This manual documents how to install and use the Multiple Precision Floating-Point Reliable Library, version 4.1.1.


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The GNU MPFR library (or MPFR for short) is free; this means that everyone is free to use it and free to redistribute it on a free basis. The library is not in the public domain; it is copyrighted and there are restrictions on its distribution, but these restrictions are designed to permit everything that a good cooperating citizen would want to do. What is not allowed is to try to prevent others from further sharing any version of this library that they might get from you.

Specifically, we want to make sure that you have the right to give away copies of the library, that you receive source code or else can get it if you want it, that you can change this library or use pieces of it in new free programs, and that you know you can do these things.

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The precise conditions of the license for the GNU MPFR library are found in the Lesser General Public License that accompanies the source code. See the file COPYING.LESSER.
1 Introduction to MPFR

MPFR is a portable library written in C for arbitrary precision arithmetic on floating-point numbers. It is based on the GNU MP library. It aims to provide a class of floating-point numbers with precise semantics. The main characteristics of MPFR, which make it differ from most arbitrary precision floating-point software tools, are:

- the MPFR code is portable, i.e., the result of any operation does not depend on the machine word size mp_bits_per_limb (64 on most current processors), possibly except in faithful rounding. It does not depend either on the machine rounding mode or rounding precision;
- the precision in bits can be set exactly to any valid value for each variable (including very small precision);
- MPFR provides the four rounding modes from the IEEE 754-1985 standard, plus away-from-zero, as well as for basic operations as for other mathematical functions. Faithful rounding (partially supported) is provided too, but the results may no longer be reproducible.

In particular, MPFR follows the specification of the IEEE 754 standard, currently IEEE 754-2019 (which will be referred to as IEEE 754 in this manual), with some minor differences, such as: there is a single NaN, the default exponent range is much wider, and subnormal numbers are not implemented (but the exponent range can be reduced to any interval, and subnormals can be emulated). For instance, computations in the binary64 format (a.k.a. double precision) can be reproduced by using a precision of 53 bits.

This version of MPFR is released under the GNU Lesser General Public License, version 3 or any later version. It is permitted to link MPFR to most non-free programs, as long as when distributing them the MPFR source code and a means to re-link with a modified MPFR library is provided.

1.1 How to Use This Manual

Everyone should read Chapter 4 [MPFR Basics], page 6. If you need to install the library yourself, you need to read Chapter 2 [Installing MPFR], page 3, too. To use the library you will need to refer to Chapter 5 [MPFR Interface], page 13.

The rest of the manual can be used for later reference, although it is probably a good idea to glance through it.
2 Installing MPFR

The MPFR library is already installed on some GNU/Linux distributions, but the development files necessary to the compilation such as mpfr.h are not always present. To check that MPFR is fully installed on your computer, you can check the presence of the file mpfr.h in /usr/include, or try to compile a small program having #include <mpfr.h> (since mpfr.h may be installed somewhere else). For instance, you can try to compile:

```c
#include <stdio.h>
#include <mpfr.h>
int main (void)
{
    printf ("MPFR library: %-12s
MPFR header: %s (based on %d.%d.%d)\n",
            mpfr_get_version (), MPFR_VERSION_STRING, MPFR_VERSION_MAJOR,
            MPFR_VERSION_MINOR, MPFR_VERSION_PATCHLEVEL);
    return 0;
}
```

with

```c
cc -o version version.c -lmpfr -lgmp
```

and if you get errors whose first line looks like

```
version.c:2:19: error: mpfr.h: No such file or directory
```

then MPFR is probably not installed. Running this program will give you the MPFR version.

If MPFR is not installed on your computer, or if you want to install a different version, please follow the steps below.

2.1 How to Install

Here are the steps needed to install the library on Unix systems (more details are provided in the INSTALL file):

1. To build MPFR, you first have to install GNU MP (version 5.0.0 or higher) on your computer. You need a C compiler, preferably GCC, but any reasonable compiler should work. And you need the standard Unix `make' command, plus some other standard Unix utility commands. Then, in the MPFR build directory, type the following commands.

2. `./configure'

This will prepare the build and set up the options according to your system. You can give options to specify the install directories (instead of the default /usr/local), threading support, and so on. See the INSTALL file and/or the output of `./configure --help' for more information, in particular if you get error messages.

3. `make'

This will compile MPFR, and create a library archive file libmpfr.a. On most platforms, a dynamic library will be produced too.

4. `make check'

This will make sure that MPFR was built correctly. If any test fails, information about this failure can be found in the tests/test-suite.log file. If you want the contents of this file to be automatically output in case of failure, you can set the 'VERBOSE' environment variable to 1 before running `make check', for instance by typing:

`VERBOSE=1 make check'
5. ‘make install’
   This will copy the files mpfr.h and mpf2mpfr.h to the directory /usr/local/include, the library files (libmpfr.a and possibly others) to the directory /usr/local/lib, the file mpfr.info to the directory /usr/local/share/info, and some other documentation files to the directory /usr/local/share/doc/mpfr (or if you passed the ‘--prefix’ option to configure, using the prefix directory given as argument to ‘--prefix’ instead of /usr/local).

2.2 Other ‘make’ Targets

There are some other useful make targets:

- ‘mpfr.info’ or ‘info’
  Create or update an info version of the manual, in mpfr.info.
  This file is already provided in the MPFR archives.
- ‘mpfr.pdf’ or ‘pdf’
  Create a PDF version of the manual, in mpfr.pdf.
- ‘mpfr.dvi’ or ‘dvi’
  Create a DVI version of the manual, in mpfr.dvi.
- ‘mpfr.ps’ or ‘ps’
  Create a PostScript version of the manual, in mpfr.ps.
- ‘mpfr.html’ or ‘html’
  Create a HTML version of the manual, in several pages in the directory doc/mpfr.html; if you want only one output HTML file, then type ‘makeinfo --html --no-split mpfr.texi’ from the ‘doc’ directory instead.
- ‘clean’
  Delete all object files and archive files, but not the configuration files.
- ‘distclean’
  Delete all generated files not included in the distribution.
- ‘uninstall’
  Delete all files copied by ‘make install’.

2.3 Build Problems

In case of problem, please read the INSTALL file carefully before reporting a bug, in particular section “In case of problem”. Some problems are due to bad configuration on the user side (not specific to MPFR). Problems are also mentioned in the FAQ https://www.mpfr.org/faq.html.

Please report problems to the MPFR mailing-list ‘mpfr@inria.fr’. See Chapter 3 [Reporting Bugs], page 5. Some bug fixes are available on the MPFR 4.1.1 web page https://www.mpfr.org/mpfr-4.1.1/.

2.4 Getting the Latest Version of MPFR

3 Reporting Bugs

If you think you have found a bug in the MPFR library, first have a look on the MPFR 4.1.1 web page https://www.mpfr.org/mpfr-4.1.1/ and the FAQ https://www.mpfr.org/faq.html: perhaps this bug is already known, in which case you may find there a workaround for it. You might also look in the archives of the MPFR mailing-list: https://sympa.inria.fr/sympa/arc/mpfr. Otherwise, please investigate and report it. We have made this library available to you, and it is not to ask too much from you to ask you to report the bugs that you find.

There are a few things you should think about when you put your bug report together.

You have to send us a test case that makes it possible for us to reproduce the bug, i.e., a small self-content program, using no other library than MPFR. Include instructions on how to run the test case.

You also have to explain what is wrong; if you get a crash, or if the results you get are incorrect and in that case, in what way.

Please include compiler version information in your bug report. This can be extracted using ‘cc -V’ on some machines, or, if you are using GCC, ‘gcc -v’. Also, include the output from ‘uname -a’ and the MPFR version (the GMP version may be useful too). If you get a failure while running ‘make’ or ‘make check’, please include the config.log file in your bug report, and in case of test failure, the tests/test-suite.log file too.

If your bug report is good, we will do our best to help you to get a corrected version of the library; if the bug report is poor, we will not do anything about it (aside of chiding you to send better bug reports).

Send your bug report to the MPFR mailing-list ‘mpfr@inria.fr’.

If you think something in this manual is unclear, or downright incorrect, or if the language needs to be improved, please send a note to the same address.
4 MPFR Basics

4.1 Headers and Libraries

All declarations needed to use MPFR are collected in the include file `mpfr.h`. It is designed to work with both C and C++ compilers. You should include that file in any program using the MPFR library:

```c
#include <mpfr.h>
```

Note, however, that prototypes for MPFR functions with `FILE *` parameters are provided only if `<stdio.h>` is included too (before `mpfr.h`):

```c
#include <stdio.h>
#include <mpfr.h>
```

Likewise `<stdarg.h>` (or `<varargs.h>`) is required for prototypes with `va_list` parameters, such as `mpfr_vprintf`.

And for any functions using `intmax_t`, you must include `<stdint.h>` or `<inttypes.h>` before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions. Moreover, under some platforms (in particular with C++ compilers), users may need to define `MPFR_USE_INTMAX_T` (and should do it for portability) before `mpfr.h` has been included; of course, it is possible to do that on the command line, e.g., with `-DMPFR_USE_INTMAX_T`.

Note: If `mpfr.h` and/or `gmp.h` (used by `mpfr.h`) are included several times (possibly from another header file), `<stdio.h>` and/or `<stdarg.h>` (or `<varargs.h>`) should be included before the first inclusion of `mpfr.h` or `gmp.h`. Alternatively, you can define `MPFR_USE_FILE` (for MPFR I/O functions) and/or `MPFR_USE_va_LIST` (for MPFR functions with `va_list` parameters) anywhere before the last inclusion of `mpfr.h`. As a consequence, if your file is a public header that includes `mpfr.h`, you need to use the latter method.

When calling a MPFR macro, it is not allowed to have previously defined a macro with the same name as some keywords (currently `do`, `while` and `sizeof`).

You can avoid the use of MPFR macros encapsulating functions by defining the `MPFR_USE_NO_MACRO` macro before `mpfr.h` is included. In general this should not be necessary, but this can be useful when debugging user code: with some macros, the compiler may emit spurious warnings with some warning options, and macros can prevent some prototype checking.

All programs using MPFR must link against both `libmpfr` and `libgmp` libraries. On a typical Unix-like system this can be done with `'-lmpfr -lgmp'` (in that order), for example:

```bash
cmake myprogram.c -lmpfr -lgmp
```

MPFR is built using Libtool and an application can use that to link if desired, see GNU Libtool.

If MPFR has been installed to a non-standard location, then it may be necessary to set up environment variables such as `C_INCLUDE_PATH` and `LIBRARY_PATH`, or use `-I` and `-L` compiler options, in order to point to the right directories. For a shared library, it may also be necessary to set up some sort of run-time library path (e.g., `LD_LIBRARY_PATH`) on some systems. Please read the INSTALL file for additional information.

Alternatively, it is possible to use `pkg-config` (a file `mpfr.pc` is provided as of MPFR 4.0):

```bash
cpkg_config myprogram.c $(pkg_config --cflags --libs mpfr)
```
Note that the `MPFR_` and `mpfr_` prefixes are reserved for MPFR. As a general rule, in order to avoid clashes, software using MPFR (directly or indirectly) and system headers/libraries should not define macros and symbols using these prefixes.

### 4.2 Nomenclature and Types

A floating-point number, or float for short, is an object representing a radix-2 floating-point number consisting of a sign, an arbitrary-precision normalized significand (also called mantissa), and an exponent (an integer in some given range); these are called regular numbers. By convention, the radix point of the significand is just before the first digit (which is always 1 due to normalization), like in the C language, but unlike in IEEE 754 (thus, for a given number, the exponent values in MPFR and in IEEE 754 differ by 1).

Like in the IEEE 754 standard, a floating-point number can also have three kinds of special values: a signed zero (+0 or −0), a signed infinity (+Inf or −Inf), and Not-a-Number (NaN). NaN can represent the default value of a floating-point object and the result of some operations for which no other results would make sense, such as 0 divided by 0 or +Inf minus +Inf; unless documented otherwise, the sign bit of a NaN is unspecified. Note that contrary to IEEE 754, MPFR has a single kind of NaN and does not have subnormals. Other than that, the behavior is very similar to IEEE 754, but there are some minor differences.

The C data type for such objects is `mpfr_t`, internally defined as a one-element array of a structure (so that when passed as an argument to a function, it is the pointer that is actually passed), and `mpfr_ptr` is the C data type representing a pointer to this structure.

The precision is the number of bits used to represent the significand of a floating-point number; the corresponding C data type is `mpfr_prec_t`. The precision can be any integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX`. In the current implementation, `MPFR_PREC_MIN` is equal to 1.

Warning! MPFR needs to increase the precision internally, in order to provide accurate results (and in particular, correct rounding). Do not attempt to set the precision to any value near `MPFR_PREC_MAX`, otherwise MPFR will abort due to an assertion failure. However, in practice, the real limitation will probably be the available memory on your platform, and in case of lack of memory, the program may abort, crash or have undefined behavior (depending on your C implementation).

An exponent is a component of a regular floating-point number. Its C data type is `mpfr_exp_t`. Valid exponents are restricted to a subset of this type, and the exponent range can be changed globally as described in Section 5.13 [Exception Related Functions], page 43. Special values do not have an exponent.

The rounding mode specifies the way to round the result of a floating-point operation, in case the exact result cannot be represented exactly in the destination (see Section 4.4 [Rounding], page 8). The corresponding C data type is `mpfr_rnd_t`.

MPFR has a global (or per-thread) flag for each supported exception and provides operations on flags (Section 4.6 [Exceptions], page 10). This C data type is used to represent a group of flags (or a mask).

### 4.3 MPFR Variable Conventions

Before you can assign to a MPFR variable, you need to initialize it by calling one of the special initialization functions. When you are done with a variable, you need to clear it out, using one of the functions for that purpose. A variable should only be initialized once, or at least cleared out between each initialization. After a variable has been initialized, it may be assigned
to any number of times. For efficiency reasons, avoid to initialize and clear out a variable in loops. Instead, initialize it before entering the loop, and clear it out after the loop has exited. You do not need to be concerned about allocating additional space for MPFR variables, since any variable has a significand of fixed size. Hence unless you change its precision, or clear and reinitialize it, a floating-point variable will have the same allocated space during all its life.

As a general rule, all MPFR functions expect output arguments before input arguments. This notation is based on an analogy with the assignment operator. MPFR allows you to use the same variable for both input and output in the same expression. For example, the main function for floating-point multiplication, \texttt{mpfr\_mul}, can be used like this: \texttt{mpfr\_mul (x, x, x, rnd)}. This computes the square of \texttt{x} with rounding mode \texttt{rnd} and puts the result back in \texttt{x}.

### 4.4 Rounding

The following rounding modes are supported:

- \texttt{MPFR\_RNDD}: round to nearest, with the even rounding rule (roundTiesToEven in IEEE 754); see details below.
- \texttt{MPFR\_RNDD}: round toward negative infinity (roundTowardNegative in IEEE 754).
- \texttt{MPFR\_RNDU}: round toward positive infinity (roundTowardPositive in IEEE 754).
- \texttt{MPFR\_RNDZ}: round toward zero (roundTowardZero in IEEE 754).
- \texttt{MPFR\_RNDA}: round away from zero.
- \texttt{MPFR\_RNDF}: faithful rounding. This feature is currently experimental. Specific support for this rounding mode has been added to some functions, such as the basic operations (addition, subtraction, multiplication, square, division, square root) or when explicitly documented. It might also work with other functions, as it is possible that they do not need modification in their code; even though a correct behavior is not guaranteed yet (corrections were done when failures occurred in the test suite, but almost nothing has been checked manually), failures should be regarded as bugs and reported, so that they can be fixed.

Note that, in particular for a result equal to zero, the sign is preserved by the rounding operation.

The \texttt{MPFR\_RNDD} mode works like roundTiesToEven from the IEEE 754 standard: in case the number to be rounded lies exactly in the middle between two consecutive representable numbers, it is rounded to the one with an even significand; in radix 2, this means that the least significant bit is 0. For example, the number 2.5, which is represented by (10.1) in binary, is rounded to (10.0) = 2 with a precision of two bits, and not to (11.0) = 3. This rule avoids the drift phenomenon mentioned by Knuth in volume 2 of The Art of Computer Programming (Section 4.2.2).

Note: In particular for a 1-digit precision (in radix 2 or other radices, as in conversions to a string of digits), one considers the significands associated with the exponent of the number to be rounded. For instance, to round the number 95 in radix 10 with a 1-digit precision, one considers its truncated 1-digit integer significand 9 and the following integer 10 (since these are consecutive integers, exactly one of them is even). 10 is the even significand, so that 95 will be rounded to 100, not to 90.

For the \textit{directed rounding modes}, a number \texttt{x} is rounded to the number \texttt{y} that is the closest to \texttt{x} such that

- \texttt{MPFR\_RNDD}: \texttt{y} is less than or equal to \texttt{x};
- \texttt{MPFR\_RNDU}: \texttt{y} is greater than or equal to \texttt{x};
- \texttt{MPFR\_RNDZ}: \texttt{|y|} is less than or equal to \texttt{|x|};
- \texttt{MPFR\_RNDA}: \texttt{|y|} is greater than or equal to \texttt{|x|}. 
The \texttt{MPFR\_RNDF} mode works as follows: the computed value is either that corresponding to \texttt{MPFR\_RNDD} or that corresponding to \texttt{MPFR\_RNDU}. In particular when those values are identical, i.e., when the result of the corresponding operation is exactly representable, that exact result is returned. Thus, the computed result can take at most two possible values, and in absence of underflow/overflow, the corresponding error is strictly less than one ulp (unit in the last place) of that result and of the exact result. For \texttt{MPFR\_RNDF}, the ternary value (defined below) and the inexact flag (defined later, as with the other flags) are unspecified, the divide-by-zero flag is as with other roundings, and the underflow and overflow flags match what would be obtained in the case the computed value is the same as with \texttt{MPFR\_RNDD} or \texttt{MPFR\_RNDU}. The results may not be reproducible.

Most MPFR functions take as first argument the destination variable, as second and following arguments the input variables, as last argument a rounding mode, and have a return value of type \texttt{int}, called the \textit{ternary value}. The value stored in the destination variable is correctly rounded, i.e., MPFR behaves as if it computed the result with an infinite precision, then rounded it to the precision of this variable. The input variables are regarded as exact (in particular, their precision does not affect the result).

As a consequence, in case of a non-zero real rounded result, the error on the result is less than or equal to $1/2$ ulp (unit in the last place) of that result in the rounding to nearest mode, and less than 1 ulp of that result in the directed rounding modes (a ulp is the weight of the least significant represented bit of the result after rounding).

Unless documented otherwise, functions returning an \texttt{int} return a ternary value. If the ternary value is zero, it means that the value stored in the destination variable is the exact result of the corresponding mathematical function. If the ternary value is positive (resp. negative), it means the value stored in the destination variable is greater (resp. lower) than the exact result. For example with the \texttt{MPFR\_RNDU} rounding mode, the ternary value is usually positive, except when the result is exact, in which case it is zero. In the case of an infinite result, it is considered as inexact when it was obtained by overflow, and exact otherwise. A NaN result (Not-a-Number) always corresponds to an exact return value. The opposite of a returned ternary value is guaranteed to be representable in an \texttt{int}.

Unless documented otherwise, functions returning as result the value 1 (or any other value specified in this manual) for special cases (like $\text{acos}(0)$) yield an overflow or an underflow if that value is not representable in the current exponent range.

