_li2` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to real part of the dilogarithm of *op*, rounded in the direction *rnd*. MPFR defines the dilogarithm function as $-\int_{t=0}^{op} \log(1-t)/t\ dt$.

`int mpfr_gamma` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the value of the Gamma function on *op*, rounded in the direction *rnd*. When *op* is a negative integer, *rop* is set to NaN.

`int mpfr_lngamma` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the value of the logarithm of the Gamma function on *op*, rounded in the direction *rnd*. When *op* is 1 or 2, set *rop* to +0 (in all rounding modes). When *op* is an infinity or a nonpositive integer, set *rop* to +Inf, following the general rules on special values. When $-2k-1 < op < -2k$, k being a nonnegative integer, set *rop* to NaN. See also `mpfr_lgamma`.

`int mpfr_lgamma` (*mpfr_t* **rop**, *int* ***signp**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the value of the logarithm of the absolute value of the Gamma function on *op*, rounded in the direction *rnd*. The sign (1 or -1) of Gamma(*op*) is returned in the object pointed to by *signp*. When *op* is 1 or 2, set *rop* to +0 (in all rounding modes). When *op* is an infinity or a nonpositive integer, set *rop* to +Inf. When *op* is NaN, −Inf or a negative integer, **signp* is undefined, and when *op* is ± 0, **signp* is the sign of the zero.

`int mpfr_digamma` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the value of the Digamma (sometimes also called Psi) function on *op*, rounded in the direction *rnd*. When *op* is a negative integer, set *rop* to NaN.

`int mpfr_zeta` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_zeta_ui` (*mpfr_t* **rop**, *unsigned long* **op**, *mpfr_rnd_t* **rnd**) [Function]
Set *rop* to the value of the Riemann Zeta function on *op*, rounded in the direction *rnd*.

`int mpfr_erf` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_erfc` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the value of the error function on *op* (resp. the complementary error function on *op*) rounded in the direction *rnd*.

`int mpfr_j0` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_j1` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_jn` (*mpfr_t* **rop**, *long* **n**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the value of the first kind Bessel function of order 0, (resp. 1 and *n*) on *op*, rounded in the direction *rnd*. When *op* is NaN, *rop* is always set to NaN. When *op* is plus or minus Infinity, *rop* is set to +0. When *op* is zero, and *n* is not zero, *rop* is set to +0 or −0 depending on the parity and sign of *n*, and the sign of *op*.

`int mpfr_y0` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_y1` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_yn` (*mpfr_t* **rop**, *long* **n**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the value of the second kind Bessel function of order 0 (resp. 1 and *n*) on *op*, rounded in the direction *rnd*. When *op* is NaN or negative, *rop* is always set to NaN. When *op* is +Inf, *rop* is set to +0. When *op* is zero, *rop* is set to +Inf or −Inf depending on the parity and sign of *n*.

`int mpfr_fma` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_t* **op3**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_fms` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_t* **op3**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to (*op1*×*op2*)+*op3* (resp. (*op1*×*op2*)−*op3*) rounded in the direction *rnd*. Concerning special values (signed zeros, infinities, NaN), these functions behave like a multiplication followed by a separate addition or subtraction. That is, the fused operation matters only for rounding.

`int mpfr_agm` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the arithmetic-geometric mean of *op1* and *op2*, rounded in the direction *rnd*. The arithmetic-geometric mean is the common limit of the sequences u_n and v_n, where u_0=*op1*, v_0=*op2*, u_{n+1} is the arithmetic mean of u_n and v_n, and v_{n+1} is the geometric mean of u_n and v_n. If any operand is negative, set *rop* to NaN.

`int mpfr_hypot` (*mpfr_t* **rop**, *mpfr_t* **x**, *mpfr_t* **y**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the Euclidean norm of *x* and *y*, i.e., $\sqrt{x^2+y^2}$, rounded in the direction *rnd*. Special values are handled as described in the ISO C99 (Section F.9.4.3) and IEEE 754-2008 (Section 9.2.1) standards: If *x* or *y* is an infinity, then +Inf is returned in *rop*, even if the other number is NaN.

`int mpfr_ai` (*mpfr_t* **rop**, *mpfr_t* **x**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the value of the Airy function Ai on *x*, rounded in the direction *rnd*. When *x* is NaN, *rop* is always set to NaN. When *x* is +Inf or −Inf, *rop* is +0. The current implementation is not intended to be used with large arguments. It works with —*x*— typically smaller than 500. For larger arguments, other methods should be used and will be implemented in a future version.

`int mpfr_const_log2` (*mpfr_t* **rop**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_const_pi` (*mpfr_t* **rop**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_const_euler` (*mpfr_t* **rop**, *mpfr_rnd_t* **rnd**) [Function]

`int mpfr_const_catalan` (*mpfr_t* **rop**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the logarithm of 2, the value of π, of Euler's constant 0.577..., of Catalan's constant 0.915..., respectively, rounded in the direction *rnd*. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use `mpfr_free_cache`.

`void mpfr_free_cache` (*void*) [Function]

Free various caches used by MPFR internally, in particular the caches used by the functions computing constants (`mpfr_const_log2`, `mpfr_const_pi`, `mpfr_const_euler` and `mpfr_const_catalan`). You should call this function before terminating a thread, even if you did not call these functions directly (they could have been called internally).

`int mpfr_sum` (*mpfr_t* **rop**, *mpfr_ptr const* **tab**[], *unsigned long int* **n**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the sum of all elements of *tab*, whose size is *n*, rounded in the direction *rnd*. Warning: for efficiency reasons, *tab* is an array of pointers to `mpfr_t`, not an array of `mpfr_t`. If the returned `int` value is zero, *rop* is guaranteed to be the exact sum; otherwise *rop* might be smaller than, equal to, or larger than the exact sum (in accordance to the rounding mode). However, `mpfr_sum` does guarantee the result is correctly rounded.

5.8 Input and Output Functions

This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a `stream` to any of these functions will make them read from `stdin` and write to `stdout`, respectively.

When using any of these functions, you must include the `<stdio.h>` standard header before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions.

`size_t mpfr_out_str` (*FILE *stream*, *int* **base**, *size_t* **n**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Output *op* on stream *stream*, as a string of digits in base *base*, rounded in the direction *rnd*. The base may vary from 2 to 62. Print *n* significant digits exactly, or if *n* is 0, enough digits so that *op* can be read back exactly (see `mpfr_get_str`).

In addition to the significant digits, a decimal point (defined by the current locale) at the right of the first digit and a trailing exponent in base 10, in the form '`eNNN`', are printed. If *base* is greater than 10, '`@`' will be used instead of '`e`' as exponent delimiter.

Return the number of characters written, or if an error occurred, return 0.

`size_t mpfr_inp_str` (*mpfr_t* **rop**, *FILE *stream*, *int* **base**, *mpfr_rnd_t* **rnd**) [Function]

Input a string in base *base* from stream *stream*, rounded in the direction *rnd*, and put the read float in *rop*.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using `mpfr_set_str`. See the documentation of `mpfr_strtofr` for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

5.9 Formatted Output Functions

5.9.1 Requirements

The class of `mpfr_printf` functions provides formatted output in a similar manner as the standard C `printf`. These functions are defined only if your system supports ISO C variadic functions and the corresponding argument access macros.

When using any of these functions, you must include the `<stdio.h>` standard header before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions.