### 4.5 Floating-Point Values on Special Numbers

This section specifies the floating-point values (of type \texttt{mpfr\_t}) returned by MPFR functions (where by “returned” we mean here the modified value of the destination object, which should not be mixed with the ternary return value of type \texttt{int} of those functions). For functions returning several values (like \texttt{mpfr\_sin\_cos}), the rules apply to each result separately.

Functions can have one or several input arguments. An input point is a mapping from these input arguments to the set of the MPFR numbers. When none of its components are NaN, an input point can also be seen as a tuple in the extended real numbers (the set of the real numbers with both infinities).

When the input point is in the domain of the mathematical function, the result is rounded as described in Section 4.4 [Rounding], page 8, (but see below for the specification of the sign of an exact zero). Otherwise the general rules from this section apply unless stated otherwise in the description of the MPFR function (Chapter 5 [MPFR Interface], page 13).
When the input point is not in the domain of the mathematical function but is in its closure in the extended real numbers and the function can be extended by continuity, the result is the obtained limit. Examples: \texttt{mpfr\_hypot} on \((+\text{Inf},0)\) gives +\text{Inf}. But \texttt{mpfr\_pow} cannot be defined on \((1,+\text{Inf})\) using this rule, as one can find sequences \((x_n,y_n)\) such that \(x_n\) goes to 1, \(y_n\) goes to +\text{Inf} and \((x_n)^{y_n}\) goes to any positive value when \(n\) goes to the infinity.

When the input point is in the closure of the domain of the mathematical function and an input argument is +0 (resp. −0), one considers the limit when the corresponding argument approaches 0 from above (resp. below), if possible. If the limit is not defined (e.g., \texttt{mpfr\_sqrt} and \texttt{mpfr\_log} on −0), the behavior is specified in the description of the MPFR function, but must be consistent with the rule from the above paragraph (e.g., \texttt{mpfr\_log} on ±0 gives −Inf).

When the result is equal to 0, its sign is determined by considering the limit as if the input point were not in the domain: If one approaches 0 from above (resp. below), the result is +0 (resp. −0); for example, \texttt{mpfr\_sin} on −0 gives −0 and \texttt{mpfr\_acos} on 1 gives +0 (in all rounding modes). In the other cases, the sign is specified in the description of the MPFR function; for example \texttt{mpfr\_max} on −0 and +0 gives +0.

When the input point is not in the closure of the domain of the function, the result is NaN. Example: \texttt{mpfr\_sqrt} on −17 gives NaN.

When an input argument is NaN, the result is NaN, possibly except when a partial function is constant on the finite floating-point numbers; such a case is always explicitly specified in Chapter 5 [MPFR Interface], page 13. Example: \texttt{mpfr\_hypot} on \((\text{NaN},0)\) gives \text{NaN}, but \texttt{mpfr\_hypot} on \((\text{NaN},+\text{Inf})\) gives +\text{Inf} (as specified in Section 5.7 [Transcendental Functions], page 27), since for any finite or infinite input \(x\), \texttt{mpfr\_hypot} on \((x, +\text{Inf})\) gives +\text{Inf}.

MPFR also tries to follow the specifications of the IEEE 754 standard on special values (IEEE 754 agree with the above rules in most cases). Any difference with IEEE 754 that is not explicitly mentioned, other than those due to the single NaN, is unintended and might be regarded as a bug. See also Chapter 7 [MPFR and the IEEE 754 Standard], page 57.

### 4.6 Exceptions

MPFR defines a global (or per-thread) flag for each supported exception. A macro evaluating to a power of two is associated with each flag and exception, in order to be able to specify a group of flags (or a mask) by OR’ing such macros.

Flags can be cleared (lowered), set (raised), and tested by functions described in Section 5.13 [Exception Related Functions], page 43.

The supported exceptions are listed below. The macro associated with each exception is in parentheses.

- **Underflow (MPFR\_FLAGS\_UNDERFLOW)**: An underflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent smaller than the minimum value of the current exponent range. (In the round-to-nearest mode, the halfway case is rounded toward zero.)

  Note: This is not the single possible definition of the underflow. MPFR chooses to consider the underflow after rounding. The underflow before rounding can also be defined. For instance, consider a function that has the exact result \(7 \times 2^{-1} \times 2^{-1}\), where \(e\) is the smallest exponent (for a significand between 1/2 and 1), with a 2-bit target precision and rounding toward positive infinity. The exact result has the exponent \(e - 1\). With the underflow before rounding, such a function call would yield an underflow, as \(e - 1\) is outside the current exponent range. However, MPFR first considers the rounded result assuming an unbounded
exponent range. The exact result cannot be represented exactly in precision 2, and here, it is rounded to $0.5 \times 2^5$, which is representable in the current exponent range. As a consequence, this will not yield an underflow in MPFR.

- **Overflow (MPFR_FLAGS_OVERFLOW):** An overflow occurs when the exact result of a function is a non-zero real number and the result obtained after the rounding, assuming an unbounded exponent range (for the rounding), has an exponent larger than the maximum value of the current exponent range. In the round-to-nearest mode, the result is infinite. Note: unlike the underflow case, there is only one possible definition of overflow here.

- **Divide-by-zero (MPFR_FLAGS_DIVBY0):** An exact infinite result is obtained from finite inputs.

- **NaN (MPFR_FLAGS_NAN):** A NaN exception occurs when the result of a function is NaN.

- **Inexact (MPFR_FLAGS_INEXACT):** An inexact exception occurs when the result of a function cannot be represented exactly and must be rounded.

- **Range error (MPFR_FLAGS_ERANGE):** A range exception occurs when a function that does not return a MPFR number (such as comparisons and conversions to an integer) has an invalid result (e.g., an argument is NaN in `mpfr_cmp`, or a conversion to an integer cannot be represented in the target type).

Moreover, the group consisting of all the flags is represented by the MPFR_FLAGS_ALL macro (if new flags are added in future MPFR versions, they will be added to this macro too).

Differences with the ISO C99 standard:

- In C, only quiet NaNs are specified, and a NaN propagation does not raise an invalid exception. Unless explicitly stated otherwise, MPFR sets the NaN flag whenever a NaN is generated, even when a NaN is propagated (e.g., in NaN + NaN), as if all NaNs were signaling.

- An invalid exception in C corresponds to either a NaN exception or a range error in MPFR.

### 4.7 Memory Handling

MPFR functions may create caches, e.g., when computing constants such as $\pi$, either because the user has called a function like `mpfr_const_pi` directly or because such a function was called internally by the MPFR library itself to compute some other function. When more precision is needed, the value is automatically recomputed; a minimum of 10% increase of the precision is guaranteed to avoid too many recomputations.

MPFR functions may also create thread-local pools for internal use to avoid the cost of memory allocation. The pools can be freed with `mpfr_free_pool` (but with a default MPFR build, they should not take much memory, as the allocation size is limited).

At any time, the user can free various caches and pools with `mpfr_free_cache` and `mpfr_free_cache2`. It is strongly advised to free thread-local caches before terminating a thread, and all caches before exiting when using tools like `valgrind` (to avoid memory leaks being reported).

MPFR allocates its memory either on the stack (for temporary memory only) or with the same allocator as the one configured for GMP: see Section “Custom Allocation” in GNU MP. This means that the application must make sure that data allocated with the current allocator will not be reallocated or freed with a new allocator. So, in practice, if an application needs to change the allocator with `mp_set_memory_functions`, it should first free all data allocated with the current allocator: for its own data, with `mpfr_clear`, etc.; for the caches and pools, with `mpfr_mp_memory_cleanup` in all threads where MPFR is potentially used. This function is currently equivalent to `mpfr_free_cache`, but `mpfr_mp_memory_cleanup` is the recommended way in case the allocation method changes in the future (for instance, one may choose to allocate the caches for floating-point constants with `malloc` to avoid freeing them if the allocator changes).
Developers should also be aware that MPFR may also be used indirectly by libraries, so that libraries based on MPFR should provide a clean-up function calling `mpfr_mp_memory_cleanup` and/or warn their users about this issue.

Note: For multithreaded applications, the allocator must be valid in all threads where MPFR may be used; data allocated in one thread may be reallocated and/or freed in some other thread.

MPFR internal data such as flags, the exponent range, the default precision, and the default rounding mode are either global (if MPFR has not been compiled as thread safe) or per-thread (thread-local storage, TLS). The initial values of TLS data after a thread is created entirely depend on the compiler and thread implementation (MPFR simply does a conventional variable initialization, the variables being declared with an implementation-defined TLS specifier).

Writers of libraries using MPFR should be aware that the application and/or another library used by the application may also use MPFR, so that changing the exponent range, the default precision, or the default rounding mode may have an effect on this other use of MPFR since these data are not duplicated (unless they are in a different thread). Therefore any such value changed in a library function should be restored before the function returns (unless the purpose of the function is to do such a change). Writers of software using MPFR should also be careful when changing such a value if they use a library using MPFR (directly or indirectly), in order to make sure that such a change is compatible with the library.

### 4.8 Getting the Best Efficiency Out of MPFR

Here are a few hints to get the best efficiency out of MPFR:

- you should avoid allocating and clearing variables. Reuse variables whenever possible, allocate or clear outside of loops, pass temporary variables to subroutines instead of allocating them inside the subroutines;
- use `mpfr_swap` instead of `mpfr_set` whenever possible. This will avoid copying the significands;
- avoid using MPFR from C++, or make sure your C++ interface does not perform unnecessary allocations or copies;
- MPFR functions work in-place: to compute \( a = a + b \) you don’t need an auxiliary variable, you can directly write `mpfr_add(a, a, b, ...)`.
5 MPFR Interface

The floating-point functions expect arguments of type `mpfr_t`.

The MPFR floating-point functions have an interface that is similar to the GNU MP functions. The function prefix for floating-point operations is `mpfr_`.

The user has to specify the precision of each variable. A computation that assigns a variable will take place with the precision of the assigned variable; the cost of that computation should not depend on the precision of variables used as input (on average).

The semantics of a calculation in MPFR is specified as follows: Compute the requested operation exactly (with “infinite accuracy”), and round the result to the precision of the destination variable, with the given rounding mode. The MPFR floating-point functions are intended to be a smooth extension of the IEEE 754 arithmetic. The results obtained on a given computer are identical to those obtained on a computer with a different word size, or with a different compiler or operating system.

MPFR does not keep track of the accuracy of a computation. This is left to the user or to a higher layer (for example, the MPFI library for interval arithmetic). As a consequence, if two variables are used to store only a few significant bits, and their product is stored in a variable with a large precision, then MPFR will still compute the result with full precision.

The value of the standard C macro `errno` may be set to non-zero after calling any MPFR function or macro, whether or not there is an error. Except when documented, MPFR will not set `errno`, but functions called by the MPFR code (libc functions, memory allocator, etc.) may do so.

5.1 Initialization Functions

An `mpfr_t` object must be initialized before storing the first value in it. The functions `mpfr_init` and `mpfr_init2` are used for that purpose.

```c
void mpfr_init2 (mpfr_t x, mpfr_prec_t prec)               [Function]
Initialize x, set its precision to be exactly prec bits and its value to NaN. (Warning: the corresponding MPF function initializes to zero instead.)
```

Normally, a variable should be initialized once only or at least be cleared, using `mpfr_clear`, between initializations. To change the precision of a variable that has already been initialized, use `mpfr_set_prec` or `mpfr_prec_round`; note that if the precision is decreased, the unused memory will not be freed, so that it may be wise to choose a large enough initial precision in order to avoid reallocations. The precision `prec` must be an integer between `MPFR_PREC_MIN` and `MPFR_PREC_MAX` (otherwise the behavior is undefined).

```c
void mpfr_init2 (mpfr_t x, ...)                         [Function]
Initialize all the `mpfr_t` variables of the given variable argument `va_list`, set their precision to be exactly `prec` bits and their value to NaN. See `mpfr_init2` for more details. The `va_list` is assumed to be composed only of type `mpfr_t` (or equivalently `mpfr_ptr`). It begins from `x`, and ends when it encounters a null pointer (whose type must also be `mpfr_ptr`).
```

```c
void mpfr_clear (mpfr_t x)                             [Function]
Free the space occupied by the significand of `x`. Make sure to call this function for all `mpfr_t` variables when you are done with them.
```
void mpfr_clears (mpfr_t x, ...) [Function]
Free the space occupied by all the mpfr_t variables of the given va_list. See mpfr_clear for more details. The va_list is assumed to be composed only of type mpfr_t (or equivalently mpfr_ptr). It begins from x, and ends when it encounters a null pointer (whose type must also be mpfr_ptr).

Here is an example of how to use multiple initialization functions (since NULL is not necessarily defined in this context, we use (mpfr_ptr) 0 instead, but (mpfr_ptr) NULL is also correct).

```c
{
    mpfr_t x, y, z, t;
    mpfr_inits2 (256, x, y, z, t, (mpfr_ptr) 0);
    ...
    mpfr_clears (x, y, z, t, (mpfr_ptr) 0);
}
```

void mpfr_init (mpfr_t x) [Function]
Initialize x, set its precision to the default precision, and set its value to NaN. The default precision can be changed by a call to mpfr_set_default_prec.

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use mpfr_init2.

void mpfr_inits (mpfr_t x, ...) [Function]
Initialize all the mpfr_t variables of the given va_list, set their precision to the default precision and their value to NaN. See mpfr_init for more details. The va_list is assumed to be composed only of type mpfr_t (or equivalently mpfr_ptr). It begins from x, and ends when it encounters a null pointer (whose type must also be mpfr_ptr).

Warning! In a given program, some other libraries might change the default precision and not restore it. Thus it is safer to use mpfr_inits2.

MPFR_DECL_INIT (name, prec) [Macro]
This macro declares name as an automatic variable of type mpfr_t, initializes it and sets its precision to be exactly prec bits and its value to NaN. name must be a valid identifier. You must use this macro in the declaration section. This macro is much faster than using mpfr_init2 but has some drawbacks:

- You must not call mpfr_clear with variables created with this macro (the storage is allocated at the point of declaration and deallocated when the brace-level is exited).
- You cannot change their precision.
- You should not create variables with huge precision with this macro.
- Your compiler must support ‘Non-Constant Initializers’ (standard in C++ and ISO C99) and ‘Token Pasting’ (standard in ISO C89). If prec is not a constant expression, your compiler must support ‘variable-length automatic arrays’ (standard in ISO C99). GCC 2.95.3 and above supports all these features. If you compile your program with GCC in C89 mode and with ‘-pedantic’, you may want to define the MPFR_USE_EXTENSION macro to avoid warnings due to the MPFR_DECL_INIT implementation.

void mpfr_set_default_prec (mpfr_prec_t prec) [Function]
Set the default precision to be exactly prec bits, where prec can be any integer between MPFR_PREC_MIN and MPFR_PREC_MAX. The precision of a variable means the number of bits used to store its significand. All subsequent calls to mpfr_init or mpfr_inits will use this
Chapter 5: MPFR Interface

precision, but previously initialized variables are unaffected. The default precision is set to 53
bits initially.

Note: when MPFR is built with the ‘--enable-thread-safe’ configure option, the default
precision is local to each thread. See Section 4.7 [Memory Handling], page 11, for more
information.

```c
mpfr_prec_t mpfr_get_default_prec (void)  [Function]
Return the current default MPFR precision in bits. See the documentation of mpfr_set_
default_prec.
```

Here is an example on how to initialize floating-point variables:

```c
{
    mpfr_t x, y;
    mpfr_init (x);  /* use default precision */
    mpfr_init2 (y, 256);  /* precision exactly 256 bits */
    ...
    /* When the program is about to exit, do ... */
    mpfr_clear (x);
    mpfr_clear (y);
    mpfr_free_cache ();  /* free the cache for constants like pi */
}
```

The following functions are useful for changing the precision during a calculation. A typical
use would be for adjusting the precision gradually in iterative algorithms like Newton-Raphson,
making the computation precision closely match the actual accurate part of the numbers.

```c
void mpfr_set_prec (mpfr_t x, mpfr_prec_t prec)  [Function]
Set the precision of x to be exactly prec bits, and set its value to NaN. The previous
value stored in x is lost. It is equivalent to a call to mpfr_clear(x) followed by a call to
mpfr_init2(x, prec), but more efficient as no allocation is done in case the current allocated
space for the significand of x is enough. The precision prec can be any integer between
MPFR_PREC_MIN and MPFR_PREC_MAX. In case you want to keep the previous value stored in
x, use mpfr_prec_round instead.

Warning! You must not use this function if x was initialized with MPFRDECL_INIT or with
mpfr_custom_init_set (see Section 5.16 [Custom Interface], page 48).

```c
mpfr_prec_t mpfr_get_prec (mpfr_t x)  [Function]
Return the precision of x, i.e., the number of bits used to store its significand.
```

5.2 Assignment Functions

These functions assign new values to already initialized floats (see Section 5.1 [Initialization
Functions], page 13).