5.9.2 Format String

The format specification accepted by `mpfr_printf` is an extension of the `printf` one. The conversion specification is of the form:

```
% [flags] [width] [.[precision]] [type] [rounding] conv
```

'`flags`', '`width`', and '`precision`' have the same meaning as for the standard `printf` (in particular, notice that the '`precision`' is related to the number of digits displayed in the base chosen by '`conv`' and not related to the internal precision of the `mpfr_t` variable). `mpfr_printf` accepts the same '`type`' specifiers as GMP (except the non-standard and deprecated '`q`', use '`ll`' instead), namely the length modifiers defined in the C standard:

'h'	`short`
'hh'	`char`
'j'	`intmax_t` or `uintmax_t`
'l'	`long` or `wchar_t`
'll'	`long long`
'L'	`long double`
't'	`ptrdiff_t`
'z'	`size_t`

and the '`type`' specifiers defined in GMP plus '`R`' and '`P`' specific to MPFR (the second column in the table below shows the type of the argument read in the argument list and the kind of '`conv`' specifier to use after the '`type`' specifier):

'F'	`mpf_t`, float conversions
'Q'	`mpq_t`, integer conversions
'M'	`mp_limb_t`, integer conversions
'N'	`mp_limb_t` array, integer conversions
'Z'	`mpz_t`, integer conversions
'P'	`mpfr_prec_t`, integer conversions
'R'	`mpfr_t`, float conversions

The '`type`' specifiers have the same restrictions as those mentioned in the GMP documentation: see Section "Formatted Output Strings" in *GNU MP*. In particular, the '`type`' specifiers (except '`R`' and '`P`') are supported only if they are supported by `gmp_printf` in your GMP build; this implies that the standard specifiers, such as '`t`', must *also* be supported by your C library if you want to use them.

The '`rounding`' field is specific to `mpfr_t` arguments and should not be used with other types.

With conversion specification not involving '`P`' and '`R`' types, `mpfr_printf` behaves exactly as `gmp_printf`.

The '`P`' type specifies that a following '`d`', '`i`', '`o`', '`u`', '`x`', or '`X`' conversion specifier applies to a `mpfr_prec_t` argument. It is needed because the `mpfr_prec_t` type does not necessarily

correspond to an `int` or any fixed standard type. The '`precision`' field specifies the minimum number of digits to appear. The default '`precision`' is 1. For example:

```
mpfr_t x;
mpfr_prec_t p;
mpfr_init (x);
...
p = mpfr_get_prec (x);
mpfr_printf ("variable x with %Pu bits", p);
```

The '`R`' type specifies that a following '`a`', '`A`', '`b`', '`e`', '`E`', '`f`', '`F`', '`g`', '`G`', or '`n`' conversion specifier applies to a `mpfr_t` argument. The '`R`' type can be followed by a '`rounding`' specifier denoted by one of the following characters:

'`U`'	round toward plus infinity
'`D`'	round toward minus infinity
'`Y`'	round away from zero
'`Z`'	round toward zero
'`N`'	round to nearest (with ties to even)
'`*`'	rounding mode indicated by the `mpfr_rnd_t` argument just before the corresponding `mpfr_t` variable.

The default rounding mode is rounding to nearest. The following three examples are equivalent:

```
mpfr_t x;
mpfr_init (x);
...
mpfr_printf ("%.128Rf", x);
mpfr_printf ("%.128RNf", x);
mpfr_printf ("%.128R*f", MPFR_RNDN, x);
```

Note that the rounding away from zero mode is specified with '`Y`' because ISO C reserves the '`A`' specifier for hexadecimal output (see below).

The output '`conv`' specifiers allowed with `mpfr_t` parameter are:

'`a`' '`A`'	hex float, C99 style
'`b`'	binary output
'`e`' '`E`'	scientific format float
'`f`' '`F`'	fixed point float
'`g`' '`G`'	fixed or scientific float

The conversion specifier '`b`' which displays the argument in binary is specific to `mpfr_t` arguments and should not be used with other types. Other conversion specifiers have the same meaning as for a `double` argument.

In case of non-decimal output, only the significand is written in the specified base, the exponent is always displayed in decimal. Special values are always displayed as `nan`, `-inf`, and `inf` for '`a`', '`b`', '`e`', '`f`', and '`g`' specifiers and `NAN`, `-INF`, and `INF` for '`A`', '`E`', '`F`', and '`G`' specifiers.

If the '`precision`' field is not empty, the `mpfr_t` number is rounded to the given precision in the direction specified by the rounding mode. If the precision is zero with rounding to nearest mode and one of the following '`conv`' specifiers: '`a`', '`A`', '`b`', '`e`', '`E`', tie case is rounded to even when it lies between two consecutive values at the wanted precision which have the same exponent, otherwise, it is rounded away from zero. For instance, 85 is displayed as `"8e+1"` and 95 is displayed as `"1e+2"` with the format specification `"%.0RNe"`. This also applies when the

'g' (resp. 'G') conversion specifier uses the 'e' (resp. 'E') style. If the precision is set to a value greater than the maximum value for an `int`, it will be silently reduced down to `INT_MAX`.

If the '`precision`' field is empty (as in `%Re` or `%.RE`) with '`conv`' specifier 'e' and 'E', the number is displayed with enough digits so that it can be read back exactly, assuming that the input and output variables have the same precision and that the input and output rounding modes are both rounding to nearest (as for `mpfr_get_str`). The default precision for an empty '`precision`' field with '`conv`' specifiers 'f', 'F', 'g', and 'G' is 6.

5.9.3 Functions

For all the following functions, if the number of characters that ought to be written exceeds the maximum limit `INT_MAX` for an `int`, nothing is written in the stream (resp. to `stdout`, to *buf*, to *str*), the function returns −1, sets the *erange* flag, and `errno` is set to `EOVERFLOW` if the `EOVERFLOW` macro is defined (such as on POSIX systems). Note, however, that `errno` might be changed to another value by some internal library call if another error occurs there (currently, this would come from the unallocation function).

`int mpfr_fprintf` (*FILE *stream*, *const char *template*, ...) [Function]
`int mpfr_vfprintf` (*FILE *stream*, *const char *template*, *va_list ap*) [Function]

Print to the stream *stream* the optional arguments under the control of the template string *template*. Return the number of characters written or a negative value if an error occurred.

`int mpfr_printf` (*const char *template*, ...) [Function]
`int mpfr_vprintf` (*const char *template*, *va_list ap*) [Function]

Print to `stdout` the optional arguments under the control of the template string *template*. Return the number of characters written or a negative value if an error occurred.

`int mpfr_sprintf` (*char *buf*, *const char *template*, ...) [Function]
`int mpfr_vsprintf` (*char *buf*, *const char *template*, *va_list ap*) [Function]

Form a null-terminated string corresponding to the optional arguments under the control of the template string *template*, and print it in *buf*. No overlap is permitted between *buf* and the other arguments. Return the number of characters written in the array *buf* *not counting* the terminating null character or a negative value if an error occurred.

`int mpfr_snprintf` (*char *buf*, *size_t n*, *const char *template*, ...) [Function]
`int mpfr_vsnprintf` (*char *buf*, *size_t n*, *const char *template*, *va_list ap*) [Function]

Form a null-terminated string corresponding to the optional arguments under the control of the template string *template*, and print it in *buf*. If *n* is zero, nothing is written and *buf* may be a null pointer, otherwise, the $n-1$ first characters are written in *buf* and the *n*-th is a null character. Return the number of characters that would have been written had *n* been sufficiently large, *not counting* the terminating null character, or a negative value if an error occurred.

`int mpfr_asprintf` (*char **str*, *const char *template*, ...) [Function]
`int mpfr_vasprintf` (*char **str*, *const char *template*, *va_list ap*) [Function]

Write their output as a null terminated string in a block of memory allocated using the allocation function (see Section 4.7 [Memory Handling], page 10). A pointer to the block is stored in *str*. The block of memory must be freed using `mpfr_free_str`. The return value is the number of characters written in the string, excluding the null-terminator, or a negative value if an error occurred, in which case the contents of *str* are undefined.

5.10 Integer and Remainder Related Functions

`int mpfr_rint` (*mpfr_t rop*, *mpfr_t op*, *mpfr_rnd_t rnd*) [Function]

`int mpfr_ceil` (*mpfr_t* **rop**, *mpfr_t* **op**) [Function]
`int mpfr_floor` (*mpfr_t* **rop**, *mpfr_t* **op**) [Function]
`int mpfr_round` (*mpfr_t* **rop**, *mpfr_t* **op**) [Function]
`int mpfr_trunc` (*mpfr_t* **rop**, *mpfr_t* **op**) [Function]

Set *rop* to *op* rounded to an integer. `mpfr_rint` rounds to the nearest representable integer in the given direction *rnd*, `mpfr_ceil` rounds to the next higher or equal representable integer, `mpfr_floor` to the next lower or equal representable integer, `mpfr_round` to the nearest representable integer, rounding halfway cases away from zero (as in the roundTiesToAway mode of IEEE 754-2008), and `mpfr_trunc` to the next representable integer toward zero.

The returned value is zero when the result is exact, positive when it is greater than the original value of *op*, and negative when it is smaller. More precisely, the returned value is 0 when *op* is an integer representable in *rop*, 1 or -1 when *op* is an integer that is not representable in *rop*, 2 or -2 when *op* is not an integer.

When *op* is NaN, the NaN flag is set as usual. In the other cases, the inexact flag is set when *rop* differs from *op*, following the ISO C99 rule for the `rint` function. If you want the behavior to be more like IEEE 754 / ISO TS 18661-1, i.e., the usual behavior where the round-to-integer function is regarded as any other mathematical function, you should use one the `mpfr_rint_*` functions instead (however it is not possible to round to nearest with the even rounding rule yet).

Note that `mpfr_round` is different from `mpfr_rint` called with the rounding to nearest mode (where halfway cases are rounded to an even integer or significand). Note also that no double rounding is performed; for instance, 10.5 (1010.1 in binary) is rounded by `mpfr_rint` with rounding to nearest to 12 (1100 in binary) in 2-bit precision, because the two enclosing numbers representable on two bits are 8 and 12, and the closest is 12. (If one first rounded to an integer, one would round 10.5 to 10 with even rounding, and then 10 would be rounded to 8 again with even rounding.)

`int mpfr_rint_ceil` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_rint_floor` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_rint_round` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_rint_trunc` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to *op* rounded to an integer. `mpfr_rint_ceil` rounds to the next higher or equal integer, `mpfr_rint_floor` to the next lower or equal integer, `mpfr_rint_round` to the nearest integer, rounding halfway cases away from zero, and `mpfr_rint_trunc` to the next integer toward zero. If the result is not representable, it is rounded in the direction *rnd*. The returned value is the ternary value associated with the considered round-to-integer function (regarded in the same way as any other mathematical function).

Contrary to `mpfr_rint`, those functions do perform a double rounding: first *op* is rounded to the nearest integer in the direction given by the function name, then this nearest integer (if not representable) is rounded in the given direction *rnd*. Thus these round-to-integer functions behave more like the other mathematical functions, i.e., the returned result is the correct rounding of the exact result of the function in the real numbers.

For example, `mpfr_rint_round` with rounding to nearest and a precision of two bits rounds 6.5 to 7 (halfway cases away from zero), then 7 is rounded to 8 by the round-even rule, despite the fact that 6 is also representable on two bits, and is closer to 6.5 than 8.

`int mpfr_frac` (*mpfr_t* **rop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the fractional part of *op*, having the same sign as *op*, rounded in the direction *rnd* (unlike in `mpfr_rint`, *rnd* affects only how the exact fractional part is rounded, not how the fractional part is generated).

`int mpfr_modf` (*mpfr_t* **iop**, *mpfr_t* **fop**, *mpfr_t* **op**, *mpfr_rnd_t* **rnd**) [Function]

Set simultaneously *iop* to the integral part of *op* and *fop* to the fractional part of *op*, rounded in the direction *rnd* with the corresponding precision of *iop* and *fop* (equivalent to `mpfr_trunc(iop, op, rnd)` and `mpfr_frac(fop, op, rnd)`). The variables *iop* and *fop* must be different. Return 0 iff both results are exact (see `mpfr_sin_cos` for a more detailed description of the return value).

`int mpfr_fmod` (*mpfr_t* **r**, *mpfr_t* **x**, *mpfr_t* **y**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_remainder` (*mpfr_t* **r**, *mpfr_t* **x**, *mpfr_t* **y**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_remquo` (*mpfr_t* **r**, *long** **q**, *mpfr_t* **x**, *mpfr_t* **y**, *mpfr_rnd_t* **rnd**) [Function]

Set r to the value of $x - ny$, rounded according to the direction *rnd*, where n is the integer quotient of x divided by y, defined as follows: n is rounded toward zero for `mpfr_fmod`, and to the nearest integer (ties rounded to even) for `mpfr_remainder` and `mpfr_remquo`.

Special values are handled as described in Section F.9.7.1 of the ISO C99 standard: If x is infinite or y is zero, r is NaN. If y is infinite and x is finite, r is x rounded to the precision of r. If r is zero, it has the sign of x. The return value is the ternary value corresponding to r.

Additionally, `mpfr_remquo` stores the low significant bits from the quotient n in $*q$ (more precisely the number of bits in a `long` minus one), with the sign of x divided by y (except if those low bits are all zero, in which case zero is returned). Note that x may be so large in magnitude relative to y that an exact representation of the quotient is not practical. The `mpfr_remainder` and `mpfr_remquo` functions are useful for additive argument reduction.

`int mpfr_integer_p` (*mpfr_t* **op**) [Function]

Return non-zero iff *op* is an integer.

5.11 Rounding Related Functions

`void mpfr_set_default_rounding_mode` (*mpfr_rnd_t* **rnd**) [Function]

Set the default rounding mode to *rnd*. The default rounding mode is to nearest initially.

`mpfr_rnd_t mpfr_get_default_rounding_mode` (*void*) [Function]

Get the default rounding mode.

`int mpfr_prec_round` (*mpfr_t* **x**, *mpfr_prec_t* **prec**, *mpfr_rnd_t* **rnd**) [Function]

Round x according to *rnd* with precision *prec*, which must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined). If *prec* is greater or equal to the precision of x, then new space is allocated for the significand, and it is filled with zeros. Otherwise, the significand is rounded to precision *prec* with the given direction. In both cases, the precision of x is changed to *prec*.

Here is an example of how to use `mpfr_prec_round` to implement Newton's algorithm to compute the inverse of a, assuming x is already an approximation to n bits:

```
mpfr_set_prec (t, 2 * n);
mpfr_set (t, a, MPFR_RNDN);          /* round a to 2n bits */
mpfr_mul (t, t, x, MPFR_RNDN);       /* t is correct to 2n bits */
mpfr_ui_sub (t, 1, t, MPFR_RNDN);    /* high n bits cancel with 1 */
```

```
mpfr_prec_round (t, n, MPFR_RNDN);  /* t is correct to n bits */
mpfr_mul (t, t, x, MPFR_RNDN);      /* t is correct to n bits */
mpfr_prec_round (x, 2 * n, MPFR_RNDN); /* exact */
mpfr_add (x, x, t, MPFR_RNDN);      /* x is correct to 2n bits */
```

Warning! You must not use this function if x was initialized with `MPFR_DECL_INIT` or with `mpfr_custom_init_set` (see Section 5.15 [Custom Interface], page 38).