```c
int mpfr_set (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_set_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)  [Function]
int mpfr_set_si (mpfr_t rop, long int op, mpfr_rnd_t rnd)  [Function]
int mpfr_set_uj (mpfr_t rop, uintmax_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_set_sj (mpfr_t rop, intmax_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_set_flt (mpfr_t rop, float op, mpfr_rnd_t rnd)  [Function]
int mpfr_set_d (mpfr_t rop, double op, mpfr_rnd_t rnd)  [Function]
```
Set the value of rop from op, rounded toward the given direction rnd. Note that the input 0 is converted to +0 by mpfr_set_ui, mpfr_set_si, mpfr_set_uj, mpfr_set_sj, mpfr_set_z, mpfr_set_q and mpfr_set_f, regardless of the rounding mode. The mpfr_set_float128 function is built only with the configure option ‘--enable-float128’, which requires the compiler or system provides the ‘_Float128’ data type (GCC 4.3 or later supports this data type); to use mpfr_set_float128, one should define the macro MPFR_WANT_FLOAT128 before including mpfr.h. If the system does not support the IEEE 754 standard, mpfr_set_flt, mpfr_set_d, mpfr_set_ld, mpfr_set_decimal64 and mpfr_set_decimal128 might not preserve the signed zeros. The mpfr_set_decimal64 and mpfr_set_decimal128 functions are built only with the configure option ‘--enable-decimal-float’, and when the compiler or system provides the ‘_Decimal64’ and ‘_Decimal128’ data type; to use those functions, one should define the macro MPFR_WANT_DECIMAL_FLOATS before including mpfr.h. mpfr_set_q might fail if the numerator (or the denominator) cannot be represented as a mpfr_t.

For mpfr_set, the sign of a NaN is propagated in order to mimic the IEEE 754 copy operation. But contrary to IEEE 754, the NaN flag is set as usual.

Note: If you want to store a floating-point constant to a mpfr_t, you should use mpfr_set_str (or one of the MPFR constant functions, such as mpfr_const_pi for π) instead of mpfr_set_flt, mpfr_set_d, mpfr_set_ld, mpfr_set_decimal64 or mpfr_set_decimal128. Otherwise the floating-point constant will be first converted into a reduced-precision (e.g., 53-bit) binary (or decimal, for mpfr_set_decimal64 and mpfr_set_decimal128) number before MPFR can work with it.

Set the value of rop from op × 2^e, rounded toward the given direction rnd. Note that the input 0 is converted to +0.

Set rop to the value of the string s in base base, rounded in the direction rnd. See the documentation of mpfr_strtofr for a detailed description of the valid string formats. Contrary to mpfr_strtofr, mpfr_set_str requires the whole string to represent a valid floating-point number.
The meaning of the return value differs from other MPFR functions: it is 0 if the entire string up to the final null character is a valid number in base \textit{base}; otherwise it is −1, and \textit{rop} may have changed (users interested in the [ternary value], page 9, should use \texttt{mpfr_strtofr} instead).

Note: it is preferable to use \texttt{mpfr_strtofr} if one wants to distinguish between an infinite \textit{rop} value coming from an infinite \textit{s} or from an overflow.

\begin{verbatim}
int mpfr_strtofr(mpfr_t rop, const char *nptr, char **endptr, int base, mpfr_rnd_t rnd)
Read a floating-point number from a string \textit{nptr} in base \textit{base}, rounded in the direction \textit{rnd}; \textit{base} must be either 0 (to detect the base, as described below) or a number from 2 to 62 (otherwise the behavior is undefined). If \textit{nptr} starts with valid data, the result is stored in \textit{rop} and \*\textit{endptr} points to the character just after the valid data (if \textit{endptr} is not a null pointer); otherwise \textit{rop} is set to zero (for consistency with \texttt{strtod}) and the value of \textit{nptr} is stored in the location referenced by \textit{endptr} (if \textit{endptr} is not a null pointer). The usual ternary value is returned.

Parsing follows the standard C \texttt{strtod} function with some extensions. After optional leading whitespace, one has a subject sequence consisting of an optional sign (‘+’ or ‘-’), and either numeric data or special data. The subject sequence is defined as the longest initial subsequence of the input string, starting with the first non-whitespace character, that is of the expected form.

The form of numeric data is a non-empty sequence of significand digits with an optional decimal-point character, and an optional exponent consisting of an exponent prefix followed by an optional sign and a non-empty sequence of decimal digits. A significand digit is either a decimal digit or a Latin letter (62 possible characters), with ‘A’ = 10, ‘B’ = 11, \ldots, ‘Z’ = 35; case is ignored in bases less than or equal to 36, in bases larger than 36, ‘a’ = 36, ‘b’ = 37, \ldots, ‘z’ = 61. The value of a significand digit must be strictly less than the base. The decimal-point character can be either the one defined by the current locale or the period (the first one is accepted for consistency with the C standard and the practice, the second one is accepted to allow the programmer to provide MPFR numbers from strings in a way that does not depend on the current locale). The exponent prefix can be ‘e’ or ‘E’ for bases up to 10, or ‘@’ in any base; it indicates a multiplication by a power of the base. In bases 2 and 16, the exponent prefix can also be ‘p’ or ‘P’, in which case the exponent, called \textit{binary exponent}, indicates a multiplication by a power of 2 instead of the base (there is a difference only for base 16); in base 16 for example ‘1p2’ represents 4 whereas ‘1@2’ represents 256. The value of an exponent is always written in base 10.

If the argument \textit{base} is 0, then the base is automatically detected as follows. If the significand starts with ‘0b’ or ‘0B’, base 2 is assumed. If the significand starts with ‘0x’ or ‘0X’, base 16 is assumed. Otherwise base 10 is assumed.

Note: The exponent (if present) must contain at least a digit. Otherwise the possible exponent prefix and sign are not part of the number (which ends with the significand). Similarly, if ‘0b’, ‘0B’, ‘0x’ or ‘0X’ is not followed by a binary/hexadecimal digit, then the subject sequence stops at the character ‘0’, thus 0 is read.

Special data (for infinities and NaN) can be ‘@inf@’ or ‘@nan@(n-char-sequence-opt)’, and if \textit{base} \leq 16, it can also be ‘infinity’, ‘inf’, ‘nan’ or ‘nan(n-char-sequence-opt)’, all case insensitive with the rules of the C locale. An \textit{n-char-sequence-opt} is a possibly empty string containing only digits, Latin letters and the underscore (0, 1, 2, \ldots, 9, a, b, \ldots, z, A, B, \ldots, Z, _). Note: one has an optional sign for all data, even NaN. For example, ‘-@nAn@(This_Is_Not_17)’ is a valid representation for NaN in base 17.
\end{verbatim}
void mpfr_set_nan (mpfr_t x) [Function]
void mpfr_set_inf (mpfr_t x, int sign) [Function]
void mpfr_set_zero (mpfr_t x, int sign) [Function]

Set the variable x to NaN (Not-a-Number), infinity or zero respectively. In mpfr_set_inf or mpfr_set_zero, x is set to positive infinity (+Inf) or positive zero (+0) iff sign is non-negative; in mpfr_set_nan, the sign bit of the result is unspecified.

void mpfr_swap (mpfr_t x, mpfr_t y) [Function]

Swap the structures pointed to by x and y. In particular, the values are exchanged without rounding (this may be different from three mpfr_set calls using a third auxiliary variable).

Warning! Since the precisions are exchanged, this will affect future assignments. Moreover, since the significand pointers are also exchanged, you must not use this function if the allocation method used for x and/or y does not permit it. This is the case when x and/or y were declared and initialized with MPFR_DECL_INIT, and possibly with mpfr_custom_init_set (see Section 5.16 [Custom Interface], page 48).

5.3 Combined Initialization and Assignment Functions

int mpfr_init_set (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_si (mpfr_t rop, long int op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_d (mpfr_t rop, double op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_ld (mpfr_t rop, long double op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_z (mpfr_t rop, mpz_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_q (mpfr_t rop, mpq_t op, mpfr_rnd_t rnd) [Macro]
int mpfr_init_set_f (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Macro]

Initialize rop and set its value from op, rounded in the direction rnd. The precision of rop will be taken from the active default precision, as set by mpfr_set_default_prec.

int mpfr_init_set_str (mpfr_t x, const char *s, int base, mpfr_rnd_t rnd) [Function]

Initialize x and set its value from the string s in base base, rounded in the direction rnd. See mpfr_set_str.

5.4 Conversion Functions

float mpfr_get_flt (mpfr_t op, mpfr_rnd_t rnd) [Function]
double mpfr_get_d (mpfr_t op, mpfr_rnd_t rnd) [Function]
long double mpfr_get_ld (mpfr_t op, mpfr_rnd_t rnd) [Function]
_Float128 mpfr_get_float128 (mpfr_t op, mpfr_rnd_t rnd) [Function]
_Decimal64 mpfr_get_decimal64 (mpfr_t op, mpfr_rnd_t rnd) [Function]
_Decimal128 mpfr_get_decimal128 (mpfr_t op, mpfr_rnd_t rnd) [Function]

Convert op to a float (respectively double, long double, _Decimal64, or _Decimal128) using the rounding mode rnd. If op is NaN, some fixed NaN (either quiet or signaling) or the result of 0.0/0.0 is returned. If op is ±Inf, an infinity of the same sign or the result of ±1.0/0.0 is returned. If op is zero, these functions return a zero, trying to preserve its sign, if possible. The mpfr_get_float128, mpfr_get_decimal64 and mpfr_get_decimal128 functions are built only under some conditions: see the documentation of mpfr_set_float128, mpfr_set_decimal64 and mpfr_set_decimal128 respectively.
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long int mpfr_get_si (mpfr_t op, mpfr_rnd_t rnd)
unsigned long int mpfr_get_ui (mpfr_t op, mpfr_rnd_t rnd)
intmax_t mpfr_get_sj (mpfr_t op, mpfr_rnd_t rnd)
uintmax_t mpfr_get_uj (mpfr_t op, mpfr_rnd_t rnd)

Convert op to a long int, an unsigned long int, an intmax_t or an uintmax_t (respectively) after rounding it to an integer with respect to rnd. If op is NaN, 0 is returned and the erange flag is set. If op is too big for the return type, the function returns the maximum or the minimum of the corresponding C type, depending on the direction of the overflow; the erange flag is set too. When there is no such range error, if the return value differs from op, i.e., if op is not an integer, the inexact flag is set. See also mpfr_fits_slong_p, mpfr_fits_ulong_p, mpfr_fits_intmax_p and mpfr_fits_uintmax_p.

double mpfr_get_d_2exp (long *exp, mpfr_t op, mpfr_rnd_t rnd)
long double mpfr_get_ld_2exp (long *exp, mpfr_t op, mpfr_rnd_t rnd)

Return d and set exp (formally, the value pointed to by exp) such that 0.5 ≤ |d| < 1 and d × 2exp equals op rounded to double (resp. long double) precision, using the given rounding mode. If op is zero, then a zero of the same sign (or an unsigned zero, if the implementation does not have signed zeros) is returned, and exp is set to 0. If op is NaN or an infinity, then the corresponding double precision (resp. long-double precision) value is returned, and exp is undefined.

int mpfr_frexp (mpfr_exp_t *exp, mpfr_t y, mpfr_t x, mpfr_rnd_t rnd)

Set exp (formally, the value pointed to by exp) and y such that 0.5 ≤ |y| < 1 and y × 2exp equals x rounded to the precision of y, using the given rounding mode. If x is zero, then y is set to a zero of the same sign and exp is set to 0. If x is NaN or an infinity, then y is set to the same value and exp is undefined.

mpfr_t mpfr_get_z_2exp (mpz_t rop, mpfr_t op)

Put the scaled significand of op (regarded as an integer, with the precision of op) into rop, and return the exponent exp (which may be outside the current exponent range) such that op exactly equals rop × 2exp. If op is zero, the minimal exponent emin is returned. If op is NaN or an infinity, the erange flag is set, rop is set to 0, and the minimal exponent emin is returned. The returned exponent may be less than the minimal exponent emin of MPFR numbers in the current exponent range; in case the exponent is not representable in the mpfr_exp_t type, the erange flag is set and the minimal value of the mpfr_exp_t type is returned.

int mpfr_get_z (mpz_t rop, mpfr_t op, mpfr_rnd_t rnd)

Convert op to a mpz_t, after rounding it with respect to rnd. If op is NaN or an infinity, the erange flag is set, rop is set to 0, and 0 is returned. Otherwise the return value is zero when rop is equal to op (i.e., when op is an integer), positive when it is greater than op, and negative when it is smaller than op; moreover, if rop differs from op, i.e., if op is not an integer, the inexact flag is set.

void mpfr_get_q (mpq_t rop, mpfr_t op)

Convert op to a mpq_t. If op is NaN or an infinity, the erange flag is set and rop is set to 0. Otherwise the conversion is always exact.

int mpfr_get_f (mpf_t rop, mpfr_t op, mpfr_rnd_t rnd)

Convert op to a mpf_t, after rounding it with respect to rnd. The erange flag is set if op is NaN or an infinity, which do not exist in MPF. If op is NaN, then rop is undefined. If op is
+Inf (resp. −Inf), then rop is set to the maximum (resp. minimum) value in the precision of the MPF number; if a future MPF version supports infinities, this behavior will be considered incorrect and will change (portable programs should assume that rop is set either to this finite number or to an infinite number). Note that since MPFR currently has the same exponent type as MPF (but not with the same radix), the range of values is much larger in MPF than in MPFR, so that an overflow or underflow is not possible.

**Function**

```c
size_t mpfr_get_str_ndigits (int b, mpfr_prec_t p)
```

Return the minimal integer \( m \) such that any number of \( p \) bits, when output with \( m \) digits in radix \( b \) with rounding to nearest, can be recovered exactly when read again, still with rounding to nearest. More precisely, we have \( m = 1 + \lceil \frac{\log 2}{\log b} \rceil \), with \( p \) replaced by \( p - 1 \) if \( b \) is a power of 2.

The argument \( b \) must be in the range 2 to 62; this is the range of bases supported by the mpfr_get_str function. Note that contrary to the base argument of this function, negative values are not accepted.

**Function**

```c
char * mpfr_get_str (char *str, mpfr_exp_t *expptr, int base, size_t n, mpfr_t op, mpfr_rnd_t rnd)
```

Convert \( op \) to a string of digits in base \([base]\), with rounding in the direction \( rnd \), where \( n \) is either zero (see below) or the number of significant digits output in the string. The argument \( base \) may vary from 2 to 62 or from \(-2 \) to \(-36 \); otherwise the function does nothing and immediately returns a null pointer.

For \( base \) in the range 2 to 36, digits and lower-case letters are used; for \(-2 \) to \(-36 \), digits and upper-case letters are used; for 37 to 62, digits, upper-case letters, and lower-case letters, in that significance order, are used. Warning! This implies that for \( base > 10 \), the successor of the digit 9 depends on \( base \). This choice has been done for compatibility with GMP’s mpfr_get_str function. Users who wish a more consistent behavior should write a simple wrapper.

If the input is NaN, then the returned string is ‘@NaN@’ and the NaN flag is set. If the input is +Inf (resp. −Inf), then the returned string is ‘@Inf@’ (resp. ‘−@Inf@’).

If the input number is a finite number, the exponent is written through the pointer expptr (for input 0, the current minimal exponent is written); the type mpfr_exp_t is large enough to hold the exponent in all cases.

The generated string is a fraction, with an implicit radix point immediately to the left of the first digit. For example, the number −3.1416 would be returned as ‘−31416’ in the string and 1 written at expptr. If \( rnd \) is to nearest, and \( op \) is exactly in the middle of two consecutive possible outputs, the one with an even significand is chosen, where both significands are considered with the exponent of \( op \). Note that for an odd base, this may not correspond to an even last digit: for example, with 2 digits in base 7, (14) and a half is rounded to (15), which is 12 in decimal, (16) and a half is rounded to (20), which is 14 in decimal, and (26) and a half is rounded to (26), which is 20 in decimal.

If \( n \) is zero, the number of digits of the significand is taken as mpfr_get_str_ndigits (\( base, p \)), where \( p \) is the precision of \( op \) (see [mpfr_get_str_ndigits], page 20).

If \( str \) is a null pointer, space for the significand is allocated using the allocation function (see Section 4.7 [Memory Handling], page 11) and a pointer to the string is returned (unless the base is invalid). To free the returned string, you must use mpfr_free_str.
If \( \text{str} \) is not a null pointer, it should point to a block of storage large enough for the significand. A safe block size (sufficient for any value) is \( \max(n + 2, 7) \) if \( n \) is not zero; if \( n \) is zero, replace it by \( \text{mpfr_get_str\_ndigits(base, p)} \), where \( p \) is the precision of \( \text{op} \), as mentioned above. The extra two bytes are for a possible minus sign, and for the terminating null character, and the value 7 accounts for ‘-@Inf@’ plus the terminating null character. The pointer to the string \( \text{str} \) is returned (unless the base is invalid).

Like in usual functions, the inexact flag is set iff the result is inexact.