`int mpfr_can_round` (*mpfr_t* **b**, *mpfr_exp_t* **err**, *mpfr_rnd_t* **rnd1**, *mpfr_rnd_t* **rnd2**, *mpfr_prec_t* **prec**) [Function]

Assuming b is an approximation of an unknown number x in the direction *rnd1* with error at most two to the power E(b)-*err* where E(b) is the exponent of b, return a non-zero value if one is able to round correctly x to precision *prec* with the direction *rnd2*, and 0 otherwise (including for NaN and Inf). This function **does not modify** its arguments.

If *rnd1* is `MPFR_RNDN`, then the sign of the error is unknown, but its absolute value is the same, so that the possible range is twice as large as with a directed rounding for *rnd1*.

Note: if one wants to also determine the correct [ternary value], page 8 when rounding b to precision *prec* with rounding mode *rnd*, a useful trick is the following:

```
if (mpfr_can_round (b, err, MPFR_RNDN, MPFR_RNDZ,
    prec + (rnd == MPFR_RNDN)))
   ...
```

Indeed, if *rnd* is `MPFR_RNDN`, this will check if one can round to *prec*+1 bits with a directed rounding: if so, one can surely round to nearest to *prec* bits, and in addition one can determine the correct ternary value, which would not be the case when b is near from a value exactly representable on *prec* bits.

`mpfr_prec_t mpfr_min_prec` (*mpfr_t* **x**) [Function]

Return the minimal number of bits required to store the significand of x, and 0 for special values, including 0. (Warning: the returned value can be less than `MPFR_PREC_MIN`.)

The function name is subject to change.

`const char * mpfr_print_rnd_mode` (*mpfr_rnd_t* **rnd**) [Function]

Return a string ("MPFR_RNDD", "MPFR_RNDU", "MPFR_RNDN", "MPFR_RNDZ", "MPFR_RNDA") corresponding to the rounding mode *rnd*, or a null pointer if *rnd* is an invalid rounding mode.

5.12 Miscellaneous Functions

`void mpfr_nexttoward` (*mpfr_t* **x**, *mpfr_t* **y**) [Function]

If x or y is NaN, set x to NaN. If x and y are equal, x is unchanged. Otherwise, if x is different from y, replace x by the next floating-point number (with the precision of x and the current exponent range) in the direction of y (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow or overflow is generated.

`void mpfr_nextabove` (*mpfr_t* **x**) [Function]
`void mpfr_nextbelow` (*mpfr_t* **x**) [Function]

Equivalent to `mpfr_nexttoward` where y is plus infinity (resp. minus infinity).

`int mpfr_min` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]
`int mpfr_max` (*mpfr_t* **rop**, *mpfr_t* **op1**, *mpfr_t* **op2**, *mpfr_rnd_t* **rnd**) [Function]

Set *rop* to the minimum (resp. maximum) of *op1* and *op2*. If *op1* and *op2* are both NaN, then *rop* is set to NaN. If *op1* or *op2* is NaN, then *rop* is set to the numeric value. If *op1* and *op2* are zeros of different signs, then *rop* is set to -0 (resp. $+0$).

`int mpfr_urandomb` (*mpfr_t* **rop**, *gmp_randstate_t* **state**) [Function]

Generate a uniformly distributed random float in the interval $0 \leq rop < 1$. More precisely, the number can be seen as a float with a random non-normalized significand and exponent 0, which is then normalized (thus if e denotes the exponent after normalization, then the least $-e$ significant bits of the significand are always 0).

Return 0, unless the exponent is not in the current exponent range, in which case *rop* is set to NaN and a non-zero value is returned (this should never happen in practice, except in very specific cases). The second argument is a `gmp_randstate_t` structure which should be created using the GMP `gmp_randinit` function (see the GMP manual).

Note: for a given version of MPFR, the returned value of *rop* and the new value of *state* (which controls further random values) do not depend on the machine word size.

`int mpfr_urandom` (*mpfr_t* **rop**, *gmp_randstate_t* **state**, *mpfr_rnd_t* **rnd**) [Function]

Generate a uniformly distributed random float. The floating-point number *rop* can be seen as if a random real number is generated according to the continuous uniform distribution on the interval [0, 1] and then rounded in the direction *rnd*.

The second argument is a `gmp_randstate_t` structure which should be created using the GMP `gmp_randinit` function (see the GMP manual).

Note: the note for `mpfr_urandomb` holds too. In addition, the exponent range and the rounding mode might have a side effect on the next random state.

The rule for the underflow flag is here "underflow before rounding" instead of the usual "underflow after rounding". The reason is that the exponent is drawn first, and if it is smaller than the minimum exponent, the significand is not drawn. To fix the behavior on the underflow flag, one would have to draw the significand in some cases, meaning that the behavior of the random generator would change, thus it would break the ABI for the MPFR 3.1 branch. However, the observed behavior always corresponds to an existing number.

`int mpfr_grandom` (*mpfr_t* **rop1**, *mpfr_t* **rop2**, *gmp_randstate_t* **state**, *mpfr_rnd_t* **rnd**) [Function]

Generate two random floats according to a standard normal gaussian distribution. If *rop2* is a null pointer, then only one value is generated and stored in *rop1*.

The floating-point number *rop1* (and *rop2*) can be seen as if a random real number were generated according to the standard normal gaussian distribution and then rounded in the direction *rnd*.

The third argument is a `gmp_randstate_t` structure, which should be created using the GMP `gmp_randinit` function (see the GMP manual).

The combination of the ternary values is returned like with `mpfr_sin_cos`. If *rop2* is a null pointer, the second ternary value is assumed to be 0 (note that the encoding of the only ternary value is not the same as the usual encoding for functions that return only one result). Otherwise the ternary value of a random number is always non-zero.

Note: the note for `mpfr_urandomb` holds too. In addition, the exponent range and the rounding mode might have a side effect on the next random state.

`mpfr_exp_t mpfr_get_exp (mpfr_t x)` [Function]

Return the exponent of x, assuming that x is a non-zero ordinary number and the significand is considered in [1/2,1). The behavior for NaN, infinity or zero is undefined.

`int mpfr_set_exp (mpfr_t x, mpfr_exp_t e)` [Function]

Set the exponent of x if e is in the current exponent range, and return 0 (even if x is not a non-zero ordinary number); otherwise, return a non-zero value. The significand is assumed to be in [1/2,1).

`int mpfr_signbit (mpfr_t op)` [Function]

Return a non-zero value iff *op* has its sign bit set (i.e., if it is negative, -0, or a NaN whose representation has its sign bit set).

`int mpfr_setsign (mpfr_t rop, mpfr_t op, int s, mpfr_rnd_t rnd)` [Function]

Set the value of *rop* from *op*, rounded toward the given direction *rnd*, then set (resp. clear) its sign bit if s is non-zero (resp. zero), even when *op* is a NaN.

`int mpfr_copysign (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)` [Function]

Set the value of *rop* from *op1*, rounded toward the given direction *rnd*, then set its sign bit to that of *op2* (even when *op1* or *op2* is a NaN). This function is equivalent to `mpfr_setsign (rop, op1, mpfr_signbit (op2), rnd)`.

`const char * mpfr_get_version (void)` [Function]

Return the MPFR version, as a null-terminated string.

`MPFR_VERSION` [Macro]
`MPFR_VERSION_MAJOR` [Macro]
`MPFR_VERSION_MINOR` [Macro]
`MPFR_VERSION_PATCHLEVEL` [Macro]
`MPFR_VERSION_STRING` [Macro]

`MPFR_VERSION` is the version of MPFR as a preprocessing constant. `MPFR_VERSION_MAJOR`, `MPFR_VERSION_MINOR` and `MPFR_VERSION_PATCHLEVEL` are respectively the major, minor and patch level of MPFR version, as preprocessing constants. `MPFR_VERSION_STRING` is the version (with an optional suffix, used in development and pre-release versions) as a string constant, which can be compared to the result of `mpfr_get_version` to check at run time the header file and library used match:

```
if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))
  fprintf (stderr, "Warning: header and library do not match\n");
```

Note: Obtaining different strings is not necessarily an error, as in general, a program compiled with some old MPFR version can be dynamically linked with a newer MPFR library version (if allowed by the library versioning system).

`long MPFR_VERSION_NUM (major, minor, patchlevel)` [Macro]

Create an integer in the same format as used by `MPFR_VERSION` from the given *major*, *minor* and *patchlevel*. Here is an example of how to check the MPFR version at compile time:

```
#if (!defined(MPFR_VERSION) || (MPFR_VERSION<MPFR_VERSION_NUM(3,0,0)))
# error "Wrong MPFR version."
#endif
```

`const char * mpfr_get_patches` (*void*) [Function]

Return a null-terminated string containing the ids of the patches applied to the MPFR library (contents of the `PATCHES` file), separated by spaces. Note: If the program has been compiled with an older MPFR version and is dynamically linked with a new MPFR library version, the identifiers of the patches applied to the old (compile-time) MPFR version are not available (however this information should not have much interest in general).

`int mpfr_buildopt_tls_p` (*void*) [Function]

Return a non-zero value if MPFR was compiled as thread safe using compiler-level Thread Local Storage (that is, MPFR was built with the `--enable-thread-safe` configure option, see `INSTALL` file), return zero otherwise.

`int mpfr_buildopt_decimal_p` (*void*) [Function]

Return a non-zero value if MPFR was compiled with decimal float support (that is, MPFR was built with the `--enable-decimal-float` configure option), return zero otherwise.

`int mpfr_buildopt_gmpinternals_p` (*void*) [Function]

Return a non-zero value if MPFR was compiled with GMP internals (that is, MPFR was built with either `--with-gmp-build` or `--enable-gmp-internals` configure option), return zero otherwise.

`const char * mpfr_buildopt_tune_case` (*void*) [Function]

Return a string saying which thresholds file has been used at compile time. This file is normally selected from the processor type.

5.13 Exception Related Functions

`mpfr_exp_t mpfr_get_emin` (*void*) [Function]
`mpfr_exp_t mpfr_get_emax` (*void*) [Function]

Return the (current) smallest and largest exponents allowed for a floating-point variable. The smallest positive value of a floating-point variable is $1/2 \times 2^{\mathrm{emin}}$ and the largest value has the form $(1 - \varepsilon) \times 2^{\mathrm{emax}}$, where ε depends on the precision of the considered variable.

`int mpfr_set_emin` (*mpfr_exp_t exp*) [Function]
`int mpfr_set_emax` (*mpfr_exp_t exp*) [Function]

Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero value when *exp* is not in the range accepted by the implementation (in that case the smallest or largest exponent is not changed), and zero otherwise. If the user changes the exponent range, it is her/his responsibility to check that all current floating-point variables are in the new allowed range (for example using `mpfr_check_range`), otherwise the subsequent behavior will be undefined, in the sense of the ISO C standard.

`mpfr_exp_t mpfr_get_emin_min` (*void*) [Function]
`mpfr_exp_t mpfr_get_emin_max` (*void*) [Function]
`mpfr_exp_t mpfr_get_emax_min` (*void*) [Function]
`mpfr_exp_t mpfr_get_emax_max` (*void*) [Function]

Return the minimum and maximum of the exponents allowed for `mpfr_set_emin` and `mpfr_set_emax` respectively. These values are implementation dependent, thus a program using `mpfr_set_emax(mpfr_get_emax_max())` or `mpfr_set_emin(mpfr_get_emin_min())` may not be portable.

`int mpfr_check_range` (*mpfr_t* `x`, *int* `t`, *mpfr_rnd_t* `rnd`) [Function]

This function assumes that x is the correctly-rounded value of some real value y in the direction *rnd* and some extended exponent range, and that t is the corresponding [ternary value], page 8. For example, one performed `t = mpfr_log (x, u, rnd)`, and y is the exact logarithm of u. Thus t is negative if x is smaller than y, positive if x is larger than y, and zero if x equals y. This function modifies x if needed to be in the current range of acceptable values: It generates an underflow or an overflow if the exponent of x is outside the current allowed range; the value of t may be used to avoid a double rounding. This function returns zero if the new value of x equals the exact one y, a positive value if that new value is larger than y, and a negative value if it is smaller than y. Note that unlike most functions, the new result x is compared to the (unknown) exact one y, not the input value x, i.e., the ternary value is propagated.

Note: If x is an infinity and t is different from zero (i.e., if the rounded result is an inexact infinity), then the overflow flag is set. This is useful because `mpfr_check_range` is typically called (at least in MPFR functions) after restoring the flags that could have been set due to internal computations.

`int mpfr_subnormalize` (*mpfr_t* `x`, *int* `t`, *mpfr_rnd_t* `rnd`) [Function]

This function rounds x emulating subnormal number arithmetic: if x is outside the subnormal exponent range, it just propagates the [ternary value], page 8 t; otherwise, it rounds x to precision `EXP(x)-emin+1` according to rounding mode *rnd* and previous ternary value t, avoiding double rounding problems. More precisely in the subnormal domain, denoting by e the value of `emin`, x is rounded in fixed-point arithmetic to an integer multiple of 2^{e-1}; as a consequence, $1.5 \times 2^{e-1}$ when t is zero is rounded to 2^e with rounding to nearest.

`PREC(x)` is not modified by this function. *rnd* and t must be the rounding mode and the returned ternary value used when computing x (as in `mpfr_check_range`). The subnormal exponent range is from `emin` to `emin+PREC(x)-1`. If the result cannot be represented in the current exponent range (due to a too small `emax`), the behavior is undefined. Note that unlike most functions, the result is compared to the exact one, not the input value x, i.e., the ternary value is propagated.

As usual, if the returned ternary value is non zero, the inexact flag is set. Moreover, if a second rounding occurred (because the input x was in the subnormal range), the underflow flag is set.

This is an example of how to emulate binary double IEEE 754 arithmetic (binary64 in IEEE 754-2008) using MPFR:

```
{
  mpfr_t xa, xb; int i; volatile double a, b;

  mpfr_set_default_prec (53);
  mpfr_set_emin (-1073); mpfr_set_emax (1024);

  mpfr_init (xa); mpfr_init (xb);

  b = 34.3; mpfr_set_d (xb, b, MPFR_RNDN);
  a = 0x1.1235P-1021; mpfr_set_d (xa, a, MPFR_RNDN);

  a /= b;
  i = mpfr_div (xa, xa, xb, MPFR_RNDN);
  i = mpfr_subnormalize (xa, i, MPFR_RNDN); /* new ternary value */
```

```
  mpfr_clear (xa); mpfr_clear (xb);
}
```

Warning: this emulates a double IEEE 754 arithmetic with correct rounding in the subnormal range, which may not be the case for your hardware.