```c
void mpfr_free_str (char *str) // [Function]
Free a string allocated by mpfr_get_str using the unallocation function (see Section 4.7 [Memory Handling], page 11). The block is assumed to be strlen(str)+1 bytes.
```

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>int mpfr_fits_ulong_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>Return non-zero if ( \text{op} ) would fit in the respective C data type, respectively unsigned long int, long int, unsigned int, int, unsigned short, short, uintmax_t, intmax_t, when rounded to an integer in the direction ( \text{rnd} ). For instance, with the MPFR_RNDU rounding mode on (-0.5), the result will be non-zero for all these functions. For MPFR_RNDF, those functions return non-zero when it is guaranteed that the corresponding conversion function (for example mpfr_get_ui for mpfr_fits_ulong_p), when called with faithful rounding, will always return a number that is representable in the corresponding type. As a consequence, for MPFR_RNDF, mpfr_fits_ulong_p will return non-zero for a non-negative number less than or equal to ULONG_MAX.</td>
</tr>
<tr>
<td>int mpfr_fits_slong_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_fits_uint_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_fits_sint_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_fits_ushort_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_fits_sshort_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_fits_uintmax_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_fits_intmax_p (mpfr_t op, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
</tbody>
</table>

5.5 Arithmetic Functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>int mpfr_add (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_add_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_add_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_add_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_add_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
<tr>
<td>int mpfr_add_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)</td>
<td>[Function]</td>
</tr>
</tbody>
</table>

Set \( \text{rop} \) to \( \text{op1} + \text{op2} \) rounded in the direction \( \text{rnd} \). The IEEE 754 rules are used, in particular for signed zeros. But for types having no signed zeros, 0 is considered unsigned (i.e., \((+0) + 0 = (+0)\) and \((-0) + 0 = (-0)\)). The mpfr_add_d function assumes that the radix of the double type is a power of 2, with a precision at most that declared by the C implementation (macro IEEE_DBL_MANT_DIG, and if not defined 53 bits).
int mpfr_sub (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_ui_sub (mpfr_t rop, unsigned long int op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_sub_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_si_sub (mpfr_t rop, long int op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_sub_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_d_sub (mpfr_t rop, double op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_sub_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd) [Function]
int mpfr_z_sub (mpfr_t rop, mpz_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_sub_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_sub_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd) [Function]

Set rop to op1 − op2 rounded in the direction rnd. The IEEE 754 rules are used, in particular for signed zeros. But for types having no signed zeros, 0 is considered unsigned (i.e., (+0) − 0 = (+0), (−0) − 0 = (−0), 0 − (+0) = (−0) and 0 − (−0) = (+0)). The same restrictions as for mpfr_add_d apply to mpfr_d_sub and mpfr_sub_d.

int mpfr_mul (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_mul_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_mul_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_mul_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd) [Function]
int mpfr_mul_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_mul_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd) [Function]

Set rop to op1 × op2 rounded in the direction rnd. When a result is zero, its sign is the product of the signs of the operands (for types having no signed zeros, 0 is considered positive). The same restrictions as for mpfr_add_d apply to mpfr_mul_d.

int mpfr_sqr (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]

Set rop to op^2 rounded in the direction rnd.

int mpfr_div (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_ui_div (mpfr_t rop, unsigned long int op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_div_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_si_div (mpfr_t rop, long int op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
int mpfr_div_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)
int mpfr_d_div (mpfr_t rop, double op1, mpfr_t op2, mpfr_rnd_t rnd)
int mpfr_div_d (mpfr_t rop, mpfr_t op1, double op2, mpfr_rnd_t rnd)
int mpfr_div_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)
int mpfr_div_q (mpfr_t rop, mpfr_t op1, mpq_t op2, mpfr_rnd_t rnd)

Set rop to op1/op2 rounded in the direction rnd. When a result is zero, its sign is the product of the signs of the operands. For types having no signed zeros, 0 is considered positive; but note that if op1 is non-zero and op2 is zero, the result might change from ±Inf to NaN in future MPFR versions if there is an opposite decision on the IEEE 754 side. The same restrictions as for mpfr_add_d apply to mpfr_d_div and mpfr_div_d.

int mpfr_sqrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
int mpfr_sqrt_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)

Set rop to $\sqrt{op}$ rounded in the direction rnd. Set rop to $-0$ if op is $-0$, to be consistent with the IEEE 754 standard (thus this differs from mpfr_rootn_ui with $n = 2$). Set rop to NaN if op is negative.

int mpfr_rec_sqrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)

Set rop to $1/\sqrt{op}$ rounded in the direction rnd. Set rop to $+Inf$ if op is $\pm 0$, $+0$ if op is $+Inf$, and NaN if op is negative. Warning! Therefore the result on $-0$ is different from the one of the rSqrt function recommended by the IEEE 754 standard (Section 9.2.1), which is $-Inf$ instead of $+Inf$.

int mpfr_cbrt (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)
int mpfr_rootn_ui (mpfr_t rop, mpfr_t op, unsigned long int n, mpfr_rnd_t rnd)

Set rop to the n-th root (with $n = 3$, the cubic root, for mpfr_cbrt) of op rounded in the direction rnd. For $n = 0$, set rop to NaN. For $n$ odd (resp. even) and op negative (including $-Inf$), set rop to a negative number (resp. NaN). If op is zero, set rop to zero with the sign obtained by the usual limit rules, i.e., the same sign as op if n is odd, and positive if n is even.

These functions agree with the rootn operation of the IEEE 754 standard. Note that it is here restricted to $n \geq 0$. Functions allowing a negative n may be implemented in the future.

int mpfr_root (mpfr_t rop, mpfr_t op, unsigned long int n, mpfr_rnd_t rnd)

This function is the same as mpfr_rootn_ui except when op is $-0$ and n is even: the result is $-0$ instead of $+0$ (the reason was to be consistent with mpfr_sqrt). Said otherwise, if op is zero, set rop to op.

This function predates IEEE 754-2008, where rootn was introduced, and behaves differently from the IEEE 754 rootn operation. It is marked as deprecated and will be removed in a future release.
int mpfr_neg (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  
Set rop to \(-op\) and the absolute value of \(op\) respectively, rounded in the direction \(rnd\). Just changes or adjusts the sign if \(rop\) and \(op\) are the same variable, otherwise a rounding might occur if the precision of \(rop\) is less than that of \(op\).

The sign rule also applies to NaN in order to mimic the IEEE 754 negate and abs operations, i.e., for mpfr_neg, the sign is reversed, and for mpfr_abs, the sign is set to positive. But contrary to IEEE 754, the NaN flag is set as usual.

int mpfr_abs (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  

int mpfr_dim (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)  
Set rop to the positive difference of \(op1\) and \(op2\), i.e., \(op1 - op2\) rounded in the direction \(rnd\) if \(op1 > op2\), \(+0\) if \(op1 \leq op2\), and NaN if \(op1\) or \(op2\) is NaN.

int mpfr_mul_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)  
int mpfr_mul_2si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)  
Set rop to \(op1 \times 2^{op2}\) rounded in the direction \(rnd\). Just increases the exponent by \(op2\) when \(rop\) and \(op1\) are identical.

int mpfr_div_2ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)  
int mpfr_div_2si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)  
Set rop to \(op1/2^{op2}\) rounded in the direction \(rnd\). Just decreases the exponent by \(op2\) when \(rop\) and \(op1\) are identical.

int mpfr_fac_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)  
Set rop to the factorial of \(op\), rounded in the direction \(rnd\).

int mpfr_fma (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_rnd_t rnd)  
int mpfr_fms (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_rnd_t rnd)  
Set rop to \((op1 \times op2) + op3\) (resp. \((op1 \times op2) - op3\)) rounded in the direction \(rnd\). Concerning special values (signed zeros, infinities, NaN), these functions behave like a multiplication followed by a separate addition or subtraction. That is, the fused operation matters only for rounding.

int mpfr_fmma (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_t op4, mpfr_rnd_t rnd)  
int mpfr_fmms (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_t op3, mpfr_t op4, mpfr_rnd_t rnd)  
Set rop to \((op1 \times op2) + (op3 \times op4)\) (resp. \((op1 \times op2) - (op3 \times op4)\)) rounded in the direction \(rnd\). In case the computation of \(op1 \times op2\) overflows or underflows (or that of \(op3 \times op4\)), the result \(rop\) is computed as if the two intermediate products were computed with rounding toward zero.
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int mpfr_hypot (mpfr_t rop, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)
Set rop to the Euclidean norm of x and y, i.e., $\sqrt{x^2 + y^2}$, rounded in the direction rnd. Special values are handled as described in the ISO C99 (Section F.9.4.3) and IEEE 754 (Section 9.2.1) standards: If x or y is an infinity, then +1Inf is returned in rop, even if the other number is NaN.

int mpfr_sum (mpfr_t rop, const mpfr_ptr tab[], unsigned long n, mpfr_rnd_t rnd)
Set rop to the sum of all elements of tab, whose size is n, correctly rounded in the direction rnd. Warning: for efficiency reasons, tab is an array of pointers to mpfr_t, not an array of mpfr_t. If n = 0, then the result is +0, and if n = 1, then the function is equivalent to mpfr_set. For the special exact cases, the result is the same as the one obtained with a succession of additions (mpfr_add) in infinite precision. In particular, if the result is an exact zero and n ≥ 1:

- if all the inputs have the same sign (i.e., all +0 or all −0), then the result has the same sign as the inputs;
- otherwise, either because all inputs are zeros with at least a +0 and a −0, or because some inputs are non-zero (but they globally cancel), the result is +0, except for the MPFR_RNDD rounding mode, where it is −0.

int mpfr_dot (mpfr_t rop, const mpfr_ptr a[], const mpfr_ptr b[], unsigned long int n, mpfr_rnd_t rnd)
Set rop to the dot product of elements of a by those of b, whose common size is n, correctly rounded in the direction rnd. Warning: for efficiency reasons, a and b are arrays of pointers to mpfr_t. This function is experimental, and does not yet handle intermediate overflows and underflows.

For the power functions (with an integer exponent or not), see [mpfr_pow], page 27, in Section 5.7 [Transcendental Functions], page 27.

5.6 Comparison Functions

int mpfr_cmp (mpfr_t op1, mpfr_t op2)     [Function]
int mpfr_cmp_ui (mpfr_t op1, unsigned long int op2)  [Function]
int mpfr_cmp_si (mpfr_t op1, long int op2)     [Function]
int mpfr_cmp_d (mpfr_t op1, double op2)        [Function]
int mpfr_cmp_ld (mpfr_t op1, long double op2)  [Function]
int mpfr_cmp_z (mpfr_t op1, mpz_t op2)         [Function]
int mpfr_cmp_q (mpfr_t op1, mpq_t op2)         [Function]
int mpfr_cmp_f (mpfr_t op1, mpf_t op2)         [Function]

Compare op1 and op2. Return a positive value if op1 > op2, zero if op1 = op2, and a negative value if op1 < op2. Both op1 and op2 are considered to their full own precision, which may differ. If one of the operands is NaN, set the erange flag and return zero.

Note: These functions may be useful to distinguish the three possible cases. If you need to distinguish two cases only, it is recommended to use the predicate functions (e.g., mpfr_equal_p for the equality) described below; they behave like the IEEE 754 comparisons, in particular when one or both arguments are NaN. But only floating-point numbers can be compared (you may need to do a conversion first).
int mpfr_cmp_ui_2exp (mpfr_t op1, unsigned long int op2, mpfr_exp_t e)

int mpfr_cmp_si_2exp (mpfr_t op1, long int op2, mpfr_exp_t e)

Compare $op1$ and $op2 \times 2^e$. Similar as above.

int mpfr_cmpabs (mpfr_t op1, mpfr_t op2)
int mpfr_cmpabs_ui (mpfr_t op1, unsigned long int op2)

Compare $|op1|$ and $|op2|$. Return a positive value if $|op1| > |op2|$, zero if $|op1| = |op2|$, and a negative value if $|op1| < |op2|$. If one of the operands is NaN, set the erange flag and return zero.

int mpfr_nan_p (mpfr_t op)
int mpfr_inf_p (mpfr_t op)
int mpfr_number_p (mpfr_t op)
int mpfr_zero_p (mpfr_t op)
int mpfr_regular_p (mpfr_t op)

Return non-zero if $op$ is respectively NaN, an infinity, an ordinary number (i.e., neither NaN nor an infinity), zero, or a regular number (i.e., neither NaN, nor an infinity nor zero). Return zero otherwise.

int mpfr_sgn (mpfr_t op)

Return a positive value if $op > 0$, zero if $op = 0$, and a negative value if $op < 0$. If the operand is NaN, set the erange flag and return zero. This is equivalent to mpfr_cmp_ui ($op$, 0), but more efficient.

int mpfr_greater_p (mpfr_t op1, mpfr_t op2)
int mpfr_greaterequal_p (mpfr_t op1, mpfr_t op2)
int mpfr_less_p (mpfr_t op1, mpfr_t op2)
int mpfr_lessequal_p (mpfr_t op1, mpfr_t op2)
int mpfr_equal_p (mpfr_t op1, mpfr_t op2)

Return non-zero if $op1 > op2$, $op1 \geq op2$, $op1 < op2$, $op1 \leq op2$, $op1 = op2$ respectively, and zero otherwise. Those functions return zero whenever $op1$ and/or $op2$ is NaN.

int mpfr_lessgreater_p (mpfr_t op1, mpfr_t op2)

Return non-zero if $op1 < op2$ or $op1 > op2$ (i.e., neither $op1$, nor $op2$ is NaN, and $op1 \neq op2$), zero otherwise (i.e., $op1$ and/or $op2$ is NaN, or $op1 = op2$).

int mpfr_unordered_p (mpfr_t op1, mpfr_t op2)

Return non-zero if $op1$ or $op2$ is a NaN (i.e., they cannot be compared), zero otherwise.

int mpfr_total_order_p (mpfr_t x, mpfr_t y)

This function implements the totalOrder predicate from IEEE 754, where $-\text{NaN} < -\text{Inf} < \text{negative finite numbers} < -0 < +0 < \text{positive finite numbers} < +\text{Inf} < +\text{NaN}$. It returns a non-zero value (true) when $x$ is smaller than or equal to $y$ for this order relation, and zero (false) otherwise. Contrary to mpfr_cmp ($x$, $y$), which returns a ternary value, mpfr_total_order_p returns a binary value (zero or non-zero). In particular, mpfr_total_order_p ($x$, $x$) returns true, mpfr_total_order_p ($-0$, $+0$) returns true and mpfr_total_order_p ($+0$, $-0$) returns false. The sign bit of NaN also matters.
5.7 Transcendental Functions

All those functions, except explicitly stated (for example mpfr_sin_cos), return a [ternary value], page 9, i.e., zero for an exact return value, a positive value for a return value larger than the exact result, and a negative value otherwise.

Important note: In some domains, computing transcendental functions (even more with correct rounding) is expensive, even in small precision, for example the trigonometric and Bessel functions does not depend only on the output precision: for instance, the memory usage of mpfr_rootn_ui is also linear in the argument \( k \), and the memory usage of the incomplete Gamma function also depends on the precision of the input \( op \). It is also theoretically possible that some functions on some particular inputs might be very hard to round (i.e. the Table Maker’s Dilemma occurs in much larger precisions than normally expected from the context), meaning that the internal precision needs to be increased even more; but it is conjectured that the needed precision has a reasonable bound.

\[
\begin{align*}
\text{int mpfr_log (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_log_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_log2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_log10 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\end{align*}
\]

Set \( rop \) to the natural logarithm of \( op \), \( \log_2 op \) or \( \log_{10} op \), respectively, rounded in the direction \( rnd \). Set \( rop \) to +0 if \( op \) is 1 (in all rounding modes), for consistency with the ISO C99 and IEEE 754 standards. Set \( rop \) to \(-\infty \) if \( op \) is ±0 (i.e., the sign of the zero has no influence on the result).

\[
\begin{align*}
\text{int mpfr_log1p (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_exp (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_exp2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_exp10 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_expm1 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_pow (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_pow_ui (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_pow_si (mpfr_t rop, mpfr_t op1, long int op2, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_pow_z (mpfr_t rop, mpfr_t op1, mpz_t op2, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_ui_pow_ui (mpfr_t rop, unsigned long int op1, unsigned long int op2, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\text{int mpfr_ui_pow (mpfr_t rop, unsigned long int op1, mpfr_t op2, mpfr_rnd_t rnd)} & \quad \text{[Function]} \\
\end{align*}
\]

Set \( rop \) to \( e^{op} - 1 \), rounded in the direction \( rnd \). Special values are handled as described in the ISO C99 and IEEE 754 standards for the \texttt{pow} function:
- \texttt{pow}(\pm 0, y) \text{ returns } \pm \infty \text{ for } y \text{ a negative odd integer.}
- \texttt{pow}(\pm 0, y) \text{ returns } +\infty \text{ for } y \text{ negative and not an odd integer.}
- \texttt{pow}(\pm 0, y) \text{ returns } \pm 0 \text{ for } y \text{ a positive odd integer.}
- \texttt{pow}(\pm 0, y) \text{ returns } +0 \text{ for } y \text{ positive and not an odd integer.}
- \texttt{pow}(-1, \pm \infty) \text{ returns } 1.
- \texttt{pow}(+1, y) \text{ returns } 1 \text{ for any } y, \text{ even a NaN.}
- \texttt{pow}(x, y) \text{ returns NaN for finite negative } x \text{ and finite non-integer } y.
- \texttt{pow}(x, -\infty) \text{ returns } +\infty \text{ for } 0 < |x| < 1, \text{ and } +0 \text{ for } |x| > 1.
- \texttt{pow}(x, +\infty) \text{ returns } +0 \text{ for } 0 < |x| < 1, \text{ and } +\infty \text{ for } |x| > 1.
- \texttt{pow}(-\infty, y) \text{ returns } -0 \text{ for } y \text{ a negative odd integer.}
- \texttt{pow}(-\infty, y) \text{ returns } +0 \text{ for } y \text{ negative and not an odd integer.}
- \texttt{pow}(-\infty, y) \text{ returns } -\infty \text{ for } y \text{ a positive odd integer.}
- \texttt{pow}(-\infty, y) \text{ returns } +\infty \text{ for } y \text{ positive and not an odd integer.}
- \texttt{pow}(+\infty, y) \text{ returns } +0 \text{ for } y \text{ negative, } +\infty \text{ for } y \text{ positive.}

Note: When 0 is of integer type, it is regarded as +0 by these functions. We do not use the usual limit rules in this case, as these rules are not used for \texttt{pow}.

\begin{verbatim}
int mpfr_cos (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_sin (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_tan (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
  Set \textit{rop} to the cosine of \textit{op}, sine of \textit{op}, tangent of \textit{op}, rounded in the direction \textit{rnd}.

int mpfr_sin_cos (mpfr_t sop, mpfr_t cop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
  Set simultaneously \textit{sop} to the sine of \textit{op} and \textit{cop} to the cosine of \textit{op}, rounded in the direction \textit{rnd} with the corresponding precisions of \textit{sop} and \textit{cop}, which must be different variables. Return 0 iff both results are exact, more precisely it returns \( s + 4c \) where \( s = 0 \) if \textit{sop} is exact, \( s = 1 \) if \textit{sop} is larger than the sine of \textit{op}, \( s = 2 \) if \textit{sop} is smaller than the sine of \textit{op}, and similarly for \textit{c} and the cosine of \textit{op}.

int mpfr_sec (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_csc (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_cot (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
  Set \textit{rop} to the secant of \textit{op}, cosecant of \textit{op}, cotangent of \textit{op}, rounded in the direction \textit{rnd}.

int mpfr_acos (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_asin (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
int mpfr_atan (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  [Function]
  Set \textit{rop} to the arc-cosine, arc-sine or arc-tangent of \textit{op}, rounded in the direction \textit{rnd}. Note that since \texttt{acos}(-1) returns the floating-point number closest to \( \pi \) according to the given rounding mode, this number might not be in the output range \( 0 \leq \textit{rop} < \pi \) of the arc-cosine function; still, the result lies in the image of the output range by the rounding function. The same holds for \texttt{asin}(-1), \texttt{asin}(1), \texttt{atan}(-\infty), \texttt{atan}(+\infty) or for \texttt{atan}(\textit{op}) with large \textit{op} and small precision of \textit{rop}.

int mpfr_atan2 (mpfr_t rop, mpfr_t y, mpfr_t x, mpfr_rnd_t rnd)  [Function]
  Set \textit{rop} to the arc-tangent2 of \textit{y} and \textit{x}, rounded in the direction \textit{rnd}: if \( x > 0 \), then \texttt{atan2}(\textit{y}, \textit{x}) returns \texttt{atan}(\textit{y}/\textit{x}); if \( x < 0 \), then \texttt{atan2}(\textit{y}, \textit{x}) returns \texttt{sign}(\textit{y}) \times (\pi - \texttt{atan}(|\textit{y}/|\textit{x}|)), thus a
number from $-\pi$ to $\pi$. As for \texttt{atan}, in case the exact mathematical result is $+\pi$ or $-\pi$, its rounded result might be outside the function output range.

\texttt{atan2}(y, 0) does not raise any floating-point exception. Special values are handled as described in the ISO C99 and IEEE 754 standards for the \texttt{atan2} function:

- \texttt{atan2}(+0, -0) returns $+\pi$.
- \texttt{atan2}(-0, -0) returns $-\pi$.
- \texttt{atan2}(+0, +0) returns $+0$.
- \texttt{atan2}(+0, x) returns $+\pi$ for $x < 0$.
- \texttt{atan2}(-0, x) returns $-\pi$ for $x < 0$.
- \texttt{atan2}(+0, x) returns $+0$ for $x > 0$.
- \texttt{atan2}(-0, x) returns $-0$ for $x > 0$.
- \texttt{atan2}(y, 0) returns $-\pi/2$ for $y < 0$.
- \texttt{atan2}(y, 0) returns $+\pi/2$ for $y > 0$.
- \texttt{atan2}(+Inf, -Inf) returns $+3\pi/4$.
- \texttt{atan2}(-Inf, -Inf) returns $-3\pi/4$.
- \texttt{atan2}(+Inf, +Inf) returns $+\pi/4$.
- \texttt{atan2}(-Inf, +Inf) returns $-\pi/4$.
- \texttt{atan2}(y, 0) returns $-\pi/2$ for finite $x$.
- \texttt{atan2}(y, -Inf) returns $+\pi$ for finite $y > 0$.
- \texttt{atan2}(y, -Inf) returns $-\pi$ for finite $y < 0$.
- \texttt{atan2}(y, +Inf) returns $+0$ for finite $y > 0$.
- \texttt{atan2}(y, +Inf) returns $-0$ for finite $y < 0$.

\begin{verbatim}
int mpfr_cosh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_sinh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_tanh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]

int mpfr_sin_cosh (mpfr_t sop, mpfr_t cop, mpfr_t op,
    mpfr_rnd_t rnd) [Function]

int mpfr_sech (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_csch (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_coth (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]

int mpfr_acosh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_asinh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_atanh (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
\end{verbatim}

Set \texttt{rop} to the hyperbolic cosine, sine or tangent of \texttt{op}, rounded in the direction \texttt{rnd}.

Set simultaneously \texttt{sop} to the hyperbolic sine of \texttt{op} and \texttt{cop} to the hyperbolic cosine of \texttt{op}, rounded in the direction \texttt{rnd} with the corresponding precision of \texttt{sop} and \texttt{cop}, which must be different variables. Return 0 if both results are exact (see \texttt{mpfr_sin_cosh} for a more detailed description of the return value).

Set \texttt{rop} to the hyperbolic secant of \texttt{op}, cosecant of \texttt{op}, cotangent of \texttt{op}, rounded in the direction \texttt{rnd}.

Set \texttt{rop} to the inverse hyperbolic cosine, sine or tangent of \texttt{op}, rounded in the direction \texttt{rnd}.
int mpfr_eint (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  
Set rop to the exponential integral of op, rounded in the direction rnd. This is the sum of Euler’s constant, of the logarithm of the absolute value of op, and of the sum for k from 1 to infinity of \( \frac{op^k}{k \cdot k!} \). For positive op, it corresponds to the Ei function at op (see formula 5.1.10 from the Handbook of Mathematical Functions from Abramowitz and Stegun), and for negative op, to the opposite of the E1 function (sometimes called eint1) at \(-op\) (formula 5.1.1 from the same reference).

int mpfr_li2 (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  
Set rop to real part of the dilogarithm of op, rounded in the direction rnd. MPFR defines the dilogarithm function as \( -\int_{t=0}^{op} \frac{\log(1-t)}{t} \, dt \).

int mpfr_gamma (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  
int mpfr_gamma_inc (mpfr_t rop, mpfr_t op, mpfr_t op2, mpfr_rnd_t rnd)  
Set rop to the value of the Gamma function on op, resp. the incomplete Gamma function on op and op2, rounded in the direction rnd. (In the literature, mpfr_gamma_inc is called upper incomplete Gamma function, or sometimes complementary incomplete Gamma function.) For mpfr_gamma (and mpfr_gamma_inc when op2 is zero), when op is a negative integer, rop is set to NaN. Note: the current implementation of mpfr_gamma_inc is slow for large values of rop or op, in which case some internal overflow might also occur.

int mpfr_lngamma (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  
int mpfr_lgamma (mpfr_t rop, int *signp, mpfr_t op, mpfr_rnd_t rnd)  
Set rop to the value of the logarithm of the Gamma function on op, rounded in the direction rnd. The sign \( (1 \text{ or } -1) \) of Gamma(op) is returned in the object pointed to by signp. When op is 1 or 2, set rop to +0 (in all rounding modes). When op is an infinity or a non-positive integer, set rop to +Inf, following the general rules on special values. When \(-2k-1 < op < -2k\), k being a non-negative integer, set rop to NaN. See also mpfr_lngamma.

int mpfr_zeta (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd)  
int mpfr_zeta_ui (mpfr_t rop, unsigned long int op, mpfr_rnd_t rnd)  
Set rop to the value of the Riemann Zeta function on op, rounded in the direction rnd.
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Set rop to the value of the error function on op (resp. the complementary error function on op) rounded in the direction rnd.

Set rop to the value of the first kind Bessel function of order 0, (resp. 1 and n) on op, rounded in the direction rnd. When op is NaN, rop is always set to NaN. When op is positive or negative infinity, rop is set to +0. When op is zero, and n is not zero, rop is set to +0 or −0 depending on the parity and sign of n, and the sign of op.

Set rop to the value of the second kind Bessel function of order 0 (resp. 1 and n) on op, rounded in the direction rnd. When op is NaN or negative, rop is always set to NaN. When op is +Inf, rop is set to +0. When op is zero, rop is set to +Inf or −Inf depending on the parity and sign of n.

Set rop to the arithmetic-geometric mean of op1 and op2, rounded in the direction rnd. The arithmetic-geometric mean is the common limit of the sequences \( u_n \) and \( v_n \), where \( u_0 = op1 \), \( u_0 = op2 \), \( u_{n+1} \) is the arithmetic mean of \( u_n \) and \( v_n \), and \( v_{n+1} \) is the geometric mean of \( u_n \) and \( v_n \). If any operand is negative and the other one is not zero, set rop to NaN. If any operand is zero and the other one is finite (resp. infinite), set rop to +0 (resp. NaN).

Set rop to the value of the Airy function Ai on x, rounded in the direction rnd. When x is NaN, rop is always set to NaN. When x is +Inf or −Inf, rop is +0. The current implementation is not intended to be used with large arguments. It works with \( |x| \) typically smaller than 500. For larger arguments, other methods should be used and will be implemented in a future version.

Set rop to the logarithm of 2, the value of \( \pi \), of Euler’s constant 0.577 . . ., of Catalan’s constant 0.915 . . ., respectively, rounded in the direction rnd. These functions cache the computed values to avoid other calculations if a lower or equal precision is requested. To free these caches, use mpfr_free_cache or mpfr_free_cache2.

5.8 Input and Output Functions

This section describes functions that perform input from an input/output stream, and functions that output to an input/output stream. Passing a null pointer for a stream to any of these functions will make them read from stdin and write to stdout, respectively.
When using a function that takes a `FILE *` argument, you must include the `<stdio.h>` standard header before `mpfr.h`, to allow `mpfr.h` to define prototypes for these functions.

### mpfr_out_str

`size_t mpfr_out_str(FILE *stream, int base, size_t n, mpfr_t op, mpfr_rnd_t rnd)`  
Output `op` on stream `stream` as a text string in base `|base|`, rounded in the direction `rnd`. The base may vary from 2 to 62 or from −2 to −36 (any other value yields undefined behavior). The argument `n` has the same meaning as in `mpfr_get_str` (see `mpfr_get_str`, page 20): Print `n` significant digits exactly, or if `n` is 0, the number `mpfr_get_str_ndigits(base, p)`, where `p` is the precision of `op` (see `mpfr_get_str_ndigits`, page 20).

If the input is NaN, +Inf, −Inf, +0, or −0, then ’@NaN@’, '@Inf@', ‘−@Inf@’, ‘0’, or ‘−0’ is output, respectively.

For the regular numbers, the format of the output is the following: the most significant digit, then a decimal-point character (defined by the current locale), then the remaining `n − 1` digits (including trailing zeros), then the exponent prefix, then the exponent in decimal. The exponent prefix is ‘e’ when `|base| ≤ 10`, and ‘@’ when `|base| > 10`. See `mpfr_get_str`, page 20, for information on the digits depending on the base.

Return the number of characters written, or if an error occurred, return 0.

### mpfr_inp_str

`size_t mpfr_inp_str(mpfr_t rop, FILE *stream, int base, mpfr_rnd_t rnd)`  
Input a string in base `base` from stream `stream`, rounded in the direction `rnd`, and put the read float in `rop`.

This function reads a word (defined as a sequence of characters between whitespace) and parses it using `mpfr_set_str`. See the documentation of `mpfr_strtofr` for a detailed description of the valid string formats.

Return the number of bytes read, or if an error occurred, return 0.

### mpfr_fpif_export

`int mpfr_fpif_export(FILE *stream, mpfr_t op)`  
Export the number `op` to the stream `stream` in a floating-point interchange format. In particular one can export on a 32-bit computer and import on a 64-bit computer, or export on a little-endian computer and import on a big-endian computer. The precision of `op` and the sign bit of a NaN are stored too. Return 0 iff the export was successful.

Note: this function is experimental and its interface might change in future versions.

### mpfr_fpif_import

`int mpfr_fpif_import(mpfr_t op, FILE *stream)`  
Import the number `op` from the stream `stream` in a floating-point interchange format (see `mpfr_fpif_export`). Note that the precision of `op` is set to the one read from the stream, and the sign bit is always retrieved (even for NaN). If the stored precision is zero or greater than `MPFR_PREC_MAX`, the function fails (it returns non-zero) and `op` is unchanged. If the function fails for another reason, `op` is set to NaN and it is unspecified whether the precision of `op` has changed to the one read from the file. Return 0 iff the import was successful.

Note: this function is experimental and its interface might change in future versions.

### mpfr_dump

`void mpfr_dump(mpfr_t op)`  
Output `op` on `stdout` in some unspecified format, then a newline character. This function is mainly for debugging purpose. Thus invalid data may be supported. Everything that is not specified may change without breaking the ABI and may depend on the environment.
The current output format is the following: a minus sign if the sign bit is set (even for NaN); ‘@NaN@’, ‘@Inf@’ or ‘0’ if the argument is NaN, an infinity or zero, respectively; otherwise the remaining of the output is as follows: ‘0.’ then the $p$ bits of the binary significand, where $p$ is the precision of the number; if the trailing bits are not all zeros (which must not occur with valid data), they are output enclosed by square brackets; the character ‘E’ followed by the exponent written in base 10; in case of invalid data or out-of-range exponent, this function outputs three exclamation marks (‘!!!’), followed by flags, followed by three exclamation marks (‘!!!’) again. These flags are: ‘N’ if the most significant bit of the significand is 0 (i.e., the number is not normalized); ‘T’ if there are non-zero trailing bits; ‘U’ if this is an UBF number (internal use only); ‘<’ if the exponent is less than the current minimum exponent; ‘>’ if the exponent is greater than the current maximum exponent.

5.9 Formatted Output Functions

5.9.1 Requirements

The class of mpfr_printf functions provides formatted output in a similar manner as the standard C printf. These functions are defined only if your system supports ISO C variadic functions and the corresponding argument access macros.

When using any of these functions, you must include the <stdio.h> standard header before mpfr.h, to allow mpfr.h to define prototypes for these functions.

5.9.2 Format String

The format specification accepted by mpfr_printf is an extension of the gmp_printf one (itself, an extension of the printf one). The conversion specification is of the form:

```
% [flags] [width] [.[precision]] [type] [rounding] conv
```

‘flags’, ‘width’, and ‘precision’ have the same meaning as for the standard printf (in particular, notice that the ‘precision’ is related to the number of digits displayed in the base chosen by ‘conv’ and not related to the internal precision of the mpfr_t variable), but note that for ‘Re’, the default precision is not the same as the one for ‘e’. mpfr_printf accepts the same ‘type’ specifiers as GMP (except the non-standard and deprecated ‘q’, use ‘ll’ instead), namely the length modifiers defined in the C standard:

- ‘h’ short
- ‘hh’ char
- ‘j’ intmax_t or uintmax_t
- ‘l’ long or wchar_t
- ‘ll’ long long
- ‘L’ long double
- ‘t’ ptrdiff_t
- ‘z’ size_t

and the ‘type’ specifiers defined in GMP, plus ‘R’ and ‘P’, which are specific to MPFR (the second column in the table below shows the type of the argument read in the argument list and the kind of ‘conv’ specifier to use after the ‘type’ specifier):

- ‘F’ mpfr_t, float conversions
- ‘Q’ mpq_t, integer conversions
- ‘M’ mp_limb_t, integer conversions
- ‘N’ mp_limb_t array, integer conversions
- ‘Z’ mpz_t, integer conversions
The ‘type’ specifiers have the same restrictions as those mentioned in the GMP documentation: see Section “Formatted Output Strings” in GNU MP. In particular, the ‘type’ specifiers (except ‘R’ and ‘P’) are supported only if they are supported by gmp_printf in your GMP build; this implies that the standard specifiers, such as ‘t’, must also be supported by your C library if you want to use them.

The ‘rounding’ field is specific to mpfr_t arguments and should not be used with other types.

With conversion specification not involving ‘P’ and ‘R’ types, mpfr_printf behaves exactly as gmp_printf.

Thus the ‘conv’ specifier ‘F’ is not supported (due to the use of ‘F’ as the ‘type’ specifier for mpf_t), except for the ‘type’ specifier ‘R’ (i.e., for mpfr_t arguments).

The ‘P’ type specifies that a following ‘d’, ‘i’, ‘o’, ‘u’, ‘x’, or ‘X’ conversion specifier applies to a mpfr_prec_t argument. It is needed because the mpfr_prec_t type does not necessarily correspond to an int or any fixed standard type. The ‘precision’ field specifies the minimum number of digits to appear. The default ‘precision’ is 1. For example:

```c
mpfr_t x;
mpfr_prec_t p;
mpfr_init (x);
...
p = mpfr_get_prec (x);
mpfr_printf ("variable x with %Pu bits", p);
```

The ‘R’ type specifies that a following ‘a’, ‘A’, ‘b’, ‘e’, ‘E’, ‘f’, ‘F’, ‘g’, ‘G’, or ‘n’ conversion specifier applies to a mpfr_t argument. The ‘R’ type can be followed by a ‘rounding’ specifier denoted by one of the following characters:

- ‘U’ round toward positive infinity
- ‘D’ round toward negative infinity
- ‘Y’ round away from zero
- ‘Z’ round toward zero
- ‘N’ round to nearest (with ties to even)
- ‘*’ rounding mode indicated by the mpfr_rnd_t argument just before the corresponding mpfr_t variable.

The default rounding mode is rounding to nearest. The following three examples are equivalent:

```c
mpfr_t x;
mpfr_init (x);
...
mpfr_printf ("%.128Rf", x);
mpfr_printf ("%.128RNf", x);
mpfr_printf ("%.128R*f", MPFR_RNDN, x);
```

Note that the rounding away from zero mode is specified with ‘Y’ because ISO C reserves the ‘A’ specifier for hexadecimal output (see below).

The output ‘conv’ specifiers allowed with mpfr_t parameter are:

- ‘a’ ‘A’ hex float, C99 style
- ‘b’ binary output
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The conversion specifier ‘b’, which displays the argument in binary, is specific to mpfr_t arguments and should not be used with other types. Other conversion specifiers have the same meaning as for a double argument.

In case of non-decimal output, only the significand is written in the specified base, the exponent is always displayed in decimal. Special values are always displayed as ‘nan’, ‘-inf’, and ‘inf’ for ‘a’, ‘b’, ‘e’, ‘f’, and ‘g’ specifiers and ‘NAN’, ‘-INF’, and ‘INF’ for ‘A’, ‘E’, ‘F’, and ‘G’ specifiers.

The mpfr_t number is rounded to the given precision in the direction specified by the rounding mode (see below if the ‘precision’ field is empty). If the precision is zero with rounding to nearest mode and one of the following ‘conv’ specifiers: ‘a’, ‘A’, ‘b’, ‘e’, ‘E’, ‘f’, and ‘g’ tie case is rounded to even when it lies between two consecutive values at the wanted precision which have the same exponent, otherwise, it is rounded away from zero. For instance, 85 is displayed as ‘8e+1’ and 95 is displayed as ‘1e+2’ with the format specification “%.0RNe”. This also applies when the ‘g’ (resp. ‘G’) conversion specifier uses the ‘e’ (resp. ‘E’) style. If the precision is set to a value greater than the maximum value for an int, it will be silently reduced down to INT_MAX.

If the ‘precision’ field is empty with ‘conv’ specifier ‘e’ and ‘E’ (as in %Re or %.RE), the chosen precision (i.e., the number of digits to be displayed after the initial digit and the decimal point) is \[ \lceil \frac{p \log 2}{\log 10} \rceil \], where \( p \) is the precision of the input variable, matching the choice done for mpfr_get_str; thus, if rounding to nearest is used, outputting the value with an empty ‘precision’ field and reading it back will yield the original value. The chosen precision for an empty ‘precision’ field with ‘conv’ specifiers ‘f’, ‘F’, ‘g’, and ‘G’ is 6.

5.9.3 Functions

For all the following functions, if the number of characters that ought to be written exceeds the maximum limit INT_MAX for an int, nothing is written in the stream (resp. to stdout, to buf, to str), the function returns −1, sets the erange flag, and errno is set to EOVERFLOW if the EOVERFLOW macro is defined (such as on POSIX systems). Note, however, that errno might be changed to another value by some internal library call if another error occurs there (currently, this would come from the unallocation function).

```
int mpfr_fprintf (FILE *stream, const char *template, ...)  [Function]
int mpfr_vfprintf (FILE *stream, const char *template, va_list ap)  [Function]

Print to the stream stream the optional arguments under the control of the template string template. Return the number of characters written or a negative value if an error occurred.

int mpfr_printf (const char *template, ...)  [Function]
int mpfr_vprintf (const char *template, va_list ap)  [Function]

Print to stdout the optional arguments under the control of the template string template. Return the number of characters written or a negative value if an error occurred.

int mpfr_sprintf (char *buf, const char *template, ...)  [Function]
int mpfr_vsprintf (char *buf, const char *template, va_list ap)  [Function]

Form a null-terminated string corresponding to the optional arguments under the control of the template string template, and print it in buf. No overlap is permitted between buf and
the other arguments. Return the number of characters written in the array \textit{buf} \textit{not counting}
the terminating null character or a negative value if an error occurred.

\begin{verbatim}
int mpfr_snprintf (char *buf, size_t n, const char *template, ...) [Function]
int mpfr_vsnprintf (char *buf, size_t n, const char *template, va_list ap) [Function]
  Form a null-terminated string corresponding to the optional arguments under the control of
  the template string \textit{template}, and print it in \textit{buf}. If \textit{n} is zero, nothing is written and \textit{buf}
  may be a null pointer, otherwise, the first \textit{n} − 1 characters are written in \textit{buf} and the \textit{n}-th one is a
  null character. Return the number of characters that would have been written had \textit{n} been
  sufficiently large, \textit{not counting} the terminating null character, or a negative value if an error
  occurred.

int mpfr_asprintf (char **str, const char *template, ...) [Function]
int mpfr_vasprintf (char **str, const char *template, va_list ap) [Function]
  Write their output as a null terminated string in a block of memory allocated using the
  allocation function (see Section 4.7 \[Memory Handling\], page 11). A pointer to the block is
  stored in \textit{str}. The block of memory must be freed using \texttt{mpfr_free_str}. The return value is
  the number of characters written in the string, excluding the null-terminator, or a negative
  value if an error occurred.
\end{verbatim}

5.10 Integer and Remainder Related Functions

\begin{verbatim}
int mpfr_rint (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_ceil (mpfr_t rop, mpfr_t op) [Function]
int mpfr_floor (mpfr_t rop, mpfr_t op) [Function]
int mpfr_round (mpfr_t rop, mpfr_t op) [Function]
int mpfr_roundeven (mpfr_t rop, mpfr_t op) [Function]
int mpfr_trunc (mpfr_t rop, mpfr_t op) [Function]
  Set \textit{rop} to \textit{op} rounded to an integer. \texttt{mpfr_rint} rounds to the nearest representable integer
  in the given direction \textit{rnd}, and the other five functions behave in a similar way with some
  fixed rounding mode:

  \begin{itemize}
  \item \texttt{mpfr\_ceil}: to the next higher or equal representable integer (like \texttt{mpfr\_rint} with
    \texttt{MPFR\_RNDU});
  \item \texttt{mpfr\_floor} to the next lower or equal representable integer (like \texttt{mpfr\_rint} with \texttt{MPFR\_RNDD});
  \item \texttt{mpfr\_round} to the nearest representable integer, rounding halfway cases away from zero
    (as in the roundTiesToAway mode of IEEE 754);
  \item \texttt{mpfr\_roundeven} to the nearest representable integer, rounding halfway cases with the
    even-rounding rule (like \texttt{mpfr\_rint} with \texttt{MPFR\_RNDN});
  \item \texttt{mpfr\_trunc} to the next representable integer toward zero (like \texttt{mpfr\_rint} with \texttt{MPFR\_RNDZ}).
  \end{itemize}

  When \textit{op} is a zero or an infinity, set \textit{rop} to the same value (with the same sign).

  The return value is zero when the result is exact, positive when it is greater than the original
  value of \textit{op}, and negative when it is smaller. More precisely, the return value is 0 when
  \textit{op} is an integer representable in \textit{rop}, 1 or −1 when \textit{op} is an integer that is not representable in \textit{rop},
  2 or −2 when \textit{op} is not an integer.
\end{verbatim}
When \(\text{op}\) is NaN, the NaN flag is set as usual. In the other cases, the inexact flag is set when \(\text{rop}\) differs from \(\text{op}\), following the ISO C99 rule for the \text{rint} function. If you want the behavior to be more like IEEE 754 / ISO TS 18661-1, i.e., the usual behavior where the round-to-integer function is regarded as any other mathematical function, you should use one of the \text{mpfr_rint\_*} functions instead.