`void mpfr_clear_underflow` (*void*) [Function]
`void mpfr_clear_overflow` (*void*) [Function]
`void mpfr_clear_divby0` (*void*) [Function]
`void mpfr_clear_nanflag` (*void*) [Function]
`void mpfr_clear_inexflag` (*void*) [Function]
`void mpfr_clear_erangeflag` (*void*) [Function]

Clear the underflow, overflow, divide-by-zero, invalid, inexact and *erange* flags.

`void mpfr_set_underflow` (*void*) [Function]
`void mpfr_set_overflow` (*void*) [Function]
`void mpfr_set_divby0` (*void*) [Function]
`void mpfr_set_nanflag` (*void*) [Function]
`void mpfr_set_inexflag` (*void*) [Function]
`void mpfr_set_erangeflag` (*void*) [Function]

Set the underflow, overflow, divide-by-zero, invalid, inexact and *erange* flags.

`void mpfr_clear_flags` (*void*) [Function]

Clear all global flags (underflow, overflow, divide-by-zero, invalid, inexact, *erange*).

`int mpfr_underflow_p` (*void*) [Function]
`int mpfr_overflow_p` (*void*) [Function]
`int mpfr_divby0_p` (*void*) [Function]
`int mpfr_nanflag_p` (*void*) [Function]
`int mpfr_inexflag_p` (*void*) [Function]
`int mpfr_erangeflag_p` (*void*) [Function]

Return the corresponding (underflow, overflow, divide-by-zero, invalid, inexact, *erange*) flag, which is non-zero iff the flag is set.

5.14 Compatibility With MPF

A header file `mpf2mpfr.h` is included in the distribution of MPFR for compatibility with the GNU MP class MPF. By inserting the following two lines after the `#include <gmp.h>` line,

```
#include <mpfr.h>
#include <mpf2mpfr.h>
```

any program written for MPF can be compiled directly with MPFR without any changes (except the `gmp_printf` functions will not work for arguments of type `mpfr_t`). All operations are then performed with the default MPFR rounding mode, which can be reset with `mpfr_set_default_rounding_mode`.

Warning: the `mpf_init` and `mpf_init2` functions initialize to zero, whereas the corresponding MPFR functions initialize to NaN: this is useful to detect uninitialized values, but is slightly incompatible with MPF.

`void mpfr_set_prec_raw` (*mpfr_t* `x`, *mpfr_prec_t* `prec`) [Function]

Reset the precision of *x* to be **exactly** *prec* bits. The only difference with `mpfr_set_prec` is that *prec* is assumed to be small enough so that the significand fits into the current allocated memory space for *x*. Otherwise the behavior is undefined.

`int mpfr_eq` (*mpfr_t* `op1`, *mpfr_t* `op2`, *unsigned long int* `op3`) [Function]

Return non-zero if *op1* and *op2* are both non-zero ordinary numbers with the same exponent and the same first *op3* bits, both zero, or both infinities of the same sign. Return zero otherwise. This function is defined for compatibility with MPF, we do not recommend to use it otherwise. Do not use it either if you want to know whether two numbers are close to each other; for instance, 1.011111 and 1.100000 are regarded as different for any value of *op3* larger than 1.

`void mpfr_reldiff` (*mpfr_t* `rop`, *mpfr_t* `op1`, *mpfr_t* `op2`, *mpfr_rnd_t* `rnd`) [Function]

Compute the relative difference between *op1* and *op2* and store the result in *rop*. This function does not guarantee the correct rounding on the relative difference; it just computes $|op1 - op2|/op1$, using the precision of *rop* and the rounding mode *rnd* for all operations.

`int mpfr_mul_2exp` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mpfr_rnd_t* `rnd`) [Function]

`int mpfr_div_2exp` (*mpfr_t* `rop`, *mpfr_t* `op1`, *unsigned long int* `op2`, *mpfr_rnd_t* `rnd`) [Function]

These functions are identical to `mpfr_mul_2ui` and `mpfr_div_2ui` respectively. These functions are only kept for compatibility with MPF, one should prefer `mpfr_mul_2ui` and `mpfr_div_2ui` otherwise.

5.15 Custom Interface

Some applications use a stack to handle the memory and their objects. However, the MPFR memory design is not well suited for such a thing. So that such applications are able to use MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows one to use MPFR in two ways:

- Either directly store a floating-point number as a `mpfr_t` on the stack.
- Either store its own representation on the stack and construct a new temporary `mpfr_t` each time it is needed.

Nothing has to be done to destroy the floating-point numbers except garbaging the used memory: all the memory management (allocating, destroying, garbaging) is left to the application.

Each function in this interface is also implemented as a macro for efficiency reasons: for example `mpfr_custom_init (s, p)` uses the macro, while `(mpfr_custom_init) (s, p)` uses the function.

Note 1: MPFR functions may still initialize temporary floating-point numbers using `mpfr_init` and similar functions. See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (`mpfr_const_pi` for example), even if they are not explicitly called. You have to call `mpfr_free_cache` each time you garbage the memory iff `mpfr_init`, through GMP Custom Allocation, allocates its memory on the application stack.

`size_t mpfr_custom_get_size` (*mpfr_prec_t* ***prec***) [Function]

Return the needed size in bytes to store the significand of a floating-point number of precision *prec*.

`void mpfr_custom_init` (*void* ****significand***, *mpfr_prec_t* ***prec***) [Function]

Initialize a significand of precision *prec*, where *significand* must be an area of `mpfr_custom_get_size (prec)` bytes at least and be suitably aligned for an array of `mp_limb_t` (GMP type, see Section 5.16 [Internals], page 39).

`void mpfr_custom_init_set` (*mpfr_t* ***x***, *int* ***kind***, *mpfr_exp_t* ***exp***, *mpfr_prec_t* ***prec***, *void* ****significand***) [Function]

Perform a dummy initialization of a `mpfr_t` and set it to:

- if `ABS(kind) == MPFR_NAN_KIND`, *x* is set to NaN;
- if `ABS(kind) == MPFR_INF_KIND`, *x* is set to the infinity of sign `sign(kind)`;
- if `ABS(kind) == MPFR_ZERO_KIND`, *x* is set to the zero of sign `sign(kind)`;
- if `ABS(kind) == MPFR_REGULAR_KIND`, *x* is set to a regular number: `x = sign(kind)*significand*2^exp`.

In all cases, it uses *significand* directly for further computing involving *x*. It will not allocate anything. A floating-point number initialized with this function cannot be resized using `mpfr_set_prec` or `mpfr_prec_round`, or cleared using `mpfr_clear`! The *significand* must have been initialized with `mpfr_custom_init` using the same precision *prec*.

`int mpfr_custom_get_kind` (*mpfr_t* ***x***) [Function]

Return the current kind of a `mpfr_t` as created by `mpfr_custom_init_set`. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`void * mpfr_custom_get_significand` (*mpfr_t* ***x***) [Function]

Return a pointer to the significand used by a `mpfr_t` initialized with `mpfr_custom_init_set`. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`mpfr_exp_t mpfr_custom_get_exp` (*mpfr_t* ***x***) [Function]

Return the exponent of *x*, assuming that *x* is a non-zero ordinary number. The return value for NaN, Infinity or zero is unspecified but does not produce any trap. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

`void mpfr_custom_move` (*mpfr_t* ***x***, *void* ****new_position***) [Function]

Inform MPFR that the significand of *x* has moved due to a garbage collect and update its new position to `new_position`. However the application has to move the significand and the `mpfr_t` itself. The behavior of this function for any `mpfr_t` not initialized with `mpfr_custom_init_set` is undefined.

5.16 Internals

A *limb* means the part of a multi-precision number that fits in a single word. Usually a limb contains 32 or 64 bits. The C data type for a limb is `mp_limb_t`.

The `mpfr_t` type is internally defined as a one-element array of a structure, and `mpfr_ptr` is the C data type representing a pointer to this structure. The `mpfr_t` type consists of four fields:

- The `_mpfr_prec` field is used to store the precision of the variable (in bits); this is not less than `MPFR_PREC_MIN`.

- The `_mpfr_sign` field is used to store the sign of the variable.
- The `_mpfr_exp` field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values n are a multiplier 2^n relative to that point. A NaN, an infinity and a zero are indicated by special values of the exponent field.
- Finally, the `_mpfr_d` field is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by `_mpfr_prec`, namely ceil(`_mpfr_prec/mp_bits_per_limb`). Non-singular (i.e., different from NaN, Infinity or zero) values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zeros.

6 API Compatibility

The goal of this section is to describe some API changes that occurred from one version of MPFR to another, and how to write code that can be compiled and run with older MPFR versions. The minimum MPFR version that is considered here is 2.2.0 (released on 20 September 2005).

API changes can only occur between major or minor versions. Thus the patchlevel (the third number in the MPFR version) will be ignored in the following. If a program does not use MPFR internals, changes in the behavior between two versions differing only by the patchlevel should only result from what was regarded as a bug or unspecified behavior.

As a general rule, a program written for some MPFR version should work with later versions, possibly except at a new major version, where some features (described as obsolete for some time) can be removed. In such a case, a failure should occur during compilation or linking. If a result becomes incorrect because of such a change, please look at the various changes below (they are minimal, and most software should be unaffected), at the FAQ and at the MPFR web page for your version (a bug could have been introduced and be already fixed); and if the problem is not mentioned, please send us a bug report (see Chapter 3 [Reporting Bugs], page 5).

However, a program written for the current MPFR version (as documented by this manual) may not necessarily work with previous versions of MPFR. This section should help developers to write portable code.

Note: Information given here may be incomplete. API changes are also described in the NEWS file (for each version, instead of being classified like here), together with other changes.

6.1 Type and Macro Changes

The official type for exponent values changed from `mp_exp_t` to `mpfr_exp_t` in MPFR 3.0. The type `mp_exp_t` will remain available as it comes from GMP (with a different meaning). These types are currently the same (`mpfr_exp_t` is defined as `mp_exp_t` with `typedef`), so that programs can still use `mp_exp_t`; but this may change in the future. Alternatively, using the following code after including `mpfr.h` will work with official MPFR versions, as `mpfr_exp_t` was never defined in MPFR 2.x:

```
#if MPFR_VERSION_MAJOR < 3
typedef mp_exp_t mpfr_exp_t;
#endif
```

The official types for precision values and for rounding modes respectively changed from `mp_prec_t` and `mp_rnd_t` to `mpfr_prec_t` and `mpfr_rnd_t` in MPFR 3.0. This change was actually done a long time ago in MPFR, at least since MPFR 2.2.0, with the following code in `mpfr.h`:

```
#ifndef mp_rnd_t
# define mp_rnd_t  mpfr_rnd_t
#endif
#ifndef mp_prec_t
# define mp_prec_t mpfr_prec_t
#endif
```

This means that it is safe to use the new official types `mpfr_prec_t` and `mpfr_rnd_t` in your programs. The types `mp_prec_t` and `mp_rnd_t` (defined in MPFR only) may be removed in the future, as the prefix `mp_` is reserved by GMP.

The precision type `mpfr_prec_t` (`mp_prec_t`) was unsigned before MPFR 3.0; it is now signed. `MPFR_PREC_MAX` has not changed, though. Indeed the MPFR code requires that `MPFR_PREC_MAX`

be representable in the exponent type, which may have the same size as `mpfr_prec_t` but has always been signed. The consequence is that valid code that does not assume anything about the signedness of `mpfr_prec_t` should work with past and new MPFR versions. This change was useful as the use of unsigned types tends to convert signed values to unsigned ones in expressions due to the usual arithmetic conversions, which can yield incorrect results if a negative value is converted in such a way. Warning! A program assuming (intentionally or not) that `mpfr_prec_t` is signed may be affected by this problem when it is built and run against MPFR 2.x.

The rounding modes `GMP_RNDx` were renamed to `MPFR_RNDx` in MPFR 3.0. However the old names `GMP_RNDx` have been kept for compatibility (this might change in future versions), using:

```
#define GMP_RNDN MPFR_RNDN
#define GMP_RNDZ MPFR_RNDZ
#define GMP_RNDU MPFR_RNDU
#define GMP_RNDD MPFR_RNDD
```

The rounding mode "round away from zero" (`MPFR_RNDA`) was added in MPFR 3.0 (however no rounding mode `GMP_RNDA` exists).

6.2 Added Functions

We give here in alphabetical order the functions that were added after MPFR 2.2, and in which MPFR version.

- `mpfr_add_d` in MPFR 2.4.
- `mpfr_ai` in MPFR 3.0 (incomplete, experimental).
- `mpfr_asprintf` in MPFR 2.4.
- `mpfr_buildopt_decimal_p` and `mpfr_buildopt_tls_p` in MPFR 3.0.
- `mpfr_buildopt_gmpinternals_p` and `mpfr_buildopt_tune_case` in MPFR 3.1.
- `mpfr_clear_divby0` in MPFR 3.1 (new divide-by-zero exception).
- `mpfr_copysign` in MPFR 2.3. Note: MPFR 2.2 had a `mpfr_copysign` function that was available, but not documented, and with a slight difference in the semantics (when the second input operand is a NaN).
- `mpfr_custom_get_significand` in MPFR 3.0. This function was named `mpfr_custom_get_mantissa` in previous versions; `mpfr_custom_get_mantissa` is still available via a macro in `mpfr.h`:

  ```
  #define mpfr_custom_get_mantissa mpfr_custom_get_significand
  ```

  Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use `mpfr_custom_get_mantissa`.
- `mpfr_d_div` and `mpfr_d_sub` in MPFR 2.4.
- `mpfr_digamma` in MPFR 3.0.
- `mpfr_divby0_p` in MPFR 3.1 (new divide-by-zero exception).
- `mpfr_div_d` in MPFR 2.4.
- `mpfr_fmod` in MPFR 2.4.
- `mpfr_fms` in MPFR 2.3.
- `mpfr_fprintf` in MPFR 2.4.
- `mpfr_frexp` in MPFR 3.1.
- `mpfr_get_flt` in MPFR 3.0.
- `mpfr_get_patches` in MPFR 2.3.
- `mpfr_get_z_2exp` in MPFR 3.0. This function was named `mpfr_get_z_exp` in previous versions; `mpfr_get_z_exp` is still available via a macro in `mpfr.h`:

```
#define mpfr_get_z_exp mpfr_get_z_2exp
```

Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use `mpfr_get_z_exp`.