Note that no double rounding is performed; for instance, 10.5 (1010.1 in binary) is rounded by \text{mpfr_rint} with rounding to nearest to 12 (1100 in binary) in 2-bit precision, because the two enclosing numbers representable on two bits are 8 and 12, and the closest is 12. (If one first rounded to an integer, one would round 10.5 to 10 with even rounding, and then 10 would be rounded to 8 again with even rounding.)

\begin{verbatim}
int mpfr_rint ceil (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_rint floor (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_rint round (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_rint roundeven (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_rint trunc (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
\end{verbatim}

Set \(\text{rop}\) to \(\text{op}\) rounded to an integer:

- \text{mpfr_rint\_ceil}: to the next higher or equal integer;
- \text{mpfr_rint\_floor}: to the next lower or equal integer;
- \text{mpfr_rint\_round}: to the nearest integer, rounding halfway cases away from zero;
- \text{mpfr_rint\_roundeven}: to the nearest integer, rounding halfway cases to the nearest even integer;
- \text{mpfr_rint\_trunc}: to the next integer toward zero.

If the result is not representable, it is rounded in the direction \(\text{rnd}\). When \(\text{op}\) is a zero or an infinity, set \(\text{rop}\) to the same value (with the same sign). The return value is the ternary value associated with the considered round-to-integer function (regarded in the same way as any other mathematical function).

Contrary to \text{mpfr_rint}, those functions do perform a double rounding: first \(\text{op}\) is rounded to the nearest integer in the direction given by the function name, then this nearest integer (if not representable) is rounded in the given direction \(\text{rnd}\). Thus these round-to-integer functions behave more like the other mathematical functions, i.e., the returned result is the correct rounding of the exact result of the function in the real numbers.

For example, \text{mpfr_rint\_round} with rounding to nearest and a precision of two bits rounds 6.5 to 7 (halfway cases away from zero), then 7 is rounded to 8 by the round-even rule, despite the fact that 6 is also representable on two bits, and is closer to 6.5 than 8.

\begin{verbatim}
int mpfr_frac (mpfr_t rop, mpfr_t op, mpfr_rnd_t rnd) [Function]
int mpfr_modf (mpfr_t iop, mpfr_t fop, mpfr_t op, mpfr_rnd_t rnd) [Function]
\end{verbatim}

Set \(\text{rop}\) to the fractional part of \(\text{op}\), having the same sign as \(\text{op}\), rounded in the direction \(\text{rnd}\) (unlike in \text{mpfr_rint}, \(\text{rnd}\) affects only how the exact fractional part is rounded, not how the fractional part is generated). When \(\text{op}\) is an integer or an infinity, set \(\text{rop}\) to zero with the same sign as \(\text{op}\).

Set simultaneously \(\text{iop}\) to the integral part of \(\text{op}\) and \(\text{fop}\) to the fractional part of \(\text{op}\), rounded in the direction \(\text{rnd}\) with the corresponding precision of \(\text{iop}\) and \(\text{fop}\) (equivalent to \text{mpfr\_trunc}(\text{iop}, \text{op}, \text{rnd}) and \text{mpfr\_frac}(\text{fop}, \text{op}, \text{rnd}))}. The variables \(\text{iop}\) and \(\text{fop}\) must be
different. Return 0 iff both results are exact (see `mpfr_sin_cos` for a more detailed description of the return value).

```c
int mpfr_fmod (mpfr_t r, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  [Function]
int mpfr_fmodquo (mpfr_t r, long int* q, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  [Function]
int mpfr_remainder (mpfr_t r, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  [Function]
int mpfr_remquo (mpfr_t r, long int* q, mpfr_t x, mpfr_t y, mpfr_rnd_t rnd)  [Function]
```

Set $r$ to the value of $x - ny$, rounded according to the direction $\text{rnd}$, where $n$ is the integer quotient of $x$ divided by $y$, defined as follows: $n$ is rounded toward zero for `mpfr_fmod` and `mpfr_fmodquo`, and to the nearest integer (ties rounded to even) for `mpfr_remainder` and `mpfr_remquo`.

Special values are handled as described in Section F.9.7.1 of the ISO C99 standard: If $x$ is infinite or $y$ is zero, $r$ is NaN. If $y$ is infinite and $x$ is finite, $r$ is $x$ rounded to the precision of $r$. If $r$ is zero, it has the sign of $x$. The return value is the ternary value corresponding to $r$.

Additionally, `mpfr_fmodquo` and `mpfr_remquo` store the low significant bits from the quotient $n$ in *q (more precisely the number of bits in a long int minus one), with the sign of $x$ divided by $y$ (except if those low bits are all zero, in which case zero is returned). Note that $x$ may be so large in magnitude relative to $y$ that an exact representation of the quotient is not practical. The `mpfr_remainder` and `mpfr_remquo` functions are useful for additive argument reduction.

```c
int mpfr_integer_p (mpfr_t op)  [Function]
    Return non-zero iff op is an integer.
```

### 5.11 Rounding-Related Functions

```c
void mpfr_set_default_rounding_mode (mpfr_rnd_t rnd)  [Function]
    Set the default rounding mode to \text{rnd}. The default rounding mode is to nearest initially.

mpfr_rnd_t mpfr_get_default_rounding_mode (void)  [Function]
    Get the default rounding mode.

int mpfr_prec_round (mpfr_t x, mpfr_prec_t prec, mpfr_rnd_t rnd)  [Function]
    Round $x$ according to $\text{rnd}$ with precision $\text{prec}$, which must be an integer between MPFR_PREC_MIN and MPFR_PREC_MAX (otherwise the behavior is undefined). If $\text{prec}$ is greater than or equal to the precision of $x$, then new space is allocated for the significand, and it is filled with zeros. Otherwise, the significand is rounded to precision $\text{prec}$ with the given direction; no memory reallocation to free the unused limbs is done. In both cases, the precision of $x$ is changed to $\text{prec}$.
```

Here is an example of how to use `mpfr_prec_round` to implement Newton’s algorithm to compute the inverse of $a$, assuming $x$ is already an approximation to $n$ bits:

```c
  mpfr_set_prec (t, 2 * n);
  mpfr_set (t, a, MPFR_RNDN);  /* round a to 2n bits */
  mpfr_mul (t, t, x, MPFR_RNDN);  /* t is correct to 2n bits */
  mpfr_ui_sub (t, 1, t, MPFR_RNDN);  /* high n bits cancel with 1 */
  mpfr_prec_round (t, n, MPFR_RNDN);  /* t is correct to n bits */
```
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mpfr_mul (t, t, x, MPFR_RNDN); /* t is correct to n bits */
mpfr_prec_round (x, 2 * n, MPFR_RNDN); /* exact */
mpfr_add (x, x, t, MPFR_RNDN); /* x is correct to 2n bits */

Warning! You must not use this function if x was initialized with MPFR_DECL_INIT or with mpfr_custom_init_set (see Section 5.16 [Custom Interface], page 48).

Function
int mpfr_can_round (mpfr_t b, mpfr_exp_t err, mpfr_rnd_t rnd1,
                    mpfr_rnd_t rnd2, mpfr_prec_t prec)

Assuming b is an approximation of an unknown number x in the direction rnd1 with error at most two to the power \( \text{EXP}(b) - err \) where \( \text{EXP}(b) \) is the exponent of b, return a non-zero value if one is able to round correctly x to precision prec with the direction rnd2 assuming an unbounded exponent range, and 0 otherwise (including for NaN and Inf). In other words, if the error on b is bounded by two to the power k ulps, and b has precision prec, you should give \( \text{err} = \text{prec} - k \). This function does not modify its arguments.

If rnd1 is MPFR_RNDN or MPFR_RNDF, the error is considered to be either positive or negative, thus the possible range is twice as large as with a directed rounding for rnd1 (with the same value of err).

When rnd2 is MPFR_RNDF, let rnd3 be the opposite direction if rnd1 is a directed rounding, and MPFR_RNDN if rnd1 is MPFR_RNDN or MPFR_RNDF. The returned value of mpfr_can_round (b, err, rnd1, MPFR_RNDF, prec) is non-zero iff after the call mpfr_set (y, b, rnd3) with y of precision prec, y is guaranteed to be a faithful rounding of x.

Note: The [ternary value], page 9, cannot be determined in general with this function. However, if it is known that the exact value is not exactly representable in precision prec, then one can use the following trick to determine the (non-zero) ternary value in any rounding mode rnd2 (note that MPFR_RNDZ below can be replaced by any directed rounding mode):

if (mpfr_can_round (b, err, MPFR_RNDN, MPFR_RNDZ, prec + (rnd2 == MPFR_RNDN)))
{
    /* round the approximation 'b' to the result 'r' of 'prec' bits
     with rounding mode 'rnd2' and get the ternary value 'inex' */
    inex = mpfr_set (r, b, rnd2);
}

Indeed, if rnd2 is MPFR_RNDN, this will check if one can round to prec + 1 bits with a directed rounding: if so, one can surely round to nearest to prec bits, and in addition one can determine the correct ternary value, which would not be the case when b is near from a value exactly representable on prec bits.

A detailed example is available in the examples subdirectory, file can_round.c.

Function
mpfr_prec_t mpfr_min_prec (mpfr_t x)

Return the minimal number of bits required to store the significand of x, and 0 for special values, including 0.

Function
const char * mpfr_print_rnd_mode (mpfr_rnd_t rnd)

Return a string ("MPFR_RNDN", "MPFR_RNDZ", "MPFR_RNDU", "MPFR_RNDD", "MPFR_RNDA", "MPFR_RNDF") corresponding to the rounding mode rnd, or a null pointer if rnd is an invalid rounding mode.
int mpfr_round_nearest_away (int (foo)(mpfr_t, type1_t, ...), mpfr_rnd_t), mpfr_t rop, type1_t op, ...) [Macro]

Given a function foo and one or more values op (which may be a mpfr_t, a long int, a double, etc.), put in rop the round-to-nearest-away rounding of foo(op,...). This rounding is defined in the same way as round-to-nearest-even, except in case of tie, where the value away from zero is returned. The function foo takes as input, from second to penultimate argument(s), the argument list given after rop, a rounding mode as final argument, puts in its first argument the value foo(op,...) rounded according to this rounding mode, and returns the corresponding ternary value (which is expected to be correct, otherwise mpfr_round_nearest_away will not work as desired). Due to implementation constraints, this function must not be called when the minimal exponent emin is the smallest possible one. This macro has been made such that the compiler is able to detect mismatch between the argument list op and the function prototype of foo. Multiple input arguments op are supported only with C99 compilers. Otherwise, for C89 compilers, only one such argument is supported.

Note: this macro is experimental and its interface might change in future versions.

unsigned long ul;
mpfr_t f, r;
/* Code that inits and sets r, f, and ul, and if needed sets emin */
int i = mpfr_round_nearest_away (mpfr_add_ui, r, f, ul);

5.12 Miscellaneous Functions

void mpfr_nexttoward (mpfr_t x, mpfr_t y) [Function]
If x or y is NaN, set x to NaN; note that the NaN flag is set as usual. If x and y are equal, x is unchanged. Otherwise, if x is different from y, replace x by the next floating-point number (with the precision of x and the current exponent range) in the direction of y (the infinite values are seen as the smallest and largest floating-point numbers). If the result is zero, it keeps the same sign. No underflow, overflow, or inexact exception is raised.

void mpfr_nextabove (mpfr_t x) [Function]
void mpfr_nextbelow (mpfr_t x) [Function]
Equivalent to mpfr_nexttoward where y is +Inf (resp. −Inf).

int mpfr_min (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
Set rop to the minimum (resp. maximum) of op1 and op2. If op1 and op2 are both NaN, then rop is set to NaN. If op1 or op2 is NaN, then rop is set to the numeric value. If op1 and op2 are zeros of different signs, then rop is set to −0 (resp. +0). As usual, the NaN flag is set only when the result is NaN, i.e., when both op1 and op2 are NaN.

Note: These functions correspond to the minimumNumber and maximumNumber operations of IEEE 754-2019 for the result. But in MPFR, the NaN flag is set only when both operands are NaN.

int mpfr_urandomb (mpfr_t rop, gmp_randstate_t state) [Function]
Generate a uniformly distributed random float in the interval 0 ≤ rop < 1. More precisely, the number can be seen as a float with a random non-normalized significand and exponent 0, which is then normalized (thus if e denotes the exponent after normalization, then the least −e significant bits of the significand are always 0).
Return 0, unless the exponent is not in the current exponent range, in which case rop is set to NaN and a non-zero value is returned (this should never happen in practice, except in very specific cases). The second argument is a gmp_randstate_t structure, which should be created using the GMP gmp_randinit function (see the GMP manual).

Note: for a given version of MPFR, the returned value of rop and the new value of state (which controls further random values) do not depend on the machine word size.

int mpfr_urandom (mpfr_t rop, gmp_randstate_t state, mpfr_rnd_t rnd)  
Generate a uniformly distributed random float. The floating-point number rop can be seen as if a random real number is generated according to the continuous uniform distribution on the interval [0, 1] and then rounded in the direction rnd.

The second argument is a gmp_randstate_t structure, which should be created using the GMP gmp_randinit function (see the GMP manual).

Note: the note for mpfr_urandomb holds too. Moreover, the exact number (the random value to be rounded) and the next random state do not depend on the current exponent range and the rounding mode. However, they depend on the target precision: from the same state of the random generator, if the precision of the destination is changed, then the value may be completely different (and the state of the random generator is different too).

int mpfr_rand (mpfr_t rop, mpfr_t state, mpfr_rnd_t rnd)
int mpfr_rnd (mpfr_t rop1, mpfr_t rop2, gmp_randstate_t state, mpfr_rnd_t rnd)

Generate one (possibly two for mpfr_grandom) random floating-point number according to a standard normal Gaussian distribution (with mean zero and variance one). For mpfr_grandom, if rop2 is a null pointer, then only one value is generated and stored in rop1.

The floating-point number rop1 (and rop2) can be seen as if a random real number were generated according to the standard normal Gaussian distribution and then rounded in the direction rnd.

The gmp_randstate_t argument should be created using the GMP gmp_randinit function (see the GMP manual).

For mpfr_grandom, the combination of the ternary values is returned like with mpfr_sin_cos. If rop2 is a null pointer, the second ternary value is assumed to be 0 (note that the encoding of the only ternary value is not the same as the usual encoding for functions that return only one result). Otherwise the ternary value of a random number is always non-zero.

Note: the note for mpfr_urandomb holds too. In addition, the exponent range and the rounding mode might have a side effect on the next random state.

Note: mpfr_rand is much more efficient than mpfr_grandom, especially for large precision. Thus mpfr_grandom is marked as deprecated and will be removed in a future release.

int mpfr_erandom (mpfr_t rop1, gmp_randstate_t state, mpfr_rnd_t rnd)

Generate one random floating-point number according to an exponential distribution, with mean one. Other characteristics are identical to mpfr_rand.
mpfr_exp_t mpfr_get_exp (mpfr_t x) [Function]
Return the exponent of x, assuming that x is a non-zero ordinary number and the significand is considered in [1/2,1). For this function, x is allowed to be outside of the current range of acceptable values. The behavior for NaN, infinity or zero is undefined.

int mpfr_set_exp (mpfr_t x, mpfr_exp_t e) [Function]
Set the exponent of x to e if x is a non-zero ordinary number and e is in the current exponent range, and return 0; otherwise, return a non-zero value (x is not changed).

int mpfr_signbit (mpfr_t op) [Function]
Return a non-zero value iff op has its sign bit set (i.e., if it is negative, −0, or a NaN whose representation has its sign bit set).

int mpfr_setsign (mpfr_t rop, mpfr_t op, int s, mpfr_rnd_t rnd) [Function]
Set the value of rop from op, rounded toward the given direction rnd, then set (resp. clear) its sign bit if s is non-zero (resp. zero), even when op is a NaN.

int mpfr_copysign (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
Set the value of rop from op1, rounded toward the given direction rnd, then set its sign bit to that of op2 (even when op1 or op2 is a NaN). This function is equivalent to mpfr_setsign (rop, op1, mpfr_signbit (op2), rnd).

const char * mpfr_get_version (void) [Function]
Return the MPFR version, as a null-terminated string.

MPFR_VERSION [Macro]
MPFR_VERSION_MAJOR [Macro]
MPFR_VERSION_MINOR [Macro]
MPFR_VERSION_PATCHLEVEL [Macro]
MPFR_VERSION_STRING [Macro]
MPFR_VERSION is the version of MPFR as a preprocessing constant. MPFR_VERSION_MAJOR, MPFR_VERSION_MINOR and MPFR_VERSION_PATCHLEVEL are respectively the major, minor and patch level of MPFR version, as preprocessing constants. MPFR_VERSION_STRING is the version (with an optional suffix, used in development and pre-release versions) as a string constant, which can be compared to the result of mpfr_get_version to check at run time the header file and library used match:

if (strcmp (mpfr_get_version (), MPFR_VERSION_STRING))
    fprintf (stderr, "Warning: header and library do not match\n");

Note: Obtaining different strings is not necessarily an error, as in general, a program compiled with some old MPFR version can be dynamically linked with a newer MPFR library version (if allowed by the library versioning system).

long MPFR_VERSION_NUM (major, minor, patchlevel) [Macro]
Create an integer in the same format as used by MPFR_VERSION from the given major, minor and patchlevel. Here is an example of how to check the MPFR version at compile time:

#if (!defined(MPFR_VERSION) || (MPFR_VERSION < MPFR_VERSION_NUM(3,0,0)))
    #error "Wrong MPFR version."
#endif
const char * mpfr_get_patches (void)  
Return a null-terminated string containing the ids of the patches applied to the MPFR library (contents of the PATCHES file), separated by spaces. Note: If the program has been compiled with an older MPFR version and is dynamically linked with a new MPFR library version, the identifiers of the patches applied to the old (compile-time) MPFR version are not available (however, this information should not have much interest in general).

int mpfr_buildopt_tls_p (void)  
Return a non-zero value if MPFR was compiled as thread safe using compiler-level Thread-Local Storage (that is, MPFR was built with the ‘--enable-thread-safe’ configure option, see INSTALL file), return zero otherwise.

int mpfr_buildopt_float128_p (void)  
Return a non-zero value if MPFR was compiled with ‘_Float128’ support (that is, MPFR was built with the ‘--enable-float128’ configure option), return zero otherwise.

int mpfr_buildopt_decimal_p (void)  
Return a non-zero value if MPFR was compiled with decimal float support (that is, MPFR was built with the ‘--enable-decimal-float’ configure option), return zero otherwise.

int mpfr_buildopt_gmpinternals_p (void)  
Return a non-zero value if MPFR was compiled with GMP internals (that is, MPFR was built with either ‘--with-gmp-build’ or ‘--enable-gmp-internals’ configure option), return zero otherwise.

int mpfr_buildopt_sharedcache_p (void)  
Return a non-zero value if MPFR was compiled so that all threads share the same cache for one MPFR constant, like mpfr_const_pi or mpfr_const_log2 (that is, MPFR was built with the ‘--enable-shared-cache’ configure option), return zero otherwise. If the return value is non-zero, MPFR applications may need to be compiled with the ‘-pthread’ option.

const char * mpfr_buildopt_tune_case (void)  
Return a string saying which thresholds file has been used at compile time. This file is normally selected from the processor type.

### 5.13 Exception Related Functions

mpfr_exp_t mpfr_get_emin (void)  
mpfr_exp_t mpfr_get_emax (void)  
Return the (current) smallest and largest exponents allowed for a floating-point variable. The smallest positive value of a floating-point variable is $1/2 \times 2^{emin}$ and the largest value has the form $(1 - \varepsilon) \times 2^{emax}$, where $\varepsilon$ depends on the precision of the considered variable.

int mpfr_set_emin (mpfr_exp_t exp)  
int mpfr_set_emax (mpfr_exp_t exp)  
Set the smallest and largest exponents allowed for a floating-point variable. Return a non-zero value when exp is not in the range accepted by the implementation (in that case the smallest or largest exponent is not changed), and zero otherwise.

For the subsequent operations, it is the user’s responsibility to check that any floating-point value used as an input is in the new exponent range (for example using mpfr_check_range). If a floating-point value outside the new exponent range is used as an input, the default
behavior is undefined, in the sense of the ISO C standard; the behavior may also be explicitly documented, such as for `mpfr_check_range`.

Note: Caches may still have values outside the current exponent range. This is not an issue as the user cannot use these caches directly via the API (MPFR extends the exponent range internally when need be).

If `emin > emax` and a floating-point value needs to be produced as output, the behavior is undefined (`mpfr_set_emin` and `mpfr_set_emax` do not check this condition as it might occur between successive calls to these two functions).

```c
mpfr_exp_t mpfr_get_emin_min (void) [Function]
mpfr_exp_t mpfr_get_emin_max (void) [Function]
mpfr_exp_t mpfr_get_emax_min (void) [Function]
mpfr_exp_t mpfr_get_emax_max (void) [Function]
```

Return the minimum and maximum of the exponents allowed for `mpfr_set_emin` and `mpfr_set_emax` respectively. These values are implementation dependent, thus a program using `mpfr_set_emax(mpfr_get_emax_max())` or `mpfr_set_emin(mpfr_get_emin_min())` may not be portable.

```c
int mpfr_check_range (mpfr_t x, int t, mpfr_rnd_t rnd) [Function]
```

This function assumes that `x` is the correctly rounded value of some real value `y` in the direction `rnd` and some extended exponent range, and that `t` is the corresponding ternary value, page 9. For example, one performed `t = mpfr_log (x, u, rnd)`, and `y` is the exact logarithm of `u`. Thus `t` is negative if `x` is smaller than `y`, positive if `x` is larger than `y`, and zero if `x` equals `y`. This function modifies `x` if needed to be in the current range of acceptable values: It generates an underflow or an overflow if the exponent of `x` is outside the current allowed range; the value of `t` may be used to avoid a double rounding. This function returns zero if the new value of `x` equals the exact one `y`, a positive value if that new value is larger than `y`, and a negative value if it is smaller than `y`. Note that unlike most functions, the new result `x` is compared to the (unknown) exact one `y`, not the input value `x`, i.e., the ternary value is propagated.

Note: If `x` is an infinity and `t` is different from zero (i.e., if the rounded result is an inexact infinity), then the overflow flag is set. This is useful because `mpfr_check_range` is typically called (at least in MPFR functions) after restoring the flags that could have been set due to internal computations.

```c
int mpfr_subnormalize (mpfr_t x, int t, mpfr_rnd_t rnd) [Function]
```

This function rounds `x` emulating subnormal number arithmetic: if `x` is outside the subnormal exponent range of the emulated floating-point system, this function just propagates the [ternary value], page 9, `t`; otherwise, if `EXP(x)` denotes the exponent of `x`, it rounds `x` to precision `EXP(x) - emin + 1` according to rounding mode `rnd` and previous ternary value `t`, avoiding double rounding problems. More precisely in the subnormal domain, denoting by `e` the value of `emin`, `x` is rounded in fixed-point arithmetic to an integer multiple of `2^e`; as a consequence, `1.5 \times 2^{e-1}` when `t` is zero is rounded to `2^e` with rounding to nearest.

The precision `PREC(x)` of `x` is not modified by this function. `rnd` and `t` must be the rounding mode and the returned ternary value used when computing `x` (as in `mpfr_check_range`).

The subnormal exponent range is from `emin` to `emin + PREC(x) - 1`. If the result cannot be represented in the current exponent range of MPFR (due to a too small `emax`), the behavior is undefined. Note that unlike most functions, the result is compared to the exact one, not the input value `x`, i.e., the ternary value is propagated.
As usual, if the returned ternary value is non zero, the inexact flag is set. Moreover, if a second rounding occurred (because the input $x$ was in the subnormal range), the underflow flag is set.

Warning! If you change $emin$ (with `mpfr_set_emin`) just before calling `mpfr_subnormalize`, you need to make sure that the value is in the current exponent range of MPFR. But it is better to change $emin$ before any computation, if possible.

This is an example of how to emulate binary64 IEEE 754 arithmetic (a.k.a. double precision) using MPFR:

```c
{  mpfr_t xa, xb; int i; volatile double a, b;

  mpfr_set_default_prec (53);
  mpfr_set_emin (-1073); mpfr_set_emax (1024);
  mpfr_init (xa); mpfr_init (xb);
  b = 34.3; mpfr_set_d (xb, b, MPFR_RNDN);
  a = 0x1.1235P-1021; mpfr_set_d (xa, a, MPFR_RNDN);
  a /= b;
  i = mpfr_div (xa, xa, xb, MPFR_RNDN);
  i = mpfr_subnormalize (xa, i, MPFR_RNDN); /* new ternary value */

  mpfr_clear (xa); mpfr_clear (xb);
}
```

Note that `mpfr_set_emin` and `mpfr_set_emax` are called early enough in order to make sure that all computed values are in the current exponent range. Warning! This emulates a double IEEE 754 arithmetic with correct rounding in the subnormal range, which may not be the case for your hardware.

Below is another example showing how to emulate fixed-point arithmetic in a specific case. Here we compute the sine of the integers 1 to 17 with a result in a fixed-point arithmetic rounded at $2^{-42}$ (using the fact that the result is at most 1 in absolute value):

```c
{  mpfr_t x; int i, inex;

  mpfr_set_emin (-41);
  mpfr_init2 (x, 42);
  for (i = 1; i <= 17; i++)
  {  mpfr_set_ui (x, i, MPFR_RNDN);
      inex = mpfr_sin (x, x, MPFR_RNDZ);
      mpfr_subnormalize (x, inex, MPFR_RNDZ);
      mpfr_dump (x);
  }
  mpfr_clear (x);
}
```

void `mpfr_clear_underflow` (void) [Function]
void `mpfr_clear_overflow` (void) [Function]
Clear (lower) the underflow, overflow, divide-by-zero, invalid, inexact and erange flags.

Clear (lower) all global flags (underflow, overflow, divide-by-zero, invalid, inexact, erange).
Note: a group of flags can be cleared by using mpfr_flags_clear.

Set (raise) the underflow, overflow, divide-by-zero, invalid, inexact and erange flags.

Return the corresponding (underflow, overflow, divide-by-zero, invalid, inexact, erange) flag, which is non-zero iff the flag is set.

The mpfr_flags_ functions below that take an argument mask can operate on any subset of the exception flags: a flag is part of this subset (or group) if and only if the corresponding bit of the argument mask is set. The MPFR_FLAGS_ macros will normally be used to build this argument. See Section 4.6 [Exceptions], page 10.

Clear (lower) the group of flags specified by mask.

Set (raise) the group of flags specified by mask.

Return the flags specified by mask. To test whether any flag from mask is set, compare the return value to 0. You can also test individual flags by AND’ing the result with MPFR_FLAGS_ macros. Example:

```c
mpfr_flags_t t = mpfr_flags_test (MPFR_FLAGS_UNDERFLOW|MPFR_FLAGS_OVERFLOW);
...
if (t) /* underflow and/or overflow (unlikely) */
{
    if (t & MPFR_FLAGS_UNDERFLOW) { /* handle underflow */ }
    if (t & MPFR_FLAGS_OVERFLOW) { /* handle overflow */ }
```
mpfr_flags_t mpfr_flags_save (void)  
Return all the flags. It is equivalent to mpfr_flags_test(MPFR_FLAGS_ALL).

void mpfr_flags_restore (mpfr_flags_t flags, mpfr_flags_t mask)  
Restore the flags specified by mask to their state represented in flags.

5.14 Memory Handling Functions

These are general functions concerning memory handling (see Section 4.7 [Memory Handling], page 11, for more information).

void mpfr_free_cache (void)  
Free all caches and pools used by MPFR internally (those local to the current thread and those shared by all threads). You should call this function before terminating a thread, even if you did not call mpfr_const_* functions directly (they could have been called internally).

void mpfr_free_cache2 (mpfr_free_cache_t way)  
Free various caches and pools used by MPFR internally, as specified by way, which is a set of flags:

- those local to the current thread if flag MPFR_FREE_LOCAL_CACHE is set;
- those shared by all threads if flag MPFR_FREE_GLOBAL_CACHE is set.

The other bits of way are currently ignored and are reserved for future use; they should be zero.

Note: mpfr_free_cache2 (MPFR_FREE_LOCAL_CACHE | MPFR_FREE_GLOBAL_CACHE) is currently equivalent to mpfr_free_cache().

void mpfr_free_pool (void)  
Free the pools used by MPFR internally. Note: This function is automatically called after the thread-local caches are freed (with mpfr_free_cache or mpfr_free_cache2).

int mpfr_mp_memory_cleanup (void)  
This function should be called before calling mp_set_memory_functions. See Section 4.7 [Memory Handling], page 11, for more information. Zero is returned in case of success, non-zero in case of error. Errors are currently not possible, but checking the return value is recommended for future compatibility.

5.15 Compatibility With MPF

A header file mpf2mpfr.h is included in the distribution of MPFR for compatibility with the GNU MP class MPF. By inserting the following two lines after the #include <gmp.h> line,

#include <mpfr.h>
#include <mpf2mpfr.h>

many programs written for MPF can be compiled directly against MPFR without any changes. All operations are then performed with the default MPFR rounding mode, which can be reset with mpfr_set_default_rounding_mode.

Warning! There are some differences. In particular:

- The precision is different: MPFR rounds to the exact number of bits (zeroing trailing bits in the internal representation). Users may need to increase the precision of their variables.
• The exponent range is also different.
• The formatted output functions (\texttt{gmp_printf}, etc.) will not work for arguments of arbitrary-precision floating-point type (\texttt{mpf_t}, which \texttt{mpf2mpfr.h} redefines as \texttt{mpfr_t}).
• The output of \texttt{mpf_out_str} has a format slightly different from the one of \texttt{mpfr_out_str} (concerning the position of the decimal-point character, trailing zeros and the output of the value 0).

\begin{verbatim}
void mpfr_set_prec_raw (mpfr_t x, mpfr_prec_t prec) [Function]
Reset the precision of \texttt{x} to be exactly \texttt{prec} bits. The only difference with \texttt{mpfr_set_prec} is that \texttt{prec} is assumed to be small enough so that the significand fits into the current allocated memory space for \texttt{x}. Otherwise the behavior is undefined.

int mpfr_eq (mpfr_t op1, mpfr_t op2, unsigned long int op3) [Function]
Return non-zero if \texttt{op1} and \texttt{op2} are both non-zero ordinary numbers with the same exponent and the same first \texttt{op3} bits, both zero, or both infinities of the same sign. Return zero otherwise. This function is defined for compatibility with MPF, we do not recommend to use it otherwise. Do not use it either if you want to know whether two numbers are close to each other; for instance, 1.011111 and 1.100000 are regarded as different for any value of \texttt{op3} larger than 1.

void mpfr_reldiff (mpfr_t rop, mpfr_t op1, mpfr_t op2, mpfr_rnd_t rnd) [Function]
Compute the relative difference between \texttt{op1} and \texttt{op2} and store the result in \texttt{rop}. This function does not guarantee the correct rounding on the relative difference; it just computes $|\texttt{op1} - \texttt{op2}|/\texttt{op1}$, using the precision of \texttt{rop} and the rounding mode \texttt{rnd} for all operations.

int mpfr_mul_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
int mpfr_div_2exp (mpfr_t rop, mpfr_t op1, unsigned long int op2, mpfr_rnd_t rnd) [Function]
These functions are identical to \texttt{mpfr_mul_2ui} and \texttt{mpfr_div_2ui} respectively. These functions are only kept for compatibility with MPF, one should prefer \texttt{mpfr_mul_2ui} and \texttt{mpfr_div_2ui} otherwise.
\end{verbatim}

\subsection{Custom Interface}
Some applications use a stack to handle the memory and their objects. However, the MPFR memory design is not well suited for such a thing. So that such applications are able to use MPFR, an auxiliary memory interface has been created: the Custom Interface.

The following interface allows one to use MPFR in two ways:

• Either directly store a floating-point number as a \texttt{mpfr_t} on the stack.
• Either store its own representation on the stack and construct a new temporary \texttt{mpfr_t} each time it is needed.

Nothing has to be done to destroy the floating-point numbers except garbageing the used memory: all the memory management (allocating, destroying, garbageing) is left to the application.

Each function in this interface is also implemented as a macro for efficiency reasons: for example \texttt{mpfr_custom_init (s, p)} uses the macro, while \texttt{(mpfr_custom_init) (s, p)} uses the function. The \texttt{mpfr_custom_init_set} macro is not usable in contexts where an expression is expected, e.g., inside \texttt{for(...) \texttt{or before a comma operator}.}
Note 1: MPFR functions may still initialize temporary floating-point numbers using \texttt{mpfr_init} and similar functions. See Custom Allocation (GNU MP).

Note 2: MPFR functions may use the cached functions (\texttt{mpfr_const_pi} for example), even if they are not explicitly called. You have to call \texttt{mpfr_free_cache} each time you garbage the memory if \texttt{mpfr_init}, through GMP Custom Allocation, allocates its memory on the application stack.

\begin{verbatim}
size_t mpfr_custom_get_size (mpfr_prec_t prec) [Function]
  Return the needed size in bytes to store the significand of a floating-point number of precision prec.

void mpfr_custom_init (void *significand, mpfr_prec_t prec) [Function]
  Initialize a significand of precision \textit{prec}, where \textit{significand} must be an area of mpfr_custom_get_size (\textit{prec}) bytes at least and be suitably aligned for an array of mp_limb_t (GMP type, see Section 5.17 [Internals], page 50).

void mpfr_custom_init_set (mpfr_t x, int kind, mpfr_exp_t exp, mpfr_prec_t prec, void *significand) [Function]
  Perform a dummy initialization of a mpfr_t and set it to:
  \begin{itemize}
    \item if \textit{\textless kind\textgreater} = MPFR_NAN_KIND, \textit{x} is set to NaN;
    \item if \textit{\textless kind\textgreater} = MPFR_INF_KIND, \textit{x} is set to the infinity of the same sign as \textit{kind};
    \item if \textit{\textless kind\textgreater} = MPFR_ZERO_KIND, \textit{x} is set to the zero of the same sign as \textit{kind};
    \item if \textit{\textless kind\textgreater} = MPFR_REGULAR_KIND, \textit{x} is set to the regular number whose sign is the one of \textit{kind}, and whose exponent and significand are given by \textit{exp} and \textit{significand}.
  \end{itemize}
  In all cases, \textit{significand} will be used directly for further computing involving \textit{x}. This function does not allocate anything. A floating-point number initialized with this function cannot be resized using mpfr_set_prec or mpfr_prec_round, or cleared using mpfr_clear! The \textit{significand} must have been initialized with mpfr_custom_init using the same precision \textit{prec}.

int mpfr_custom_get_kind (mpfr_t x) [Function]
  Return the current kind of a mpfr_t as created by mpfr_custom_init_set. The behavior of this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.

void * mpfr_custom_get_significand (mpfr_t x) [Function]
  Return a pointer to the significand used by a mpfr_t initialized with mpfr_custom_init_set. The behavior of this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.

mpfr_exp_t mpfr_custom_get_exp (mpfr_t x) [Function]
  Return the exponent of \textit{x}, assuming that \textit{x} is a non-zero ordinary number and the significand is considered in [1/2,1). But if \textit{x} is NaN, infinity or zero, contrary to mpfr_get_exp (where the behavior is undefined), the return value is here an unspecified, valid value of the mpfr_exp_t type. The behavior of this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.

void mpfr_custom_move (mpfr_t x, void *new_position) [Function]
  Inform MPFR that the significand of \textit{x} has moved due to a garbage collect and update its new position to \textit{new_position}. However, the application has to move the significand and the mpfr_t itself. The behavior of this function for any mpfr_t not initialized with mpfr_custom_init_set is undefined.
\end{verbatim}
5.17 Internals

A *limb* means the part of a multi-precision number that fits in a single word. Usually a limb contains 32 or 64 bits. The C data type for a limb is *mp_limb_t*.

The *mpfr_t* type is internally defined as a one-element array of a structure, and *mpfr_ptr* is the C data type representing a pointer to this structure. The *mpfr_t* type consists of four fields:

- The *_mpfr_prec* field is used to store the precision of the variable (in bits); this is not less than *MPFR_PREC_MIN*.
- The *_mpfr_sign* field is used to store the sign of the variable.
- The *_mpfr_exp* field stores the exponent. An exponent of 0 means a radix point just above the most significant limb. Non-zero values $n$ are a multiplier $2^n$ relative to that point. A NaN, an infinity and a zero are indicated by special values of the exponent field.
- Finally, the *_mpfr_d* field is a pointer to the limbs, least significant limbs stored first. The number of limbs in use is controlled by *_mpfr_prec*, namely ceil(*_mpfr_prec/mp_bits_per_limb*). Non-singular (i.e., different from NaN, infinity or zero) values always have the most significant bit of the most significant limb set to 1. When the precision does not correspond to a whole number of limbs, the excess bits at the low end of the data are zeros.
6 API Compatibility

The goal of this section is to describe some API changes that occurred from one version of MPFR to another, and how to write code that can be compiled and run with older MPFR versions. The minimum MPFR version that is considered here is 2.2.0 (released on 20 September 2005).

API changes can only occur between major or minor versions. Thus the patchlevel (the third number in the MPFR version) will be ignored in the following. If a program does not use MPFR internals, changes in the behavior between two versions differing only by the patchlevel should only result from what was regarded as a bug or unspecified behavior.

As a general rule, a program written for some MPFR version should work with later versions, possibly except at a new major version, where some features (described as obsolete for some time) can be removed. In such a case, a failure should occur during compilation or linking. If a result becomes incorrect because of such a change, please look at the various changes below (they are minimal, and most software should be unaffected), at the FAQ and at the MPFR web page for your version (a bug could have been introduced and be already fixed); and if the problem is not mentioned, please send us a bug report (see Chapter 3 [Reporting Bugs], page 5).

However, a program written for the current MPFR version (as documented by this manual) may not necessarily work with previous versions of MPFR. This section should help developers to write portable code.

Note: Information given here may be incomplete. API changes are also described in the NEWS file (for each version, instead of being classified like here), together with other changes.