- `mpfr_grandom` in MPFR 3.1.
- `mpfr_j0`, `mpfr_j1` and `mpfr_jn` in MPFR 2.3.
- `mpfr_lgamma` in MPFR 2.3.
- `mpfr_li2` in MPFR 2.4.
- `mpfr_min_prec` in MPFR 3.0.
- `mpfr_modf` in MPFR 2.4.
- `mpfr_mul_d` in MPFR 2.4.
- `mpfr_printf` in MPFR 2.4.
- `mpfr_rec_sqrt` in MPFR 2.4.
- `mpfr_regular_p` in MPFR 3.0.
- `mpfr_remainder` and `mpfr_remquo` in MPFR 2.3.
- `mpfr_set_divby0` in MPFR 3.1 (new divide-by-zero exception).
- `mpfr_set_flt` in MPFR 3.0.
- `mpfr_set_z_2exp` in MPFR 3.0.
- `mpfr_set_zero` in MPFR 3.0.
- `mpfr_setsign` in MPFR 2.3.
- `mpfr_signbit` in MPFR 2.3.
- `mpfr_sinh_cosh` in MPFR 2.4.
- `mpfr_snprintf` and `mpfr_sprintf` in MPFR 2.4.
- `mpfr_sub_d` in MPFR 2.4.
- `mpfr_urandom` in MPFR 3.0.
- `mpfr_vasprintf`, `mpfr_vfprintf`, `mpfr_vprintf`, `mpfr_vsprintf` and `mpfr_vsnprintf` in MPFR 2.4.
- `mpfr_y0`, `mpfr_y1` and `mpfr_yn` in MPFR 2.3.
- `mpfr_z_sub` in MPFR 3.1.

6.3 Changed Functions

The following functions have changed after MPFR 2.2. Changes can affect the behavior of code written for some MPFR version when built and run against another MPFR version (older or newer), as described below.

- `mpfr_check_range` changed in MPFR 2.3.2 and MPFR 2.4. If the value is an inexact infinity, the overflow flag is now set (in case it was lost), while it was previously left unchanged. This is really what is expected in practice (and what the MPFR code was expecting), so that the previous behavior was regarded as a bug. Hence the change in MPFR 2.3.2.
- `mpfr_get_f` changed in MPFR 3.0. This function was returning zero, except for NaN and Inf, which do not exist in MPF. The *erange* flag is now set in these cases, and `mpfr_get_f` now returns the usual ternary value.
- `mpfr_get_si`, `mpfr_get_sj`, `mpfr_get_ui` and `mpfr_get_uj` changed in MPFR 3.0. In previous MPFR versions, the cases where the *erange* flag is set were unspecified.
- `mpfr_get_z` changed in MPFR 3.0. The return type was `void`; it is now `int`, and the usual ternary value is returned. Thus programs that need to work with both MPFR 2.x and 3.x must not use the return value. Even in this case, C code using `mpfr_get_z` as the second

or third term of a conditional operator may also be affected. For instance, the following is correct with MPFR 3.0, but not with MPFR 2.x:

```
bool ? mpfr_get_z(...) : mpfr_add(...);
```

On the other hand, the following is correct with MPFR 2.x, but not with MPFR 3.0:

```
bool ? mpfr_get_z(...) : (void) mpfr_add(...);
```

Portable code should cast `mpfr_get_z(...)` to `void` to use the type `void` for both terms of the conditional operator, as in:

```
bool ? (void) mpfr_get_z(...) : (void) mpfr_add(...);
```

Alternatively, `if ... else` can be used instead of the conditional operator.

Moreover the cases where the *erange* flag is set were unspecified in MPFR 2.x.

- `mpfr_get_z_exp` changed in MPFR 3.0. In previous MPFR versions, the cases where the *erange* flag is set were unspecified. Note: this function has been renamed to `mpfr_get_z_2exp` in MPFR 3.0, but `mpfr_get_z_exp` is still available for compatibility reasons.
- `mpfr_strtofr` changed in MPFR 2.3.1 and MPFR 2.4. This was actually a bug fix since the code and the documentation did not match. But both were changed in order to have a more consistent and useful behavior. The main changes in the code are as follows. The binary exponent is now accepted even without the `0b` or `0x` prefix. Data corresponding to NaN can now have an optional sign (such data were previously invalid).
- `mpfr_strtofr` changed in MPFR 3.0. This function now accepts bases from 37 to 62 (no changes for the other bases). Note: if an unsupported base is provided to this function, the behavior is undefined; more precisely, in MPFR 2.3.1 and later, providing an unsupported base yields an assertion failure (this behavior may change in the future).
- `mpfr_subnormalize` changed in MPFR 3.1. This was actually regarded as a bug fix. The `mpfr_subnormalize` implementation up to MPFR 3.0.0 did not change the flags. In particular, it did not follow the generic rule concerning the inexact flag (and no special behavior was specified). The case of the underflow flag was more a lack of specification.
- `mpfr_urandom` and `mpfr_urandomb` changed in MPFR 3.1. Their behavior no longer depends on the platform (assuming this is also true for GMP's random generator, which is not the case between GMP 4.1 and 4.2 if `gmp_randinit_default` is used). As a consequence, the returned values can be different between MPFR 3.1 and previous MPFR versions. Note: as the reproducibility of these functions was not specified before MPFR 3.1, the MPFR 3.1 behavior is *not* regarded as backward incompatible with previous versions.

6.4 Removed Functions

Functions `mpfr_random` and `mpfr_random2` have been removed in MPFR 3.0 (this only affects old code built against MPFR 3.0 or later). (The function `mpfr_random` had been deprecated since at least MPFR 2.2.0, and `mpfr_random2` since MPFR 2.4.0.)

6.5 Other Changes

For users of a C++ compiler, the way how the availability of `intmax_t` is detected has changed in MPFR 3.0. In MPFR 2.x, if a macro `INTMAX_C` or `UINTMAX_C` was defined (e.g. when the `__STDC_CONSTANT_MACROS` macro had been defined before `<stdint.h>` or `<inttypes.h>` has been included), `intmax_t` was assumed to be defined. However this was not always the case (more precisely, `intmax_t` can be defined only in the namespace `std`, as with Boost), so that compilations could fail. Thus the check for `INTMAX_C` or `UINTMAX_C` is now disabled for C++ compilers, with the following consequences:

- Programs written for MPFR 2.x that need `intmax_t` may no longer be compiled against MPFR 3.0: a `#define MPFR_USE_INTMAX_T` may be necessary before `mpfr.h` is included.

- The compilation of programs that work with MPFR 3.0 may fail with MPFR 2.x due to the problem described above. Workarounds are possible, such as defining `intmax_t` and `uintmax_t` in the global namespace, though this is not clean.

The divide-by-zero exception is new in MPFR 3.1. However it should not introduce incompatible changes for programs that strictly follow the MPFR API since the exception can only be seen via new functions.

As of MPFR 3.1, the `mpfr.h` header can be included several times, while still supporting optional functions (see Section 4.1 [Headers and Libraries], page 6).

Contributors

The main developers of MPFR are Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier, Philippe Théveny and Paul Zimmermann.

Sylvie Boldo from ENS-Lyon, France, contributed the functions `mpfr_agm` and `mpfr_log`. Sylvain Chevillard contributed the `mpfr_ai` function. David Daney contributed the hyperbolic and inverse hyperbolic functions, the base-2 exponential, and the factorial function. Alain Delplanque contributed the new version of the `mpfr_get_str` function. Mathieu Dutour contributed the functions `mpfr_acos`, `mpfr_asin` and `mpfr_atan`, and a previous version of `mpfr_gamma`. Laurent Fousse contributed the `mpfr_sum` function. Emmanuel Jeandel, from ENS-Lyon too, contributed the generic hypergeometric code, as well as the internal function `mpfr_exp3`, a first implementation of the sine and cosine, and improved versions of `mpfr_const_log2` and `mpfr_const_pi`. Ludovic Meunier helped in the design of the `mpfr_erf` code. Jean-Luc Rémy contributed the `mpfr_zeta` code. Fabrice Rouillier contributed the `mpfr_xxx_z` and `mpfr_xxx_q` functions, and helped to the Microsoft Windows porting. Damien Stehlé contributed the `mpfr_get_ld_2exp` function.

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