6.1 Type and Macro Changes

The official type for exponent values changed from \texttt{mp_exp_t} to \texttt{mpfr\_exp\_t} in MPFR 3.0. The type \texttt{mp\_exp\_t} will remain available as it comes from GMP (with a different meaning). These types are currently the same (\texttt{mpfr\_exp\_t} is defined as \texttt{mp\_exp\_t} with \texttt{typedef}), so that programs can still use \texttt{mp\_exp\_t}; but this may change in the future. Alternatively, using the following code after including \texttt{mpfr.h} will work with official MPFR versions, as \texttt{mpfr\_exp\_t} was never defined in MPFR 2.x:

```c
#ifndef MPFR_VERSION_MAJOR
#define MPFR_VERSION_MAJOR 3
#endif

#ifndef mp\_exp\_t
#define mp\_exp\_t mpfr\_exp\_t
#endif
```

The official types for precision values and for rounding modes respectively changed from \texttt{mp\_prec\_t} and \texttt{mp\_rnd\_t} to \texttt{mpfr\_prec\_t} and \texttt{mpfr\_rnd\_t} in MPFR 3.0. This change was actually done a long time ago in MPFR, at least since MPFR 2.2.0, with the following code in \texttt{mpfr.h}:

```c
#ifndef mp\_rnd\_t
#define mp\_rnd\_t mpfr\_rnd\_t
#endif

#ifndef mp\_prec\_t
#define mp\_prec\_t mpfr\_prec\_t
#endif
```

This means that it is safe to use the new official types \texttt{mpfr\_prec\_t} and \texttt{mpfr\_rnd\_t} in your programs. The types \texttt{mp\_prec\_t} and \texttt{mp\_rnd\_t} (defined in MPFR only) may be removed in the future, as the prefix \texttt{mp\_} is reserved by GMP.

The precision type \texttt{mpfr\_prec\_t} (\texttt{mp\_prec\_t}) was unsigned before MPFR 3.0; it is now signed. \texttt{MPFR\_PREC\_MAX} has not changed, though. Indeed the MPFR code requires that \texttt{MPFR\_PREC\_MAX}
be representable in the exponent type, which may have the same size as `mpfr_prec_t` but has always been signed. The consequence is that valid code that does not assume anything about the signedness of `mpfr_prec_t` should work with past and new MPFR versions. This change was useful as the use of unsigned types tends to convert signed values to unsigned ones in expressions due to the usual arithmetic conversions, which can yield incorrect results if a negative value is converted in such a way. Warning! A program assuming (intentionally or not) that `mpfr_prec_t` is signed may be affected by this problem when it is built and run against MPFR 2.x.

The rounding modes `GMP_RNDx` were renamed to `MPFR_RNDx` in MPFR 3.0. However, the old names `GMP_RNDx` have been kept for compatibility (this might change in future versions), using:

```c
#define GMP_RNDN MPFR_RNDN
#define GMP_RNDZ MPFR_RNDZ
#define GMP_RNDU MPFR_RNDU
#define GMP_RNDD MPFR_RNDD
```

The rounding mode “round away from zero” (`MPFR_RNDA`) was added in MPFR 3.0 (however, no rounding mode `GMP_RNDA` exists). Faithful rounding (`MPFR_RNDF`) was added in MPFR 4.0, but currently, it is partially supported.

The flags-related macros, whose name starts with `MPFR_FLAGS_`, were added in MPFR 4.0 (for the new functions `mpfr_flags_clear`, `mpfr_flags_restore`, `mpfr_flags_set` and `mpfr_flags_test`, in particular).

### 6.2 Added Functions

We give here in alphabetical order the functions (and function-like macros) that were added after MPFR 2.2, and in which MPFR version.

- `mpfr_add_d` in MPFR 2.4.
- `mpfr_ai` in MPFR 3.0 (incomplete, experimental).
- `mpfr_asprintf` in MPFR 2.4.
- `mpfr_beta` in MPFR 4.0 (incomplete, experimental).
- `mpfr_buildopt_decimal_p` in MPFR 3.0.
- `mpfr_buildopt_float128_p` in MPFR 4.0.
- `mpfr_buildopt_gmpinternals_p` in MPFR 3.1.
- `mpfr_buildopt_sharedcache_p` in MPFR 4.0.
- `mpfr_buildopt_tls_p` in MPFR 3.0.
- `mpfr_buildopt_tune_case` in MPFR 3.1.
- `mpfr_clear_divby0` in MPFR 3.1 (new divide-by-zero exception).
- `mpfr_cmpabs_ui` in MPFR 4.1.
- `mpfr_copysign` in MPFR 2.3. Note: MPFR 2.2 had a `mpfr_copysign` function that was available, but not documented, and with a slight difference in the semantics (when the second input operand is a NaN).
- `mpfr_custom_get_significand` in MPFR 3.0. This function was named `mpfr_custom_get_mantissa` in previous versions; `mpfr_custom_get_mantissa` is still available via a macro in `mpfr.h`:

  ```c
  #define mpfr_custom_get_mantissa mpfr_custom_get_significand
  ```

  Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use `mpfr_custom_get_mantissa`.
- `mpfr_d_div` and `mpfr_d_sub` in MPFR 2.4.
• mpfr_digamma in MPFR 3.0.
• mpfr_divby0_p in MPFR 3.1 (new divide-by-zero exception).
• mpfr_div_d in MPFR 2.4.
• mpfr_dot in MPFR 4.1 (incomplete, experimental).
• mpfr_erandom in MPFR 4.0.
• mpfr_flags_clear, mpfr_flags_restore, mpfr_flags_save, mpfr_flags_set and mpfr_flags_test in MPFR 4.0.
• mpfr_fmma and mpfr_fmms in MPFR 4.0.
• mpfr_fmod in MPFR 2.4.
• mpfr_fmodquo in MPFR 4.0.
• mpfr_fms in MPFR 2.3.
• mpfr_fprintf in MPFR 2.4.
• mpfr_free_cache2 in MPFR 4.0.
• mpfr_free_pool in MPFR 4.0.
• mpfr_frexp in MPFR 3.1.
• mpfr_gamma_inc in MPFR 4.0.
• mpfr_get_decimal128 in MPFR 4.1.
• mpfr_get_float128 in MPFR 4.0 if configured with ‘--enable-float128’.
• mpfr_get_flt in MPFR 3.0.
• mpfr_get_patches in MPFR 2.3.
• mpfr_get_q in MPFR 4.0.
• mpfr_get_str_ndigits in MPFR 4.1.
• mpfr_get_z_2exp in MPFR 3.0. This function was named mpfr_get_z_exp in previous versions; mpfr_get_z_exp is still available via a macro in mpfr.h:
  #define mpfr_get_z_exp mpfr_get_z_2exp
Thus code that needs to work with both MPFR 2.x and MPFR 3.x should use mpfr_get_z_exp.
• mpfr_grandom in MPFR 3.1.
• mpfr_j0, mpfr_j1 and mpfr_jn in MPFR 2.3.
• mpfr_lgamma in MPFR 2.3.
• mpfr_li2 in MPFR 2.4.
• mpfr_log_ui in MPFR 4.0.
• mpfr_min_prec in MPFR 3.0.
• mpfr_modf in MPFR 2.4.
• mpfr_mp_memory_cleanup in MPFR 4.0.
• mpfr_mul_d in MPFR 2.4.
• mpfr_nrandom in MPFR 4.0.
• mpfr_printf in MPFR 2.4.
• mpfr_rec_sqrt in MPFR 2.4.
• mpfr_round_nearest_away in MPFR 3.0.
• mpfr_remainder and mpfr_remquo in MPFR 2.3.
• mpfr_rint_roundeven and mpfr_roundeven in MPFR 4.0.
• mpfr_round_nearest_away in MPFR 4.0.
• mpfr_rootn_ui in MPFR 4.0.
• mpfr_set_decimal128 in MPFR 4.1.
• mpfr_set_divby0 in MPFR 3.1 (new divide-by-zero exception).
• mpfr_set_float128 in MPFR 4.0 if configured with ‘--enable-float128’.
• mpfr_set_flt in MPFR 3.0.
• mpfr_set_z_2exp in MPFR 3.0.
• mpfr_set_zero in MPFR 3.0.
• mpfr_setsign in MPFR 2.3.
• mpfr_signbit in MPFR 2.3.
• mpfr_sinh_cosh in MPFR 2.4.
• mpfr_snprintf and mpfr_sprintf in MPFR 2.4.
• mpfr_sub_d in MPFR 2.4.
• mpfr_total_order_p in MPFR 4.1.
• mpfr_urandom in MPFR 3.0.
• mpfr_vasprintf, mpfr_vfprintf, mpfr_vprintf, mpfr_vsprintf and mpfr_vsnprintf in MPFR 2.4.
• mpfr_y0, mpfr_y1 and mpfr_yn in MPFR 2.3.
• mpfr_z_sub in MPFR 3.1.

6.3 Changed Functions

The following functions and function-like macros have changed after MPFR 2.2. Changes can affect the behavior of code written for some MPFR version when built and run against another MPFR version (older or newer), as described below.

• The formatted output functions (mpfr_printf, etc.) have slightly changed in MPFR 4.1 in the case where the precision field is empty: trailing zeros were not output with the conversion specifier ‘e’ / ‘E’ (the chosen precision was not fully specified and it depended on the input value), and also on the value zero with the conversion specifiers ‘f’ / ‘F’ / ‘g’ / ‘G’ (this could partly be regarded as a bug); they are now kept in a way similar to the formatted output functions from C.

• mpfr_abs, mpfr_neg and mpfr_set changed in MPFR 4.0. In previous MPFR versions, the sign bit of a NaN was unspecified; however, in practice, it was set as now specified except for mpfr_neg with a reused argument: mpfr_neg(x,x,rnd).

• mpfr_check_range changed in MPFR 2.3.2 and MPFR 2.4. If the value is an inexact infinity, the overflow flag is now set (in case it was lost), while it was previously left unchanged. This is really what is expected in practice (and what the MPFR code was expecting), so that the previous behavior was regarded as a bug. Hence the change in MPFR 2.3.2.

• mpfr_eint changed in MPFR 4.0. This function now returns the value of the E1/eint1 function for negative argument (before MPFR 4.0, it was returning NaN).

• mpfr_get_f changed in MPFR 3.0. This function was returning zero, except for NaN and Inf, which do not exist in MPF. The error flag is now set in these cases, and mpfr_get_f now returns the usual ternary value.

• mpfr_get_si, mpfr_get_sj, mpfr_get_ui and mpfr_get_uj changed in MPFR 3.0. In previous MPFR versions, the cases where the error flag is set were unspecified.

• mpfr_get_str changed in MPFR 4.0. This function now sets the NaN flag on NaN input (to follow the usual MPFR rules on NaN and IEEE 754 recommendations on string conversions from Subclause 5.12.1) and sets the inexact flag when the conversion is inexact.
• **mpfr_get_z** changed in MPFR 3.0. The return type was void; it is now int, and the usual ternary value is returned. Thus programs that need to work with both MPFR 2.x and 3.x must not use the return value. Even in this case, C code using mpfr_get_z as the second or third term of a conditional operator may also be affected. For instance, the following is correct with MPFR 3.0, but not with MPFR 2.x:

```c
bool ? mpfr_get_z(...) : mpfr_add(...);
```

On the other hand, the following is correct with MPFR 2.x, but not with MPFR 3.0:

```c
bool ? mpfr_get_z(...) : (void) mpfr_add(...);
```

Portable code should cast mpfr_get_z(...) to void to use the type void for both terms of the conditional operator, as in:

```c
bool ? (void) mpfr_get_z(...) : (void) mpfr_add(...);
```

Alternatively, if ... else can be used instead of the conditional operator. Moreover, the cases where the **erange** flag is set were unspecified in MPFR 2.x.

• **mpfr_get_z_exp** changed in MPFR 3.0. In previous MPFR versions, the cases where the **erange** flag is set were unspecified. Note: this function has been renamed to mpfr_get_z_2exp in MPFR 3.0, but mpfr_get_z_exp is still available for compatibility reasons.

• **mpfr_out_str** changed in MPFR 4.1. The argument **base** can now be negative (from −2 to −36), in order to follow mpfr_get_str and GMP’s mpf_out_str functions.

• **mpfr_set_exp** changed in MPFR 4.0. Before MPFR 4.0, the exponent was set whatever the contents of the MPFR object in argument. In practice, this could be useful as a low-level function when the MPFR number was being constructed by setting the fields of its internal structure, but the API does not provide a way to do this except by using internals. Thus, for the API, this behavior was useless and could quickly lead to undefined behavior due to the fact that the generated value could have an invalid format if the MPFR object contained a special value (NaN, infinity or zero).

• **mpfr_strtofr** changed in MPFR 2.3.1 and MPFR 2.4. This was actually a bug fix since the code and the documentation did not match. But both were changed in order to have a more consistent and useful behavior. The main changes in the code are as follows. The binary exponent is now accepted even without the ‘0b’ or ‘0x’ prefix. Data corresponding to NaN can now have an optional sign (such data were previously invalid).

• **mpfr_strtofr** changed in MPFR 3.0. This function now accepts bases from 37 to 62 (no changes for the other bases). Note: if an unsupported base is provided to this function, the behavior is undefined; more precisely, in MPFR 2.3.1 and later, providing an unsupported base yields an assertion failure (this behavior may change in the future).

• **mpfr_subnormalize** changed in MPFR 3.1. This was actually regarded as a bug fix. The mpfr_subnormalize implementation up to MPFR 3.0.0 did not change the flags. In particular, it did not follow the generic rule concerning the inexact flag (and no special behavior was specified). The case of the underflow flag was more a lack of specification.

• **mpfr_sum** changed in MPFR 4.0. The mpfr_sum function has completely been rewritten for MPFR 4.0, with an update of the specification: the sign of an exact zero result is now specified, and the return value is now the usual ternary value. The old mpfr_sum implementation could also take all the memory and crash on inputs of very different magnitude.

• **mpfr_urandom** and **mpfr_urandomb** changed in MPFR 3.1. Their behavior no longer depends on the platform (assuming this is also true for GMP’s random generator, which is not the case between GMP 4.1 and 4.2 if gmp_randinit_default is used). As a consequence, the returned values can be different between MPFR 3.1 and previous MPFR versions. Note: as the reproducibility of these functions was not specified before MPFR 3.1, the MPFR 3.1 behavior is not regarded as backward incompatible with previous versions.

• **mpfr_urandom** changed in MPFR 4.0. The next random state no longer depends on the current exponent range and the rounding mode. The exceptions due to the rounding of
the random number are now correctly generated, following the uniform distribution. As a consequence, the returned values can be different between MPFR 4.0 and previous MPFR versions.

- Up to MPFR 4.1.0, some macros of the Section 5.16 [Custom Interface], page 48, had undocumented limitations. In particular, their arguments may be evaluated multiple times or none.

### 6.4 Removed Functions

Functions `mpfr_random` and `mpfr_random2` have been removed in MPFR 3.0 (this only affects old code built against MPFR 3.0 or later). (The function `mpfr_random` had been deprecated since at least MPFR 2.2.0, and `mpfr_random2` since MPFR 2.4.0.)

Macros `mpfr_add_one_ulp` and `mpfr_sub_one_ulp` have been removed in MPFR 4.0. They were no longer documented since MPFR 2.1.0 and were announced as deprecated since MPFR 3.1.0.

Function `mpfr_grandom` is marked as deprecated in MPFR 4.0. It will be removed in a future release.

### 6.5 Other Changes

For users of a C++ compiler, the way how the availability of `intmax_t` is detected has changed in MPFR 3.0. In MPFR 2.x, if a macro `INTMAX_C` or `UINTMAX_C` was defined (e.g. when the `__STDC_CONSTANT_MACROS` macro had been defined before `<stdint.h>` or `<inttypes.h>` has been included), `intmax_t` was assumed to be defined. However, this was not always the case (more precisely, `intmax_t` can be defined only in the namespace `std`, as with Boost), so that compilations could fail. Thus the check for `INTMAX_C` or `UINTMAX_C` is now disabled for C++ compilers, with the following consequences:

- Programs written for MPFR 2.x that need `intmax_t` may no longer be compiled against MPFR 3.0: a `#define MPFR_USE_INTMAX_T` may be necessary before `mpfr.h` is included.
- The compilation of programs that work with MPFR 3.0 may fail with MPFR 2.x due to the problem described above. Workarounds are possible, such as defining `intmax_t` and `uintmax_t` in the global namespace, though this is not clean.

The divide-by-zero exception is new in MPFR 3.1. However, it should not introduce incompatible changes for programs that strictly follow the MPFR API since the exception can only be seen via new functions.

As of MPFR 3.1, the `mpfr.h` header can be included several times, while still supporting optional functions (see Section 4.1 [Headers and Libraries], page 6).

The way memory is allocated by MPFR should be regarded as well-specified only as of MPFR 4.0.
7 MPFR and the IEEE 754 Standard

This section describes differences between MPFR and the IEEE 754 standard, and behaviors that are not specified yet in IEEE 754.

The MPFR numbers do not include subnormals. The reason is that subnormals are less useful than in IEEE 754 as the default exponent range in MPFR is large and they would have made the implementation more complex. However, subnormals can be emulated using mpfr_subnormalize.

MPFR has a single NaN. The behavior is similar either to a signaling NaN or to a quiet NaN, depending on the context. For any function returning a NaN (either produced or propagated), the NaN flag is set, while in IEEE 754, some operations are quiet (even on a signaling NaN).

The mpfr_rec_sqrt function differs from IEEE 754 on −0, where it gives +Inf (like for +0), following the usual limit rules, instead of −Inf.

The mpfr_root function predates IEEE 754-2008, where rootn was introduced, and behaves differently from the IEEE 754 rootn operation. It is deprecated and mpfr_rootn_ui should be used instead.

Operations with an unsigned zero: For functions taking an argument of integer or rational type, a zero of such a type is unsigned unlike the floating-point zero (this includes the zero of type unsigned long, which is a mathematical, exact zero, as opposed to a floating-point zero, which may come from an underflow and whose sign would correspond to the sign of the real non-zero value). Unless documented otherwise, this zero is regarded as +0, as if it were first converted to a MPFR number with mpfr_set_ui or mpfr_set_si (thus the result may not agree with the usual limit rules applied to a mathematical zero). This is not the case of addition and subtraction (mpfr_add_ui, etc.), but for these functions, only the sign of a zero result would be affected, with +0 and −0 considered equal. Such operations are currently out of the scope of the IEEE 754 standard, and at the time of specification in MPFR, the Floating-Point Working Group in charge of the revision of IEEE 754 did not want to discuss issues with non-floating-point types in general.

Note also that some obvious differences may come from the fact that in MPFR, each variable has its own precision. For instance, a subtraction of two numbers of the same sign may yield an overflow; idem for a call to mpfr_set, mpfr_neg or mpfr_abs, if the destination variable has a smaller precision.
Contributors

The main developers of MPFR are Guillaume Hanrot, Vincent Lefèvre, Patrick Pélissier, Philippe Théveny and Paul Zimmermann.

Sylvie Boldo from ENS-Lyon, France, contributed the functions mpfr_agm and mpfr_log. Sylvain Chevillard contributed the mpfr_ai function. David Daney contributed the hyperbolic and inverse hyperbolic functions, the base-2 exponential, and the factorial function. Alain Delplanque contributed the new version of the mpfr_get_str function. Matthieu Dutour contributed the functions mpfr_acos, mpfr_asin and mpfr_atan, and a previous version of mpfr_gamma. Laurent Fousse contributed the original version of the mpfr_sum function (used up to MPFR 3.1). Emmanuel Jeandel, from ENS-Lyon too, contributed the generic hypergeometric code, as well as the internal function mpfr_exp3, a first implementation of the sine and cosine, and improved versions of mpfr_const_log2 and mpfr_const_pi. Ludovic Meunier helped in the design of the mpfr_erf code. Jean-Luc Rémy contributed the mpfr_zeta code. Fabrice Rouillier contributed the mpfr_XXX_z and mpfr_XXX_q functions, and helped to the Microsoft Windows porting. Damien Stehlé contributed the mpfr_get ld_2exp function. Charles Karney contributed the mpfr_nrandom and mpfr_erandom functions.

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References


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